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Periodic disturbance estimation based adaptive robust control of marine vehicles

Deniz Kurtoglu [a], Baris Bidikli [b], Enver Tatlicioglu [c,*], Erkan Zergeroglu [d]

[a] Department of Electrical & Electronics Engineering, Izmir Institute of Technology, Izmir, 35430, Turkey
[b] Department of Mechatronics Engineering, Izmir Katip Celebi University, Izmir, 35620, Turkey
[c] Department of Electrical & Electronics Engineering, Ege University, Izmir, 35100, Turkey
[d] Department of Computer Engineering, Gebze Technical University, Kocaeli, 41400, Turkey

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ABSTRACT

Tracking control of marine vessels in the presence of parametric uncertainty and additive periodic disturbances is considered. For optimal estimation of environmental forces, periodic disturbance estimation method inspired from Fourier series expansion have been applied. Stability of the closed–loop system and the convergence of the tracking error under the closed–loop operation are established via Lyapunov based arguments. Simulation studies are provided to support the theoretical results and the effectiveness of the proposed method.

1. Introduction

In an effort to utilize the environmental resources of oceans, in a more effective way, research on dynamical positioning of marine systems used in offshore operations have gained momentum in recent years. Significant amount of those operations are for offshore oil and gas drilling, underwater pipeline and cable laying, offshore wind farm constructing turbines, and also for marine rescue and wreck investigation.

Automatic control of marine vessels, to our best knowledge, has started with the invention of electrically driven gyroscopes in early marine control systems (Fossen, 2002). These early controllers were followed by linear controller approaches applied onto the simplified system model of marine vessels obtained from the linearization about pre–specified yaw angles (Fossen and Grovlen, 1998). Proportional integral derivative (PID) type controller (Balchen et al., 1980) and linear optimal control laws (Balchen et al., 1976), (Grimble et al., 1980), (Sorensen et al., 1996) are some examples of the linear controller approaches. Sliding mode controllers as in (Agostinho et al., 2009), (Tannuri et al., 2010) and H_∞ control (Katebi et al., 2001) were also presented as a feasible solution for the control of the linearized system model.

Problems inherited by linearization motivated researchers to apply nonlinear control techniques for the accurate control of marine vehicles. Singular perturbation theory supported robust nonlinear control law was proposed for the control of an underwater vehicle in (De Wit et al., 1998). In (Fjellstad and Fossen, 2014), position regulation of underwater vehicles was provided via nonlinear proportional derivative (PD) type controller while higher order sliding mode controller was proposed for the similar purpose in marine vessel control in (Tannuri and Agostinho, 2010). A full–state feedback nonlinear robust control design was used to provide position tracking control of marine vessels in (Bidikli et al., 2017a). In (Zhang et al., 2017), a novel robust model predictive control method was proposed for the path following control of underactuated marine vessels. Interpolation of the Riccati equation solution based robust H_∞ control design was realized for the control of underwater vehicles in (Zhang et al., 2018). In (Chen et al., 2013), positioning of a marine vessel was provided via a robust adaptive control design having dynamic control allocation. Trajectory tracking control of fully actuated marine vessels was provided via a robust adaptive finite time tracking control design in (Wang et al., 2016). In (Du et al., 2015), a high gain observer based robust adaptive output feedback controller was designed to ensure dynamic positioning of ships while an observer based robust adaptive nonlinear control design was realized to ensure the position tracking control of marine vessels in (Bidikli et al., 2017b). Path following of an underactuated marine vessel was provided via a fuzzy unknown observer based robust adaptive controller in the presence of unmodeled dynamics, uncertainties and disturbances in (Wang et al., 2019). In (Zhang et al., 2019), dynamic positioning system of marine vessels was designed by utilizing a Lyapunov based robust adaptive control approach.

* Corresponding author.
E-mail address: enver.tatlicioglu@ege.edu.tr (E. Tatlicioglu).

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As highlighted in many studies on control of marine vehicles, parametric uncertainty and environmental disturbances are crucial cases that have to be taken into account. In some of the control studies about these type of systems, it was observed that designing a robust controller may not cope with external disturbances and parametric uncertainty together. In these type of situations, the designed robust controller can be supported with an extra tool as it was preferred in some of the aforementioned robust control designs. In (Balchen et al., 1980), PID controller was cascaded with a low pass filter, in (Balchen et al., 1976), (Grimble et al., 1980) and (Sorensen et al., 1996) linear optimal control laws were used in conjunction with Kalman filtering techniques, in (Fjellstad and Fossen, 2014) nonlinear PD type controller was supported with an extended Kalman filter to reach the mentioned aim. Disturbance observer (Zhang et al., 2017), (Wang et al., 2016) and fuzzy unknown observer (Wang et al., 2019) are among the preferred tools to cope with the effects of external disturbances in marine control systems. From the examination of these studies, one may conclude that using extra tools to estimate external disturbances usually provide increased control performance compared to the classical robust controllers. However, the use of the aforementioned extra tools generally increase the complexity of controller's structure. It is clear that, proposing a novel control approach that can cope with the external disturbances and parametric uncertainty without needing additional tools is a valid contribution for this research area.

To address this open research problem by compensating for external disturbances and parametric uncertainties, recently in (Du et al., 2018), external disturbances of a marine vehicle dynamics were modeled as periodic disturbances and a robust adaptive controller was designed that yielded a globally uniformly ultimately bounded tracking result. Inspired mainly from the idea of modeling low frequency disturbances of the ocean environment as summation of a series of sinusoidal components having different frequencies, amplitudes and phases in (Du et al., 2018), in this study we have designed a robust controller that ensures the position tracking of marine vessel in the presence of parametric uncertainty and unknown but periodic external disturbances. In contrast to the widely used repetitive learning controller where the period of periodic signal is required to be known exactly, in this work, by using a Fourier series expansion–like method, the external disturbance is modeled without needing additional information. Then, backstepping control technique is utilized to design a robust control structure supported with a periodic disturbance estimation method. At this point, it should be noted that, disturbance estimation is not made available by the use of an extra tool, but is constructed as a part of the controller designed. To the authors' best knowledge a robust control structure that efficiently provides the position tracking control of marine vessels subject to periodic disturbance without using additional information about the type of the disturbance which also does not utilize additional tools for disturbance estimation, has been designed for the first time. While the concept of representing disturbances and noise terms using a Fourier series expansion–like method is firstly applied to marine vessel control in this work, this method was previously applied to robotic systems in (Delibasi et al., Cansever), (Delibasi et al., 2006). The robotic systems are commonly represented with similar dynamical equations to that of marine vehicles however the periodic disturbances affecting the robotic systems are commonly modeled with known periods which allows the use of repetitive learning techniques. However, the use of this method on marine vehicles where the main source of external disturbance terms being oceanic waves is, to our best knowledge, is limited to the proposed work. When compared with (Du et al., 2018), in this work, asymptotic convergence of the error signals is achieved without requiring an initial bound or projection type bounding requirements on the parameter estimation signals.

The remaining parts of the paper is organized as follows: Section 2 represents a dynamic model of the 3 degree of freedom marine vessel while Section 3 shows the error system and the control objective. Stability analysis and numerical results are presented in Sections 4 and 5, respectively. Section 6 contains concluding remarks.

2. System model

The mathematical model for the dynamically positioned ship has the following form (Fossen, 2002)

$$\dot{\eta} = J\nu \tag{1}$$

$$M\dot{\nu} + D\nu + d = \tau \tag{2}$$

where $\eta(t)$, $\nu(t) \in \mathbb{R}^3$ represent the position and the velocity vectors of the ship, respectively, $d(t) \in \mathbb{R}^3$ is the vector that contains uncertain periodic disturbances, $\tau(t) \in \mathbb{R}^3$ denotes the vector of control inputs, $M \in \mathbb{R}^{3\times3}$ is the constant, positive definite, symmetric, uncertain mass inertia matrix, $D \in \mathbb{R}^{3\times3}$ is the constant uncertain damping matrix, $J(\eta) \in \mathbb{R}^{3\times3}$ represents the orthogonal rotation matrix between the earth and body fixed coordinate frames that satisfies $J^{-1} = J^T$ and has the following structure

$$J(\eta) = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In (1) and (2), $\eta \triangleq [x(t) \quad y(t) \quad \varphi(t)]^T$ contains translational positions denoted by $x(t), y(t) \in \mathbb{R}$ and the rotation about yaw angle of the ship denoted by $\varphi(t) \in \mathbb{R}$ while $\nu \triangleq [u(t) \quad v(t) \quad r(t)]^T$ contains translational velocities denoted by $u(t), v(t) \in \mathbb{R}$ and the rotational velocity about yaw angle denoted by $r(t) \in \mathbb{R}$.

After taking the time derivative of (1) and using the orthogonality property of the rotation matrix, the following expression is obtained

$$\dot{\nu} = J^T\left(\ddot{\eta} - \dot{J}J^T\dot{\eta}\right). \tag{3}$$

Therefore, (2) can be rewritten as

$$MJ^T\left(\ddot{\eta} - \dot{J}J^T\dot{\eta}\right) + DJ^T\dot{\eta} + d = \tau. \tag{4}$$

In view of (4), the mathematical model of the ship consisting of the desired position $\eta_d(t) \in \mathbb{R}^3$ and its time derivatives can be written as follows

$$Y_d\theta = MJ^T(\eta_d)\left[\ddot{\eta}_d - \dot{J}(\eta_d)J^T(\eta_d)\dot{\eta}_d\right] + DJ^T(\eta_d)\dot{\eta}_d \tag{5}$$

where $Y_d(\eta_d, \dot{\eta}_d, \ddot{\eta}_d) \in \mathbb{R}^{3\times p}$ is a function of desired position and its time derivatives and $\theta \in \mathbb{R}^p$ is an uncertain parameter vector.

3. Error system development and control input design

Our control objective is to make $\eta(t)$ track a sufficiently smooth, bounded desired trajectory under the restriction that the dynamic model is uncertain (*i.e.*, the entries of M and D are not known) and the additive uncertain periodic disturbances are effective onto it.

The position tracking error, denoted by $z_1(t) \in \mathbb{R}^3$, is defined as

$$z_1 \triangleq \eta - \eta_d. \tag{6}$$

An auxiliary error, denoted by $z_2(t) \in \mathbb{R}^3$, is defined as

$$z_2 \triangleq \nu - \alpha \tag{7}$$

where $\alpha(t) \in \mathbb{R}^3$ is an auxiliary input like term designed as

$$\alpha = J^T\left(\dot{\eta}_d - K_1z_1\right) \tag{8}$$

where $K_1 \in \mathbb{R}^{3\times3}$ is a positive definite diagonal control gain matrix. Substituting (7) and (8) into the time derivative of (6) yields

$$\dot{z}_1 = -K_1z_1 + Jz_2 \tag{9}$$

where (1) and orthogonality of $J(\eta)$ were made use of. Following the foot steps of the literature on backstepping based control design, for the stability analysis, consider a non–negative function, denoted by $V_1(t) \in \mathbb{R}^3$,

$$V_1 \triangleq \frac{1}{2} z_1^T z_1. \tag{10}$$

The time derivative of $V_1(t)$ is obtained as

$$\dot{V}_1 = -z_1^T K_1 z_1 + z_1^T J z_2 \tag{11}$$

where (9) was substituted. To obtain the dynamics for $z_2(t)$, taking the time derivative of $z_2(t)$ yields

$$\dot{z}_2 = \dot{\nu} - \dot{\alpha}. \tag{12}$$

The time derivative of (8) is taken to obtain

$$\dot{\alpha} = \dot{J}^T \left(\dot{\eta}_d - K_1 z_1\right) + J^T \left(\ddot{\eta}_d - K_1 \dot{z}_1\right) \tag{13}$$

Which contains the time derivative of rotation matrix obtained as $\dot{J} = JS(r)$ where $S(r) \in \mathbb{R}^{3\times3}$ is a skew symmetric matrix defined as

$$S(r) \triangleq \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

Satisfying $S^T = -S$. By utilizing the given property, the time derivative of α is rearranged as

$$\dot{\alpha} = -SJ^T \dot{\eta}_d + SJ^T K_1 z_1 + J^T \ddot{\eta}_d - J^T K_1 \dot{\eta} + J^T K_1 \dot{\eta}_d. \tag{15}$$

Premultiplying (12) with M and utilizing (2) and (15) yields

$$M\dot{z}_2 = \tau - D\nu - d + MSJ^T \dot{\eta}_d - MSJ^T K_1 z_1 \tag{16}$$

$$-MJ^T \ddot{\eta}_d + MJ^T K_1 \dot{\eta} - MJ^T K_1 \dot{\eta}_d.$$

To obtain a compact form of (16), $W(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d, \ddot{\eta}_d) \in \mathbb{R}^{3\times p}$ is introduced as

$$W\theta = D\nu - MSJ^T \dot{\eta}_d + MSJ^T K_1 z_1 \tag{17}$$

$$+MJ^T \ddot{\eta}_d - MJ^T K_1 \dot{\eta} + MJ^T K_1 \dot{\eta}_d$$

By which (16) is rewritten as

$$M\dot{z}_2 = \tau - d - W\theta. \tag{18}$$

Assumption 1. By assuming that the disturbance is periodic, it can be expressed in the following form by utilizing Fourier series expansion–like techniques (Delibasi et al., Cansever) (Delibasi et al., 2006),

$$d = E^T Tanh(z_2) + \sum_{\ell=1}^{h} D_\ell^T Cos(\ell z_2) + \sum_{\ell=1}^{h} F_\ell^T Sin(\ell z_2) \tag{19}$$

where $E \in \mathbb{R}^{3\times3}$ is unknown, mean value disturbance weights, $D_\ell, F_\ell \in \mathbb{R}^{3\times3}$, $\ell = 1, \cdots, h$ are constant matrices with unknown parameters and $h \in \mathbb{R}^+$ is harmonic limit of the approximation with $\ell = 1, \cdots, h$ representing different error frequencies where

$$Tanh(z_2) = [\tanh(z_{21}) \quad \tanh(z_{22}) \quad \tanh(z_{23})]^T \tag{20}$$

$$Sin(\ell z_2) = [\sin(\ell z_{21}) \quad \sin(\ell z_{22}) \quad \sin(\ell z_{23})]^T \tag{21}$$

$$Cos(\ell z_2) = [\cos(\ell z_{21}) \quad \cos(\ell z_{22}) \quad \cos(\ell z_{23})]^T \tag{22}$$

While $z_2(t) = [z_{21} \quad z_{22} \quad z_{23}]^T$.
Substituting (19) into (18) yields,

$$M\dot{z}_2 = -W\theta + \tau - E^T \mathrm{Tanh}(z_2) - \sum_{\ell=1}^{h} D_\ell^T \mathrm{Cos}(\ell z_2) - \sum_{\ell=1}^{h} F_\ell^T \mathrm{Sin}(\ell z_2). \tag{23}$$

Based on the error system development and the subsequent stability analysis, the control input is designed as

$$\tau = -K_2 z_2 - J^T z_1 + Y_d \widehat{\theta} + \widehat{E}^T \mathrm{Tanh}(z_2) + \sum_{\ell=1}^{h} \widehat{D_\ell}^T \mathrm{Cos}(\ell z_2) + \sum_{\ell=1}^{h} \widehat{F_\ell}^T \mathrm{Sin}(\ell z_2) \tag{24}$$

where $K_2 \in \mathbb{R}^{3\times3}$ is a positive definite control gain matrix, $\widehat{\theta}(t) \in \mathbb{R}^p$ is the estimate of uncertain model parameters, $\widehat{E}(t) \in \mathbb{R}^{3\times3}$, $\widehat{D_\ell}(t) \in \mathbb{R}^{3\times3}$, $\widehat{F}_\ell(t) \in \mathbb{R}^{3\times3}$ stand for the estimates of E, D_ℓ and F_ℓ for $\ell = 1, \cdots, h$, respectively, that are designed as follows

$$\dot{\widehat{\theta}} = -\Gamma Y_d^T z_2 \tag{25}$$

$$\dot{\widehat{E}} = -\psi \mathrm{Tanh}(z_2) z_2^T \tag{26}$$

$$\dot{\widehat{D}}_\ell = -\psi_\ell \mathrm{Cos}(\ell z_2) z_2^T, \ell = 1, \cdots, h \tag{27}$$

$$\dot{\widehat{F}}_\ell = -\psi_\ell \mathrm{Sin}(\ell z_2) z_2^T, \ell = 1, \cdots, h \tag{28}$$

where $\Gamma \in \mathbb{R}^{p\times p}$, ψ, $\psi_\ell \in \mathbb{R}^{3\times3}$ $\ell = 1, \cdots, h$ are positive definite, diagonal adaptive gain matrices.

After substituting (24) into (23), the closed loop error system for z_2 yields

$$M\dot{z}_2 = -W\theta + Y_d \widehat{\theta} - K_2 z_2 - J^T z_1 - \tilde{E}^T \mathrm{Tanh}(z_2) - \sum_{\ell=1}^{h} \tilde{D}_\ell^T \mathrm{Cos}(\ell z_2) - \sum_{\ell=1}^{h} \tilde{F}_\ell^T \mathrm{Sin}(\ell z_2) \tag{29}$$

where $\tilde{\theta}(t) \in \mathbb{R}^p$, $\tilde{E}(t)$, $\tilde{D}_\ell(t)$, $\tilde{F}_\ell(t) \in \mathbb{R}^{3\times3}$ $\ell = 1, \cdots, h$ are defined as

$$\tilde{\theta} \triangleq \theta - \widehat{\theta} \tag{30}$$

$$\tilde{E} \triangleq E - \widehat{E} \tag{31}$$

$$\tilde{D}_\ell \triangleq D_\ell - \widehat{D}_\ell \tag{32}$$

$$\tilde{F}_\ell \triangleq F_\ell - \widehat{F}_\ell. \tag{33}$$

To quantify the difference between the previously defined parametrizations $Y_d\theta$ and $W\theta$, the auxiliary term $\chi(t) \in \mathbb{R}^3$ is defined as

$$\chi \triangleq Y_d \theta - W\theta \tag{34}$$

Whose norm can be proven to be upper bounded as given below

$$||\chi|| \le c_1 ||z_1|| + c_2 ||z_2|| + c_3 ||z_1||^2 \tag{35}$$

where c_1, c_2, $c_3 \in \mathbb{R}$ are positive bounding constants. In view of (34), (29) is obtained as

$$M\dot{z}_2 = \chi - Y_d \tilde{\theta} - K_2 z_2 - J^T z_1 - \tilde{E}^T \mathrm{Tanh}(z_2) - \sum_{\ell=1}^{h} \tilde{D}_\ell^T \mathrm{Cos}(\ell z_2) - \sum_{\ell=1}^{h} \tilde{F}_\ell^T \mathrm{Sin}(\ell z_2). \tag{36}$$

4. Stability analysis

Theorem 1. *The control input in (24), consisting of the parameter estimation update rules in (25)–(28), ensures the stability in the sense that*

$$||z_1(t)||, ||z_2(t)|| \to 0 \text{ as } t \to \infty \tag{37}$$

And provided that

$$\min\{\lambda_{\min}\{K_1\},\lambda_{\min}\{K_2\}\} > \max\left\{\frac{c_1}{2}+\frac{c_3}{4\delta}2V(0),\frac{c_1}{2}+c_2+\delta\right\} \tag{38}$$

Is satisfied where δ is a positive damping constant and $V(0)$ represents the initial value of the subsequently designed Lyapunov function $V(t)$.

Proof 1. *To prove the above result, the non–negative Lyapunov function* $V(z_1,z_2,\tilde{\theta},\tilde{E},\tilde{D}_\ell,\tilde{F}_\ell)\in\mathbb{R}$ *is defined as*

$$V \triangleq V_1+\frac{1}{2}z_2^TMz_2+\frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} +\frac{1}{2}tr\left\{\tilde{E}^T\psi^{-1}\tilde{E}\right\}+\frac{1}{2}tr\left\{\sum_{\ell=1}^{h}\tilde{D}_\ell^T\psi_\ell^T\tilde{D}_\ell\right\} +\frac{1}{2}tr\left\{\sum_{\ell=1}^{h}\tilde{F}_\ell^T\psi_\ell^{-1}\tilde{F}_\ell\right\} \tag{39}$$

where $tr\{\cdot\}$ is the trace operator.

The time derivative of the Lyapunov function (39) is obtained as

$$\dot{V}=\dot{V}_1+z_2^TM\dot{z}_2+\tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}+tr\left\{\tilde{E}^T\psi^{-1}\dot{\tilde{E}}\right\} +tr\left\{\sum_{\ell=1}^{h}\tilde{D}_\ell^T\psi_\ell^{-1}\dot{\tilde{D}}_\ell\right\} +tr\left\{\sum_{\ell=1}^{h}\tilde{F}_\ell^T\psi_\ell^{-1}\dot{\tilde{F}}_\ell\right\}. \tag{40}$$

Utilizing *(11), (36)* and *(30)–(33) along with θ,* E, D_ℓ, F_ℓ being constant yields

$$\begin{aligned}\dot{V}=&-z_1^TK_1z_1+z_1^TJz_2+z_2^T\Big[\chi-Y_d\tilde{\theta}\\&-K_2z_2-J^Tz_1-\tilde{E}^TTanh(z_2)\\&-\sum_{\ell=1}^{h}\tilde{D}_\ell^TCos(\ell z_2)-\sum_{\ell=1}^{h}\tilde{F}_\ell^TSin(\ell z_2)\Big]\\&+\tilde{\theta}^TY_d^Tz_2+tr\left\{\tilde{E}^TTanh(z_2)z_2^T\right\}\\&+tr\left\{\sum_{\ell=1}^{h}\tilde{D}_\ell^TCos(\ell z_2)z_2^T\right\}\\&+tr\left\{\sum_{\ell=1}^{h}\tilde{F}_\ell^TSin(\ell z_2)z_2^T\right\}.\end{aligned} \tag{41}$$

By using the trace property $tr\{a^Tbc^T\}=tr\{c^Ta^Tb\}$, $\dot{V}$ is rearranged as

$$\dot{V}=-z_1^TK_1z_1+z_2^T\chi-z_2^TK_2z_2. \tag{42}$$

Therefore by utilizing (35), the upper bound for the right hand side of (42) can be obtained as

$$\begin{aligned}\dot{V}\le&-\lambda_{\min}\{K_1\}||z_1||^2-\lambda_{\min}\{K_2\}||z_2||^2+c_1z_1||z_2||\\&+c_2z_2^2+c_3\left\|z_1^2\right\|||z_2||\\\le&-\lambda_{\min}\{K_1\}||z_1||^2-\lambda_{\min}\{K_2\}z_2^2+\frac{c_1}{2}||z_1||^2\\&+\frac{c_1}{2}||z_2||^2+c_2\left\|z_2^2\right\|+\frac{c_3}{4\delta}||z_1||^4+\delta||z_2||^2\\=&-\left[\lambda_{\min}\{K_1\}-\frac{c_1}{2}-\frac{c_3}{4\delta}||z_1||^2\right]||z_1||^2\\&-\left[\lambda_{\min}\{K_2\}-\frac{c_1}{2}-c_2-\delta\right]||z_2||^2\\\le&-\left[\lambda_{\min}\{K_1\}-\frac{c_1}{2}-\frac{c_3}{4\delta}2V(t)\right]||z_1||^2\\&-\left[\lambda_{\min}(K_2)-\frac{c_1}{2}-c_2-\delta\right]||z_2||^2\end{aligned} \tag{43}$$

Provided that

$$\min\{\lambda_{\min}\{K_1\},\lambda_{\min}\{K_2\}\}-\max\left\{\frac{c_1}{2}+\frac{c_3}{4\delta}2V,\frac{c_1}{2}+c_2\right\}>0 \tag{44}$$

Then

$$\dot{V}\le-\beta\left(||z_1||^2+||z_2||^2\right) \tag{45}$$

For some $\beta>0$. Since the maximum value $V(t)$ that can take is its initial value $V(0)$ then a more conservative relationship is obtained as

$$\min\{\lambda_{\min}\{K_1\},\lambda_{\min}\{K_2\}\} > \max\left\{\frac{c_1}{2}+\frac{c_3}{4\delta}2V(0),\frac{c_1}{2}+c_2+\delta\right\}. \tag{46}$$

Provided that (46) is satisfied then $z_1(t)$ and $z_2(t)$ go to zero as time increases.

5. Numerical results

To validate the performance of the proposed controller along with the periodic disturbance estimation method, a numerical simulation with Matlab Simulink is conducted for trajectory tracking problem of a ship model. The parameters of the inertia and damping matrices in the ship model in (2) are (Fossen and Grovlen, 1998)

$$M=\begin{bmatrix}1.0852&0&0\\0&2.0575&-0.4087\\0&-0.4087&0.2153\end{bmatrix},D=\begin{bmatrix}0.08656&0&0\\0&0.0762&0.1510\\0&0.0151&0.0031\end{bmatrix}. \tag{47}$$

The desired trajectory was selected as follows

$$\eta_d=\begin{bmatrix}10\sin(0.2t)\left(1-\exp\left(-0.3t^3\right)\right)[m]\\10\cos(0.2t)\left(1-\exp\left(-0.3t^3\right)\right)[m]\\5\sin(0.2t)\left(1-\exp\left(-0.3t^3\right)\right)[\deg]\end{bmatrix} \tag{48}$$

With the initial positions $\eta(0)=[1\quad -1\quad 1]^T$ while the initial velocities were set to zero and the periodic disturbance was adjusted as $d(t)=[\sin(t)\quad \sin(t)\quad \sin(t)]^T$. For the harmonic limit $h=5$ and the following gains

$$K_1=\mathrm{diag}\{7.25\quad 7.25\quad 8\}$$

$$K_2=\mathrm{diag}\{6\quad 4\quad 4.5\}$$

$$\Gamma=\mathrm{diag}\{4\quad 5\quad 6\quad 4\quad 5\quad 7\quad 8\quad 5\quad 6\}$$

$$\phi=\mathrm{diag}\{2\quad 4\quad 6\}$$

$$\phi_\ell=\mathrm{diag}\{2\quad 4\quad 6\},\ \ell=1,\cdots,5.$$

The results are presented in Figs. 1–4 where Figs. 1–3 present the tracking error $z_1(t)$, the auxiliary error $z_2(t)$ and the input torque $\tau(t)$, respectively, while the entries of the parameter estimate vector are presented in Fig. 4. From Fig. 1, it is clear that the tracking objective is met.

Additionally, numerical simulations were conducted for different values of h. Specially, for $h=0,1,3,5$, simulations were run and $\mathscr{L}_2$ norms and maximum values of the entries of z_1 were evaluated and presented in Table 1. From the results given in Table 1, it is clear that as h increases, the $\mathscr{L}_2$ norm of the entries of the tracking error decreases.

A comparison was made with the robust adaptive controller in (Du et al., 2018). The robust adaptive controller of (Du et al., 2018) was run on the model given in this paper where the following gains were adjusted after trial and error

$$K_1=\mathrm{diag}\{27\quad 25\quad 22\}$$

$$K_2=\mathrm{diag}\{15\quad 16\quad 14\}$$

$$G=\mathrm{diag}\{-1.205,-1.260,-1.315,-1.370,-1.425,-1.480\\-1.535,-1.590,-1.645,-1.655,-1.710,-1.765\}$$

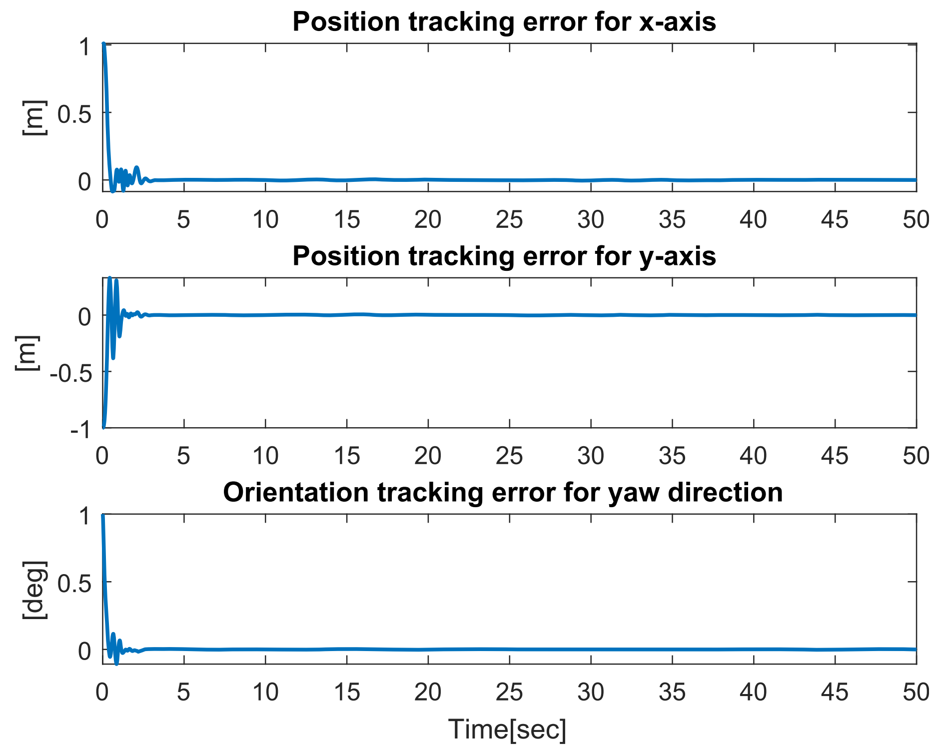

Fig. 1. The tracking error $z_1(t)$.

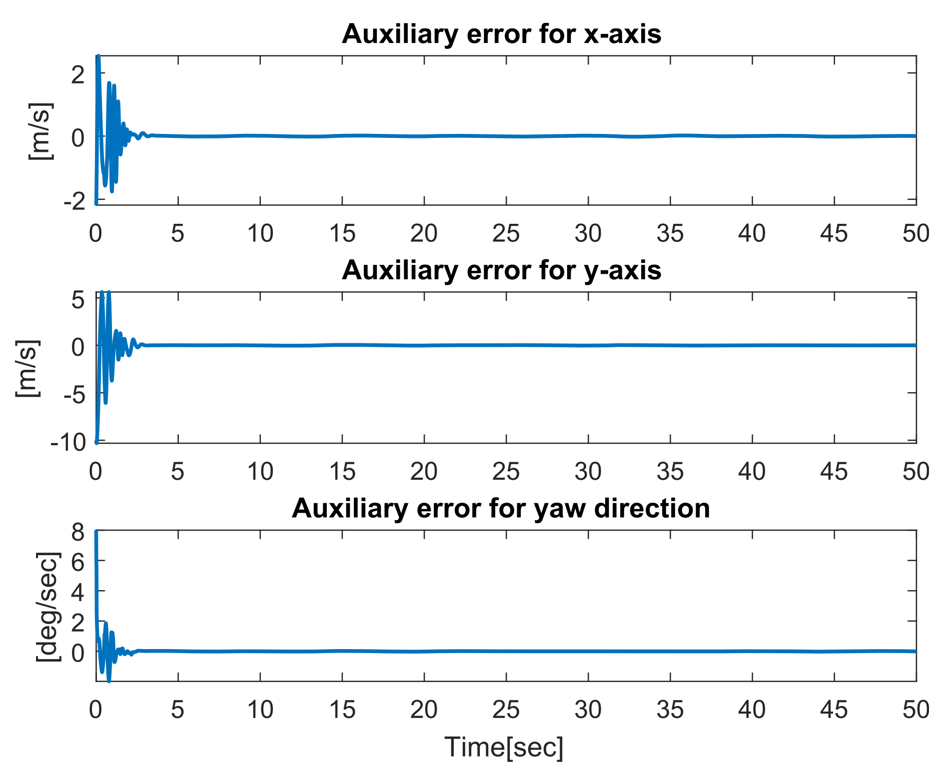

Fig. 2. The auxiliary error $z_2(t)$.

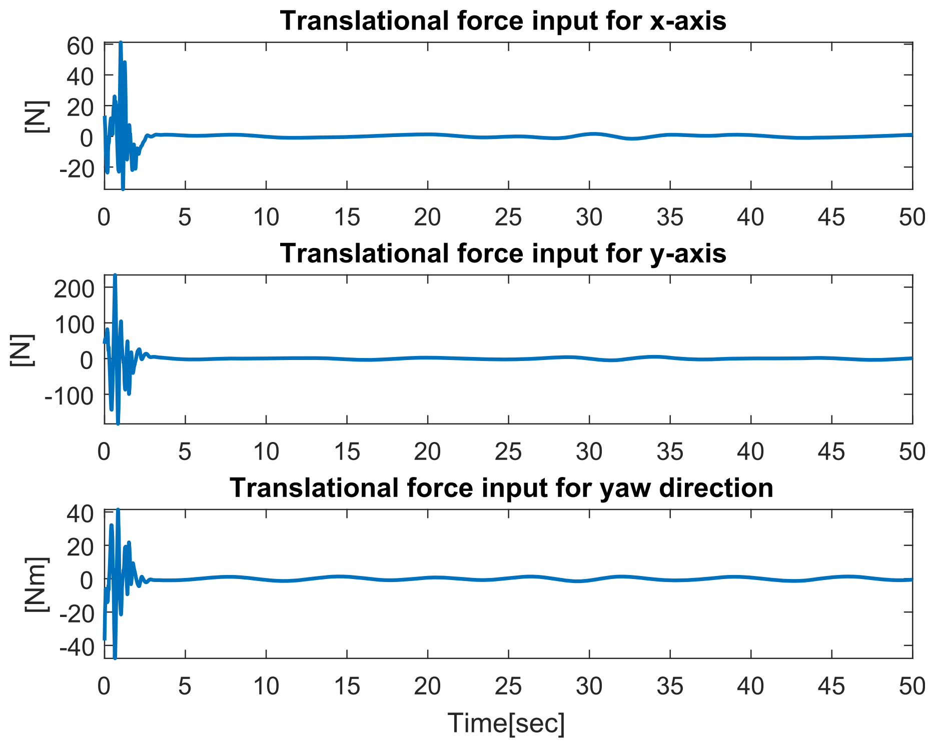

Fig. 3. The input torque $\tau(t)$.

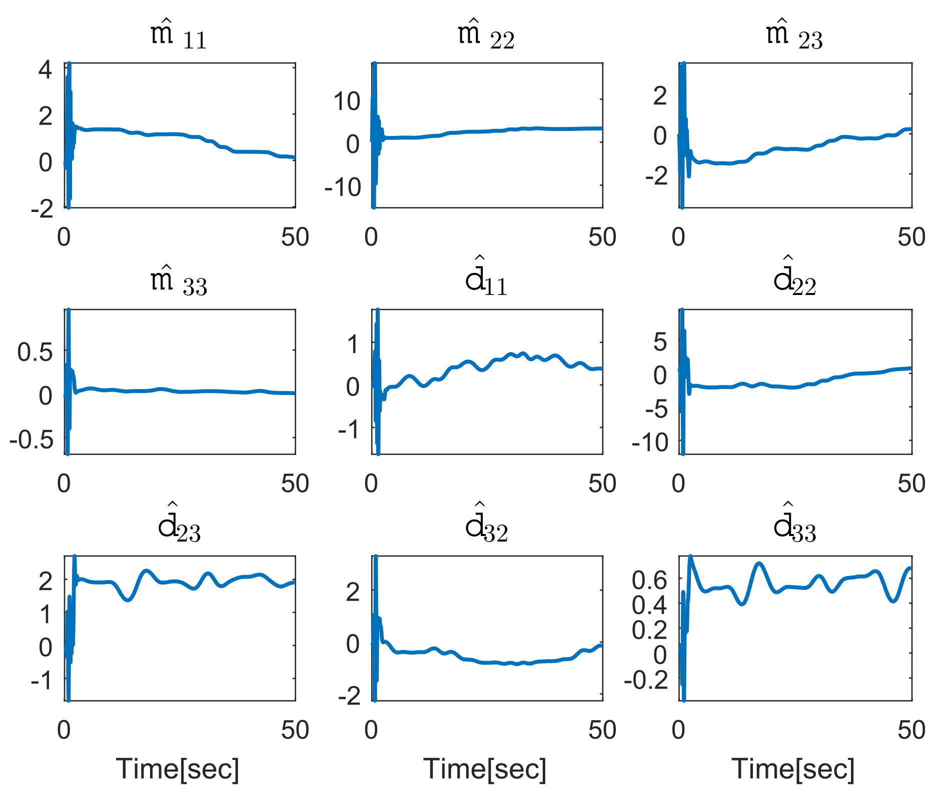

Fig. 4. Estimations of constant values of D and M matrices.

Table 1
Comparison table for different harmonic limits of the approximation.

Harmonic Limit	States	max value of z_1	$\mathscr{L}_2$ norm of z_1
$h = 0$	Linear position x	1.0119	0.6513
	Linear position y	0.5671	0.6059
	Yaw angle ψ	1	0.3443
$h = 1$	Linear position x	1.0117	0.5108
	Linear position y	0.4274	0.5233
	Yaw angle ψ	1	0.3224
$h = 3$	Linear position x	1.0114	0.4894
	Linear position y	0.3596	0.4942
	Yaw angle ψ	1	0.3163
$h = 5$	Linear position x	1.0112	0.4805
	Linear position y	0.3305	0.4784
	Yaw angle ψ	1	0.3133

Table 2
Performances of the controllers.

Cases	States	$\mathscr{L}_2$ norm of z_1	$\mathscr{L}_2$ norm of τ
Results obtained with the control input torque in (24)	Linear position x	0.4805	29.1535
	Linear position y	0.4784	113.0200
	Yaw angle ψ	0.3133	25.2283
Results obtained with the control input torque in (Du et al., 2018)	Linear position x	0.4807	56.7218
	Linear position y	0.5751	199.7748
	Yaw angle ψ	0.3200	46.9422

$$Q_{\alpha_1} = 6 \times 10^{-4} I_6$$

$$Q_{\alpha_2} = 7 \times 10^{-4} I_6$$

$$Q_{\alpha_3} = 5.5 \times 10^{-4} I_6$$

$$Q_{\kappa_1} = 2 \times 10^{-4} I_{228}$$

$$Q_{\kappa_1} = 2 \times 10^{-4} I_{228}$$

$$Q_{\kappa_1} = 2 \times 10^{-4} I_{228}$$

$$\mathrm{L}. = \begin{bmatrix} 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \end{bmatrix}^T.$$

The results obtained are compared with the results of the proposed controller in $\mathscr{L}_2$ norm sense and are presented in Table 2. As can be seen from the numerical validations, the proposed controller performs as good as the controller in (Du et al., 2018).

6. Conclusions

In this paper, a robust control design supported with a periodic disturbance estimation was addressed for the position tracking control of marine vessels. A backstepping control technique was utilized to realize the control design while the disturbance estimation was realized via a Fourier series expansion based method. Lyapunov based arguments were utilized to prove that the designed controller guarantees the convergence of the tracking error in the presence of parametric uncertainty and unknown periodic external disturbances. The presented theoretical results were supported with simulation studies. In these studies, it was assumed that the system is disturbed by sinusoidal perturbations. As a result of different simulation studies conducted for different values of harmonic limit, it was observed that the designed controller can be efficiently used to reach the control purpose. We would like to point out that, motivated by the resemblance between the repetitive structure of oceanic waves and sinusoidal disturbance terms, estimation of wave dependent disturbances in the marine vessel dynamical system via a Fourier series representation is novel. However our assumption in this study is validated only with numerical studies. In order for a more effective validation experimental studies with actual oceanic conditions is required. Unfortunately we were unable to conduct experimental studies to further prove our assumption. The proposed controller needs measurements of all system states as feedback information. Designing the output feedback version of the proposed controller may be focused as a future work to prevent possible problems that may be encountered in velocity measurement.

CRediT authorship contribution statement

Deniz Kurtoglu: Software, Formal analysis, Investigation, Writing - original draft. **Baris Bidikli:** Software, Formal analysis, Writing - review & editing. **Enver Tatlicioglu:** Conceptualization, Methodology, Formal analysis, Writing - original draft, Writing - review & editing, Supervision. **Erkan Zergeroglu:** Conceptualization, Methodology, Formal analysis, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.oceaneng.2020.108351.

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