

Casting the Swarms Problem in the Ensembles Context

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Abstract

Robotic swarms have been modeled in a myriad of ways. One property of the swarms is their multitude. As their numbers increase to uncountable numbers, the thermostistical mechanics may come into play. Authors took advantage of this fact so as to generate global statistics for the swarm. Three distinct ensembles are explained and formulated. When isolated, the swarms behave as if microcanonical ensemble reigns. But when a predator or a prey appears, transitions are observed depending on the conditions. Therefore, both the formulations and the transitions are all contingent. Finally, observed probabilities were discussed.

Keywords: Swarm modeling, Statistical physics, Ensembles, Robotics

Sürüler Probleminin Topluluk Bağlamı Açısından Modellenmesi

Öz

Sürü robotları yüzlerce farklı şekilde modellenmiştir. Kalabalık olmaları sürülerin bir özelliğidir. Sayılamayacak kadar çok sayıya ulaştıklarında, termo-istatistiksel mekanik devreye girebilir. Yazarlar bu avantajı kullanarak sürü robotları için evrensel istatistik oluşturmak istediler. Üç temel topluluk açıklandı ve formüle edildi. Sürüler izole edildiklerinde mikrokanojik uyum ortama hakim olurken, ortama av veya avcı girişi olur ise, duruma bağlı olarak değişimler gözlemlenir. Bu yüzden formülasyonlar ve geçişler şarta bağlıdır. Son olarak gözlemlenen olasılıklar tartışıldı.

Anahtar Kelimeler: Sürü modellemesi, İstatistiksel fizik, Genel uyumlar, Robotik

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1. INTRODUCTION

Swarms so far have been modeled through a multitude of techniques in literature. These techniques range from stability analyses to modeling biological inspired swarms, and particle swarm optimization. To exemplify, Rauch, Millanos, and Chialvo have studied pattern formation by mean-field stability analysis [1]. They generated a Langevin like equation for motion of particles. Another modeling study published by Matinoli, Easton, and Agassanun mentions distributed control of robots in a case study of collaboratively completing a given task. They introduced a research which concerned with probabilistic and non-spatial model of swarm robotics. This research was not interested in spatial distribution or trajectories of robots [2]. Additionally, Chen and Fang have investigated collective behavior of social foraging swarms with the help of Lyapunov stability theory [3]. Arlotti, Deutsch, and Lachowicz have proposed a Boltzmann-type mathematical swarm model for groups of agents moving orderly into the same direction [4]. More over most of the other publications have made use of Particle Swarm Optimization (PSO) [5-9]. Even though there is a plethora of literature modeling dynamics down to particle scale, none of them has dealt with particle locations specified by probability functionals. Probability functionals may be acquired in many ways. One of these ways is by means of thermostatical mechanics.

When one scours through the articles about modeling by ensembles, it may easily be seen that ensembles are being used in different branches of science from astrophysics to food chemistry. To name a few, Zhang has created a canonical ensemble model in order to examine the statistical significance of the quantum tunneling radiation spectrum [10,12]. Roman and Dukelsky have introduced grand canonical and canonical ensemble modeling in order to detect the low energy excited states in the pairing model of superconductivity (Bardeen-Cooper-Schrieffer model) [11]. The main objective of Nogawa, Ito, and Watanabe was to investigate the evaporation-condensation transition of the Potts model. In

accordance with this purpose, an intrinsically system-size-dependent discrete transition between supersaturation state and phase-separation state has been surveyed in the microcanonical ensemble [13]. Wang and Yang have aimed at improving the capability of numerical calculation on statistical model with large lattice sizes by means of microcanonical ensemble theory [14]. Hilbert and Dunkel have introduced one-dimensional evaporation model and they have analyzed this model by calculating thermodynamic functions both for microcanonical and canonical ensembles. By doing so, they have analogized and exemplified the differences between the microcanonical and canonical ensembles [15]. Alkhimov has submitted a d-dimensional model of the canonical ensemble of open self-avoiding strings [16]. Knani, Khalfaoui, Hachicha, Ben Lamine, and Mathlouthi have used grand canonical ensemble in food chemistry for modeling of water vapor adsorption on food products [17]. Knani, Mathlouthi, and Ben Lamine have delved into the peripheral mechanism of taste perception by the aid of grand canonical ensemble [18]. But none yet may be found on swarm modeling.

In this paper, the authors have modeled the swarm behavior using ensembles within the thermostatical mechanics framework. The classical statistical mechanics have three formally posed ensembles, i.e. microcanonical, canonical and grand canonical ensembles. Each of the three ensembles will be explained below.

2. ENSEMBLES

In statistical mechanics, a little is known about a system when a system is defined as one particle. This knowledge however, is not enough to describe exact state the system is in. Therefore, using an N number of same structure particle system and observing average behavior in distinct probable states provides more information. Such systems are called ensembles [19].

Statistical mechanics does not govern the cases where one or more but limited number of particles is involved. Knowing how many particles hit

boundary walls of a system is more of a concern than knowing when one specific particle hits the walls [20].

2.1. Microcanonical Ensemble

Microcanonical ensemble can be described as a system insulated from energy and particle transfer, with a known internal energy denoted by “U” [21]. All state probabilities are equal to each other and is equal to $p_i = 1/W$, where W is number of the states. Also, its temperature is given as it follows.

$$\frac{1}{T} \equiv \frac{\partial S_{BG}}{\partial U} = k \frac{\partial \ln W}{\partial U} \quad (1)$$

T = Temperature

S_{BG} = Boltzmann-Gibbs Entropy

U = Total Energy of the System

k = Boltzmann constant

W = Number of States

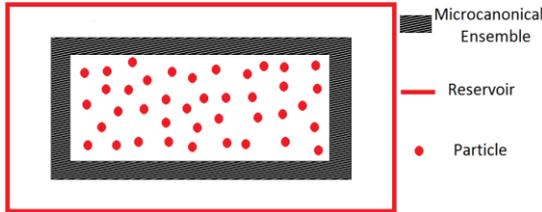


Figure 1. Representation of microcanonical ensemble

It can be imagined as an insulated box, shown in Figure 1, with N particles in it, and walls to be assumed rigid and smooth in order to talk about energy conservation when particles bump to the walls of box [20].

2.2. Canonical Ensemble

Canonical ensemble, in Figure 2, is a system that consists of N many particles and is in thermal equilibrium with a heat bath. Energy transfer is allowed with the heat bath, but particle transfer is impermissible. Heat bath can be described as a system with a relatively large heat capacity, in so much that its temperature remains constant in spite

of any energy transfer [22]. Since the energy transfer can be shown between the system and the heat bath, exact energy of the system is unknown.

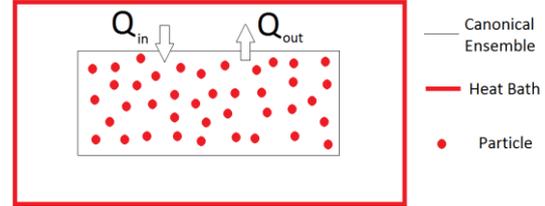


Figure 2. Representation of canonical ensemble

However, mean energy U of the system can be calculated as it follows.

$$U = \sum_{i=1}^W p_i E_i \quad (2)$$

$$p_i = \frac{\exp(-\beta E_i)}{Z_{BG}} \quad (3)$$

$$Z_{BG} \equiv \sum_{i=1}^W \exp(-\beta E_i) \quad (4)$$

$$\beta = \frac{1}{kT} \quad (5)$$

$$\frac{1}{T} = \frac{\partial S_{BG}}{\partial U} \quad (6)$$

$$F_{BG} = U - TS_{BG} = -\frac{1}{\beta} \ln Z_{BG} \quad (7)$$

$$U = -\frac{\partial}{\partial \beta} \ln Z_{BG} \quad (8)$$

U = Mean Energy of the System

p_i = Probability of The States

E_i = Energy Eigenvalue

W = Number of States

β = Lagrange Parameter

Z_{BG} = Partition Function

F_{BG} = Helmholtz Free Energy

2.3. Grand Canonical Ensemble

Grand canonical ensemble is a statistical ensemble whose particle number N is also given in form of average as well as its mean energy U. It means, a grand canonical ensemble can exchange particles with a reservoir. This reservoir can also act as a

heat bath, thus allows energy transfer as well [23]. Simply, a grand canonical can be seen in Figure 3 as an open system in contact with a reservoir [22].

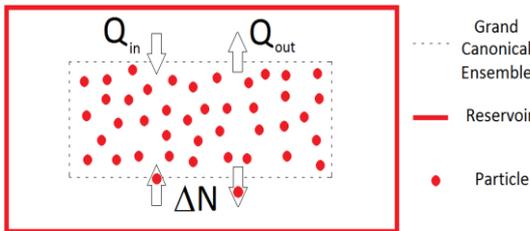


Figure 3. Representation of grand canonical ensemble

3. ENERGY AND MASS CONCEPTS IN SWARMS

Next, the authors wish to proceed by representing the swarms as a collection of particles, with a difference that they will be cast in ensembles. As may be read below, the differences and equivalences in ensembles appear as the occurrence of both the prey and the hunter. Prey and the hunter have an effect on swarm, and it can be described as a form of energy.

Other factors, such as environmental interactions, leadership disputes, etc. may also be described as disturbance and/or energy, and are omitted for the clarity of the thesis of this paper.

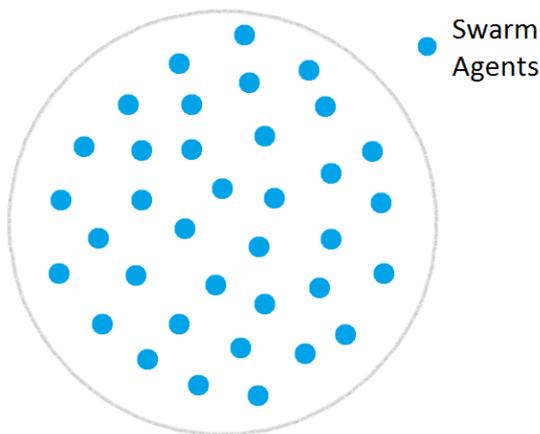


Figure 4. Swarm in microcanonical state

A swarm shows microcanonical characteristics when it is away from interactions of prey and/or hunter. There are no external influences on the swarm, and the swarm acts as if it is an isolated system, Figure 4.

All by itself, a microcanonical ensemble model would simply state that any formation is equally likely, and no preference of a particular configuration is observed. Its internal energy is constant; a minute temperature fluctuation is observed. Temperature may substitute for any state of the swarm, such as average speed, average distance, etc.

Aside from environmental interactions, leadership disputes, and all other psychologic reasons can be described as energy.

A swarm shows microcanonical characteristics when it is away from interactions of prey or hunter. There is no influence on swarm, and swarm acts as if it is an isolated system.

4. SWARM CAST IN ENSEMBLES, AND ENSEMBLE TRANSITIONS

4.1. Microcanonical to Canonical Ensemble Transitions

Transitions from the microcanonical to the canonical ensemble, and therefore the representation swarms in the canonical ensemble framework are given in the following two categories. These categories are heat dissipation and contraction, and heating and expansion as follows.

4.1.1. Heat Dissipation and Contraction: The Prey

When the swarm finds a prey, probabilities in phase space change. Probabilities of the states close to the prey builds up, Figure 5, and it becomes likely to see more agents around the prey. Interactions between a swarm and a prey force the swarm to congregate at the location where the prey is. This shrinking in effective swarm radius

is an expected result of the energy dissipation from the system. Then it can be said that the presence of prey (food) has an effect of cooling on system.

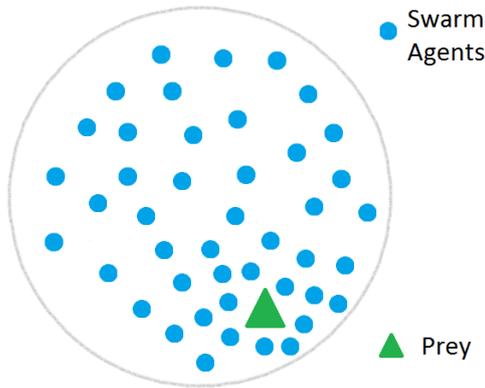


Figure 5. Swarm in canonical state, interaction with prey

4.1.2. Heating and Expansion: The Predator

When it comes to the interactions between the swarm and the predator, the swarm tries to get away and expand. This scene is similar to heating of the system. The state probabilities around the predator go to lower values, Figure 6, and the agents may tend to isolate the hunter, leaving a void around it, or flee from that location depending on the swarm size.

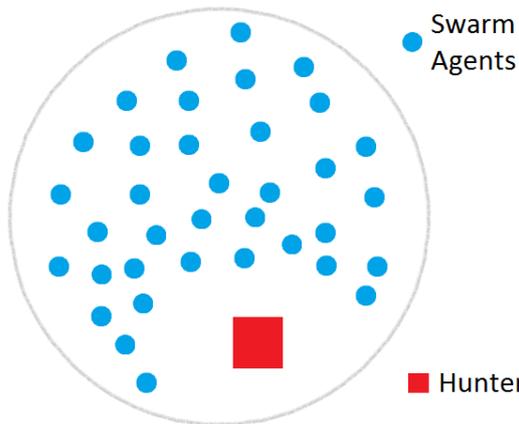


Figure 6. Swarm in canonical state, interaction with hunter

4.2. Canonical to Grand Canonical Ensemble Transition

There may be multiple ways where a grand canonical ensemble appears. One such case may be observed when two swarms start mingling. If the agents of these two swarms are identical, but assumes a predator role, not only an exchange of particles but loss of agents may be observed. Or simply that the swarm loses an agent, since it is a particle transfer, a loss or a gain of an agent can be defined as another transition, in this case, from canonical to grand canonical ensemble.

4.3. The Probabilities

The probabilities change after the transition from microcanonical to canonical ensemble. The probabilities that were originally uniform now become nonuniform. The probabilities of the states in canonical ensemble are functions of energy and the pseudo temperatures of those states, hence nonuniform. Temperature gradient (or speed gradient) is the cause of the new probability distribution.

Local interactions can be described as energy fluctuations when mass of hunter or prey is negligible compared to total mass of the swarm. This may very well be pronounced as *ensemble equivalence*.

5. CONCLUSION

In this study, the behavior of the collection of particles i.e. swarms is evaluated in thermostatical mechanics framework. Statistical ensembles are utilized to describe the behaviors of those particles and, hence this study focuses on modeling the swarm problem in the statistical ensembles. The authors deal with the swarm problem in three widely known statistical ensembles i.e. microcanonical, canonical and grand-canonical, each of which approaches the swarms in a different point of view. The initial state of the swarms is described within microcanonical ensemble where the agents of the swarms are isolated, Figure 4. In the following

scenarios, the transition from microcanonical to canonical and then to grand canonical ensembles are discussed in this paper. The scenarios of the transitions between the ensembles are illustrated and explained over predator, prey and swarm agents. It is assumed that the effects of prey and predator emerge in the form of energy. Thus, in the canonical ensemble context, the interaction between the swarm and the prey or predator would change the probabilities in the phase space, exhibiting the heating or cooling effects on the system, similar to Figure 5, and Figure 6. In grand canonical case, exchange of particles is also crucial, and in this paper, in the form of loss or gain of the agents is considered in the grand canonical ensemble. The authors believe that this new approach could deliver the researchers a tool that could bring in also a new insight regarding swarm behavior.

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