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# Reply to the discussion on "The modified Vlasov model on a nonhomogeneous and nonlinear soil layer"

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## Author's reply

The discussion suggests that the paper has left out four references (Haldar and Basu, 2013; Haldar and Basu, 2016; Elhuni and Basu, 2019; Elhuni and Basu, 2021), and the research work in the paper is not new. The authors would like to state that the literature review in the paper was kept as vast as and to stay as close as possible to the model given in the paper, which is a gross model. The study aimed to offer a method using a previously developed formulation for accounting for soil nonlinearity (Vallabhan and Das, 1988,1991a,1991b). The primary concentration in the literature was given to the most relevant works on the modified Vlasov model that employs the same concept for the solution method and algorithm, which lays the foundation of the paper (Vallabhan and Das 1988,1991a,1991b; Asik, 1999; Asik and Vallabhan,2001). On the other hand, as clearly indicated in the study, the literature in the field is extensive, including the works suggested by the discussion. Despite the author's efforts, it was not possible to review or/ and mention all the works previously conducted to solve the problem with different approaches. In this respect, the previous studies by Haldar and Basu (2013) and Elhuni and Basu (2019) mentioned in the discussion are not directly related to the problem considered in the paper. Haldar and Basu (2016) proposed a framework incorporating the finite difference and the finite element methods. They obtained the solution by solving the beam deflection and the soil displacements outside the beam domain simultaneously and developed an algorithm, which is a distinctly different concept than the one taken in the paper. Elhuni and Basu (2021) also presented a new formulation that considers different governing differential equations for beam deflection and soil displacement outside the beam domain than the original equations followed in the paper. Since the problem investigated in the paper attracts the attention of researchers in the field, it is expected that various attempts to propose new models such as the one offered in the paper will continue.

Another point the discussion raises is the formulation used in the study. The method proposed in the paper did not attempt to change the variational formulation presented by Vallabhan and Das (1988,1991a,1991b). The paper clearly assumes that the original

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Available online 24 April 2023 0266-352X/© 2023 Published by Elsevier Ltd. governing equations are valid to introduce the nonlinearity only in the existing  $2t_i$  and  $k_i$  parameters appearing in the beam deflection equation with the logic behind decreasing complexity and computational cost. Similar parameter reduction or modulus reduction-based approaches using existing governing equations have been applied to various problems in geotechnical engineering (Schnabel and Bolton, 1972; Yankelevsky et al., 1989; Mayne et al., 2001; Niazi and Mayne, 2014). The original variational formulation was presented in the study for the sake of completeness and to distinguish the changes in soil parameters and the new form of the finite-difference equations for the beam, including these parameters. We would like to clarify the assumption used in the paper to better address the discussion. The paper discusses, in general, how the problem geometry can be discretized and how reduction curves can be assigned to all the nodes in soil layers. However, the formulation considers only the nodes and soil layers under the beam. The new formulation of the finite difference equations Eq. (13)–(15) given in the paper allows for different  $2t_i$  and  $k_i$  values at each point only on the beam. In contrast, the soil displacement outside the beam domain was not included in the iteration scheme. The paper did not give the governing differential equations and did not present new formulations for these regions to prevent misconception. The soil modulus reduction curves were employed to update soil modulus depending on the accumulated strain level at nodes only underneath the footing, which is the discretized region, as clearly presented in the formulation in the section "Mathematical Formulation and the New Algorithm". Following the new formulation, we refer to the soil nodes and layers under the beam throughout the explanation of the algorithm and examples.

Although not used in the paper,  $Q_n$  and  $M_n$  are the prescribed shear force and moment, respectively, at the end of the beam as described in Vallabhan and Das (1988, 1991b) and given in the paper for completeness. The formulation given in the paper considered the shear force term coming from the soil domain at the sides of the beam, which was included in Eq. (15) in the paper. The shear force term was calculated with the new soil parameters calculated iteratively at the end of the beam and the end displacement determined by the beam deflection equation, as shown in the paper. Eq. (15) means that the shear force term assumes a solution with the beam-end soil parameters at the boundary for the out-of-beam soil domain. This was a gross approximation assuming the beam-end soil parameters for that region. The modified finite difference equations presented in the paper for the boundary already considered the boundary conditions and did not use the soil parameters belonging to the soil domain outside the beam. Since the boundary conditions were expressed at the end of the beam, the corresponding soil parameters at that point were used.

Similarly, Vallabhan and Das (1988) and Vallabhan and Das (1991a) used the same decay function for both of their works, which assumed constant and linearly increasing soil modulus, respectively. The paper employed this original initial form of the decay function in the analyses, while it was recalculated in each iteration considering the updated deflection values obtained using nonlinear soil modulus. However, the comment in the discussion drew our attention to a typing error in Eq. (11). We have corrected this and published it as a corrigendum below for Eq. (11). Eq. (12) uses the beam-end soil parameters at the endpoints to introduce the lumped effect of the soil displacement outside the beam, which was a consistent approach with the shear force term in Eq. (15). Therefore, the equations and decay function were given correctly in their original forms, while the algorithm used in the paper was consistent with those equations and in conjunction with the assumption of the validity of the original governing equations, as explained above.

With respect to the concept of the examples, since the new method was an extension of a previously developed work to include soil nonlinearity, the initial comparison with the original method presented that the new algorithm captures the response of the original method using linear elastic soil if the modulus reduction curve was set constant to the initial ratio. The FEM analysis results were in parallel with the model results, and the comparison was provided with a discussion on the proposed model's limitations. Fig. 10 in the paper showed that reduced modulus values were used with increasing strain by plotting these values at the end of each convergent load step, and the deflections were calculated with the reduced modulus values. Parametric comparisons were chosen to be presented in terms of normalized deflection, different M values, and quadratically increasing soil modulus. Overall, with the simplifying assumptions in the model, the examples covered in the paper aimed to present the soil nonlinearity effect on the footing response by providing comparisons with the original method for various cases.

In conclusion, the paper introduces a practical method to model a complex problem; however, it still presents a better concept than Winkler-based models to consider soil continuity and nonlinearity under the beam.

CORRIGENDUM

Eq. (11) was not correct as published. The corrected form of the equation is

$$\emptyset(z) = \frac{\sinh\gamma\left(1 - \frac{z}{H}\right)}{\sinh\gamma}$$

#### **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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