2023 International Balkan Conference on Communications and Networking (BalkanCom) | 979-8-3503-3910-9/23/$31.00 ©2023 IEEE | DOI: 10.1109/BalkanCom58402.2023.10167911

A Framework for Physical Layer Network Coding with Multiple Antennas for BPSK

1st Mert İlgüy
Electrical and Electronics Eng. Dept.
Izmir Institute of Technology
İzmir, Türkiye
mertilguy@iyte.edu.tr

2nd Berna Özbek
Electrical and Electronics Eng. Dept.
Izmir Institute of Technology
İzmir, Türkiye
bernaozbek@iyte.edu.tr

***Abstract*—Physical layer network coding (PNC) is combined with multiple antennas to increase the spectral efficiency of wireless communication systems. In this work, we present a PNC framework including both uplink and downlink for binary phase shift keying (BPSK). In the uplink, we propose a scheme for detecting network-coded symbol (NCS) with reduced complexity. For the downlink, we propose a transmission scheme of NCS through maximum ratio transmission (MRT) by defining the precoding vector as an average of users' channels. The bit-error-rate (BER) performances and the comparison results with the conventional scheme in both downlink and uplink are provided for the proposed low-complexity PNC framework.**

***Index Terms*—Physical Layer Network Coding, Low-complexity scheme**

I. INTRODUCTION

Physical layer network coding (PNC) is a promising technique to exchange data between two devices through a relay while increasing spectral efficiency [1]. Two phases are required for this data exchange: The first phase, the data transmission from user nodes to the relay node, is called the multiple access channel (MAC) phase. In the broadcast channel (BC) phase, the relay node broadcasts the network coded symbol (NCS) to both user nodes. For the network coding (NC), three time slots are required to complete the overall framework, while a simultaneous transmission is performed for the PNC scheme with only two time slots. Then, the PNC scheme has significantly increased spectral efficiency compared to the NC and the conventional four time slot transmission, which can be a primary technique for developing 6G applications [2]. This paper examines the PNC framework, where the relay node is called the base station (BS), including both uplink and downlink transmissions. In the MAC phase, all users transmit the corresponding binary data to the BS, and the BS decides the NCS. In the BC phase, the BS broadcasts the NCS to two users. Then, the network coding operation is applied to each user to obtain the other user's data.

The PNC scheme has been analyzed to enhance spectral efficiency in [2]. The comparison of the PNC and conventional schemes in terms of BER and sum data rate has been shown in [3]. In [4], [5], different quadrature amplitude modulation (QAM) schemes have been considered for the PNC, where the BER results of M-QAM are examined through the closed-form expressions to analyze the effect of the modulation scheme on the performance of the detection of NCS.

Several recent advances have been analyzed in PNC research, including the use of more efficient codes such as polar codes and low-density parity-check (LDPC) codes [6], [7], as well as the development of new signal processing techniques such as iterative decoding, and successive interference cancellation (SIC) [8]. Moreover, in [9], a deep neural network (DNN) technique is adopted to an asynchronous PNC system to increase the system's throughput.

For the detection of NCS at the BS, mainly log-likelihood ratio (LLR) based detection has been given after applying zero-forcing (ZF) or minimum mean squared error (MMSE) through a sum difference matrix [10]–[13]. In [10], the combination of PNC and multiple antenna technology has been provided to enhance the performance of the system. Then, the LLR is performed with the help of sum and difference signals, and the NCS is decided by using the LLR value. In [12], the multiple antenna technology has been extended to a multi-user massive multiple input multiple output (MIMO) system. In [13], deep learning techniques have been used for the PNC with multiple antennas using LLR-based detection. Since the LLR-based detection needs information about noise variances, it brings complexity to the overall system.

This paper considers a novel PNC framework, including downlink and uplink phases, to reduce computational complexity for 6G communications. To be specific, the contributions of this paper are listed as

- For the uplink phase, we propose a low-complexity NCS detection instead of LLR-based detection.
- For the downlink phase, a novel precoding scheme based on maximum ratio combining (MRT) is proposed instead of ZF precoding while transmitting NCS to all users to improve the BER performance.

The remainder of this paper is organized as follows. Section II gives the system model for the PNC scheme. The proposed uplink and downlink PNC schemes with

979-8-3503-3910-9/23/$31.00 ©2023 IEEE

reduced complexity are given in Section III. The complexity analysis is given in Section IV. The simulation results are provided in Section V. Finally, the conclusion is discussed in Section VI.

II. SYSTEM MODEL

A PNC system is considered, including $K = 2$ users having one antenna and one BS equipped with $M = 2$ antennas as illustrated in Figure 1. The information is exchanged between two users at a low data rate through the BS to implement a reduced complexity framework. The BS aims to decide the NCS correctly and broadcast it to both users. Binary phase shift keying (BPSK) is used for all data transmissions.

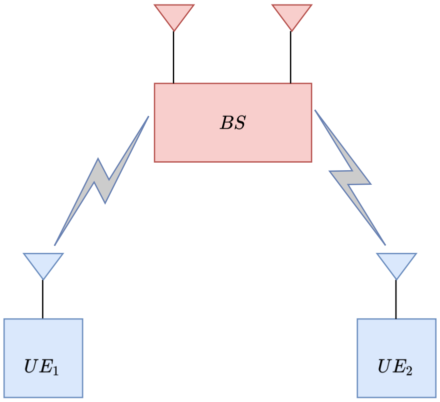

Fig. 1: System Model for the PNC scheme with two users and two antennas.

The composite transmit symbol vector is defined by,

$$\mathbf{s} = \left[s_1, s_2\right]^T \tag{1}$$

where s_i $i \in 1,2$ is the BPSK transmitted symbol of ith user. The composite uplink channel matrix is given by,

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2] \tag{2}$$

where $\mathbf{h}_1 \in \mathbb{C}^{M\times 1}$ and $\mathbf{h}_2 \in \mathbb{C}^{M\times 1}$ are the uplink channel vectors between the first and second user and the BS respectively. Each element of the channel vectors is experienced with Rayleigh fading.

The received signal at the BS is expressed by,

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{3}$$

where $\mathbf{n}$ is the additive white Gaussian noise (AWGN) vector at the BS, with Gaussian distributed complex elements with zero mean and σ^2 variance.

Conventional Uplink SIMO-PNC

The manipulated composite transmit symbol vector whose elements are the sum and difference of the users' transmit symbols is defined as:

$$\hat{\mathbf{s}} = \mathbf{D}\mathbf{s} \tag{4}$$

where the sum-difference matrix is [10]:

$$\mathbf{D} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{5}$$

In addition to that, the manipulated composite channel matrix $\hat{\mathbf{H}}$ is given by,

$$\hat{\mathbf{H}} = \mathbf{H}\mathbf{D}^{-1} \tag{6}$$

Then, the received vector in (3) is re-written by,

$$\mathbf{r} = \hat{\mathbf{H}}\hat{\mathbf{s}} + \mathbf{n} \tag{7}$$

At the BS, the received signal is equalized by,

$$\mathbf{y} = \mathbf{G}\mathbf{r} \tag{8}$$

where $\mathbf{G} = (\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1}\hat{\mathbf{H}}^H$ is the ZF equalization matrix.

The LLR-based detection can be utilized to detect the NCS [10] by

$$\mathrm{LLR} = \log\left(\frac{\exp\left(-\frac{2}{\sigma_2^2}\right)\left(\exp\left(\frac{2y_2}{\sigma_2^2}\right) + \exp\left(-\frac{2y_2}{\sigma_2^2}\right)\right)}{\exp\left(-\frac{2}{\sigma_1^2}\right)\left(\exp\left(\frac{2y_1}{\sigma_1^2}\right) + \exp\left(-\frac{2y_1}{\sigma_1^2}\right)\right)}\right) \tag{9}$$

where σ_1^2 and σ_2^2 are variances after equalization belonging to the first and second users:

$$\sigma_k^2 = \{\mathbf{G}\mathbf{G}^H\}_{k,k}\sigma^2, \quad k \in \{1,2\} \tag{10}$$

Finally, for the LLR-based detection, the NCS is determined by:

$$s_R = \begin{cases} 1 & \mathrm{LLR} \geq 0 \\ -1 & \text{otherwise} \end{cases} \tag{11}$$

We refer all BER expressions and derivations to [12].

However, LLR-based detection might have a higher computational complexity. The next section proposes a reduced complexity PNC framework, including both downlink and uplink phases.

Conventional Downlink MISO-PNC

For the conventional downlink MISO-PNC scheme, a ZF precoding is used to transmit the NCS symbol. Then the received signal is given by,

$$\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\mathbf{W}^d\tilde{\mathbf{s}} + \mathbf{n} \tag{12}$$

where $\tilde{\mathbf{r}} = [\tilde{r}_1, \tilde{r}_2]$ is the downlink received signal, $\tilde{\mathbf{H}} = [\tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_2]$ is the downlink composite channel matrix where $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{h}}_2$ are the downlink channel vectors of users and $\tilde{\mathbf{s}} = \left[\tilde{s}_1, \tilde{s}_2\right]^T$ is the downlink composite transmit symbol for the conventional system. Here, since we consider the PNC system, the transmitted symbols for two users are the same as $\tilde{s}_1 = \tilde{s}_2 = s_R$.

Then, the precoding matrix for the conventional precoding system $\mathbf{W}'$ is given by,

$$\mathbf{W}' = \tilde{\mathbf{H}}^H(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H)^{-1} \tag{13}$$

After, the normalized precoding matrix is obtained by,

$$\mathbf{W}^d = \frac{\mathbf{W}'}{||\mathbf{W}'||_F} \tag{14}$$

Then, the estimated NCS in ith user $\tilde{b}^R$ is given by:

$$\tilde{b}^R = \begin{cases} 0, & \mathrm{Re}(\tilde{r}_i) > 0 \\ \\ 1, & \mathrm{Re}(\tilde{r}_i) \leq 0 \end{cases} \tag{15}$$

where $\tilde{r}_i$ is the received signal at the ith user.

III. Proposed Framework for PNC

Firstly, we propose an uplink single input multiple output (SIMO)-PNC with reduced complexity for NCS detection. Then, we propose a downlink multiple input single output (MISO)-PNC for the transmission of NCS.

A. Uplink SIMO-PNC With Reduced Complexity

In the literature, the sum and differences are estimated by the BS and mapped to the NCS using the LLR. The sum and difference signals are also used in the proposed low-complexity approach, but we do not employ LLR detection. To have reduced complexity, the squared differences of the sum and difference signals are defined at the BS.

In Table I, it is shown that exclusive-OR (XOR) operation in the bit level can be represented by multiplication operation in signal level. Then, we can derive that the squared difference of the sum and difference signals results in a multiplication operation using simple algebraic steps. First, the squared difference of the sum and difference of transmit symbols is taken as:

TABLE I: Bit and Signal Level Mapping

b_1	b_2	$b_1 \oplus b_2$	s_1	s_2	$s_R = s_1 s_2$
0	0	0	1	1	1
0	1	1	1	-1	-1
1	0	1	-1	1	-1
1	1	0	-1	-1	1

$$s^m = (s_1 + s_2)^2 - (s_1 - s_2)^2 \tag{16}$$

After expanding the squared terms, s^m can be revised as:

$$s^m = s_1^2 + 2s_1s_2 + s_2^2 - (s_1^2 - 2s_1s_2 + s_2^2) \tag{17}$$

After the simplifications in (17), the expression for the difference becomes:

$$s^m = 4s_1s_2 \tag{18}$$

However, the sum and difference of the transmitted symbols' exact value are unknown, and only the equalized signals y_1 and y_2 are available at the BS. Therefore, the squared difference of the equalized signals y^m, is determined as follows:

$$y^m = (y_1)^2 - (y_2)^2 \tag{19}$$

where y_1 and y_2 are the equalized signals of the users and y^m is the squared difference of them.

By defining the equalized signals explicitly, y^m is re-expressed as:

$$y^m = (s_1 + s_2 + n_1)^2 - (s_1 - s_2 + n_2)^2 \tag{20}$$

where n_i is the noise coefficient at ith antenna of the BS.

After applying the algebraic operations, y^m becomes:

$$y^m = 4s_1s_2 + 2s_1(n_1 - n_2) + 2s_2(n_1 + n_2) + (n_1^2 - n_2^2) \tag{21}$$

When we focus on the multiplication term of the transmitted symbols, we can consider the remaining terms as a simple noise term denoted by n^m. Then, (21) is arranged as:

$$y^m = 4s_1s_2 + n^m \tag{22}$$

Finally, we propose to estimate the NCS s_R as follows:

$$s_R = \begin{cases} 1, & \mathrm{Re}(y^m) > 0 \\ -1, & \mathrm{Re}(y^m) \leq 0 \end{cases} \tag{23}$$

B. Downlink MISO-PNC

In the downlink phase, while transmitting the NCS, we employ MRT precoding scheme where the precoding vector is set to be a weighted average of two users' channel coefficients.

Then, the precoding vector $\mathbf{w} \in \mathbb{C}^{K \times 1}$ is determined given by:

$$\mathbf{w} = \frac{\sqrt{\alpha}\mathbf{g}_1 + \sqrt{1-\alpha}\mathbf{g}_2}{||\sqrt{\alpha}\mathbf{g}_1 + \sqrt{1-\alpha}\mathbf{g}_2||} \tag{24}$$

where α is a parameter between 0 and 1 and the normalized ith channel vector is

$$\mathbf{g}_i = \frac{\mathbf{h}_i}{||\mathbf{h}_i||}, i \in \{1, 2\} \tag{25}$$

ith user's received signal is given by:

$$r_i^{DL} = \mathbf{h}_i^H \mathbf{w} s_R + z_i, \tag{26}$$

where z_i is the AWGN in ith user whose mean and variance are zero and σ^2, respectively.

As a result, the NCS in ith user is decided by the following rule:

$$\hat{b}^R = \begin{cases} 0, & \mathrm{Re}(r_i^{DL}) > 0 \\ \\ 1, & \mathrm{Re}(r_i^{DL}) \leq 0 \end{cases} \tag{27}$$

After that, each user obtains the other user's data by performing XOR operation between its own data and detected NCS:

$$\hat{b}_1 = \hat{b}^R \oplus b_2 \tag{28}$$

$$\hat{b}_2 = \hat{b}^R \oplus b_1 \tag{29}$$

where $\hat{b}_1$ and $\hat{b}_2$ are the estimated bits of other user at the considered user.

IV. COMPLEXITY ANALYSIS

In order to illustrate that the proposed scheme has lower complexity than the LLR-based NCS detection, we examine the complexity based on the big $\mathcal{O}$ function. The proposed scheme includes taking square operation, difference operation and decision operation to detect NCS. Since all of these operations are functioned on the scalar values, the overall complexity of the proposed system becomes $\mathcal{O}(1)$. On the other hand, LLR-based detection includes logarithmic, exponential, difference and decision operations. While detecting the NCS, both the proposed and the LLR-based scheme have the complexity of $\mathcal{O}(1)$. However, the LLR-based scheme requires determining noise variance after equalization through the multiplication of two matrices of $\mathbf{G} \in \mathbb{C}^{K \times M}$ and $\mathbf{G}^H \in \mathbb{C}^{M \times K}$. The complexity of the multiplication of two matrices having the dimension of $m \times n$ and $n \times p$ dimensions, respectively, is $\mathcal{O}(mnp)$. Consequently, the complexity of the LLR-based detection is $\mathcal{O}(K^2M)$ while the proposed algorithms has only $\mathcal{O}(1)$.

V. PERFORMANCE RESULTS

We provide performance results by considering BER results' cumulative distribution function (CDF). The users' signal-to-noise ratio (SNR) is uniformly distributed between 0 dB and 30 dB. Each element of the channel vectors is generated through Rayleigh distribution. While comparing the proposed framework with the conventional case, the LLR-based detection is applied in the uplink phase, and the ZF precoding is performed in the downlink phase.

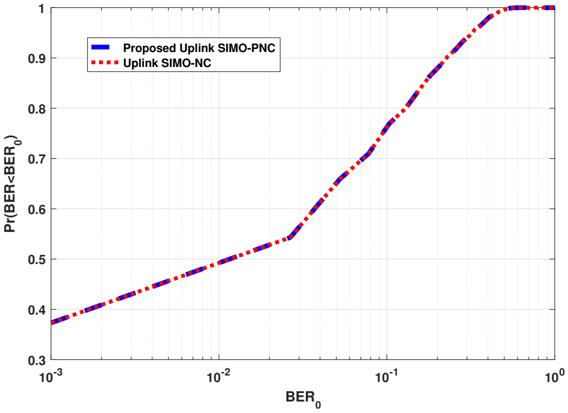

Fig. 2: CDF of BER for the proposed uplink SIMO-PNC and the uplink SIMO-NC.

In Figure 2, the CDF of BER results based on the detected NCS for the uplink is shown for the proposed uplink SIMO-PNC and the uplink SIMO-NC. It is demonstrated that the proposed PNC and NC schemes give the same BER performance while the PNC has doubled spectral efficiency compared to the NC scheme.

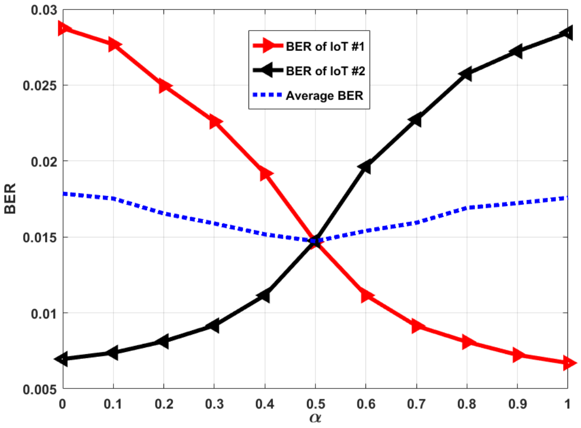

Fig. 3: BER versus α for the SNR of users are uniformly distributed between 0dB and 30dB.

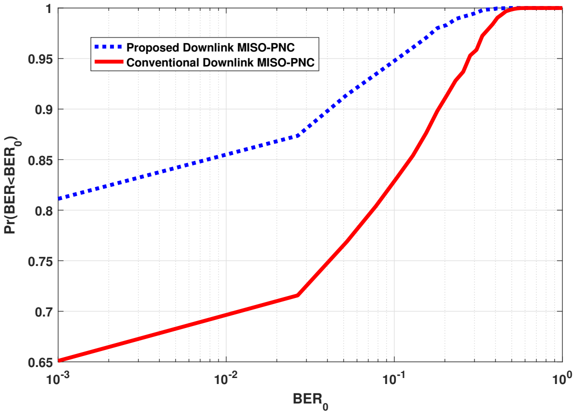

Fig. 4: CDF of BER of the proposed downlink MISO-PNC and the conventional downlink MISO-PNC.

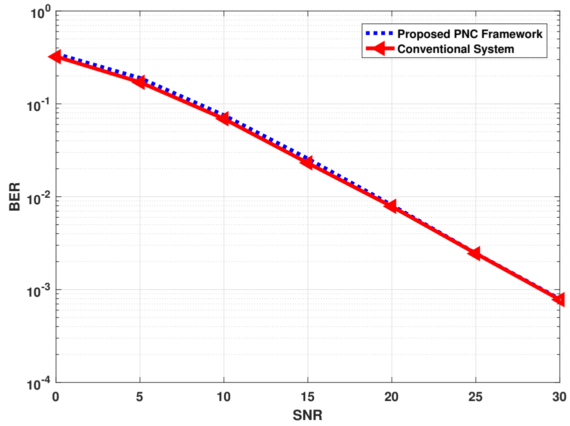

Fig. 5: Comparison of BER results for the proposed PNC framework and the conventional system.

In Figure 3, the BER of each user and the average BER of the PNC system are shown with respect to α values for the proposed downlink MISO-PNC. It is concluded that when the α gets closer to the one, the second user's BER is increased. On the other hand, when α gets closer to zero, the first user's BER is increased. Since it is observed that the average BER of two users reaches its lowest value at $\alpha = 0.5$, α is chosen as 0.5 for the remaining simulation results.

The CDF of BER for the proposed downlink MISO-PNC and conventional downlink MISO-PNC with ZF precoding scheme is illustrated in Figure 4. It is shown that the proposed downlink MISO-PNC outperforms significantly compared to the conventional scheme for the downlink case.

In Figure 5, for the end-to-end transmission scheme, the BER results are plotted for the proposed PNC framework and conventional system with LLR-based detection in the uplink and ZF precoding in the downlink. It is shown that the proposed and the conventional schemes achieve the same BER performance while the proposed PNC scheme has 80% lower complexity than the conventional PNC scheme.

VI. CONCLUSION

In this paper, we have proposed the reduced complexity PNC scheme with BPSK for detecting NCS in the uplink and transmitting NCS in the downlink. The proposed low-complexity detection scheme gives the same performance as the NC scheme for the uplink while doubling spectral efficiency. For the downlink phase, the proposed downlink MISO-PNC with MRT precoding significantly outperforms the conventional downlink MISO-PNC scheme with ZF precoding. The proposed PNC framework gives the same BER performance as the conventional scheme while it has much lower complexity since it employs algebraic operations and the MRT precoding instead of LLR-based detection and the ZF precoding. As a future work, the proposed PNC scheme will be extended to the case of higher-order modulations with more than two users and two antennas at the BS.

ACKNOWLEDGMENT

This work has been funded by the European Union Horizon 2020, RISE 2018 scheme (H2020-MSCA-RISE-2018) under the Marie Sklodowska-Curie grant agreement No. 823903 (RECENT).

REFERENCES

[1] Z. Wang, L. Liu, S. Zhang, P. Dong, Q. Yang and T. Wang, ”PNC Enabled IIoT: A General Framework for Channel-Coded Asymmetric Physical-Layer Network Coding,” in IEEE Transactions on Wireless Communications, 2022, doi: 10.1109/TWC.2022.3183407.

[2] S. Zhang, S. C. Liew, and P. P. Lam, “Physical-layer network coding, ”in Proceedings of the 12th Annual International Conference on Mobile Computing and Networking, New York, NY, USA, Sep. 2006, p. 358–365.

[3] R. H. Y. Louie, Y. Li, and B. Vucetic, “Performance analysis of physical layer network coding in two-way relay channels,” in GLOBECOM IEEE Global Telecommunications Conference, Honolulu, HI, USA, Nov. 2009, pp. 1–6.

[4] M. Lu, Z. Shen, D. Guo, and X. Wang, “A new physical layer network coding de-noising mapping based on M-QAM,” in First International Conference on Electronics Instrumentation and Information Systems (EIIS), Harbin,China, Jun. 2017, pp. 1–5.

[5] V. Namboodiri, K. Venugopal, and B. Rajan, “Physical Layer Network Coding for Two-Way Relaying with QAM,” IEEE Transactions on Wireless Communications, vol. 12, no. 10, pp. 5074–5086, Oct. 2013.

[6] Z. Xie, P. Chen, Z. Mei, S. Long, K. Cai and Y. Fang, ”Polar-Coded Physical Layer Network Coding Over Two-Way Relay Channels,” in IEEE Communications Letters, vol. 23, no. 8, pp. 1301-1305, Aug. 2019, doi: 10.1109/LCOMM.2019.2922633.

[7] B. Sun, G. Wang, W. Yang and L. Zheng, ”Joint LDPC and Physical Layer Network Coding for Two-way Relay Channels with different frequency offsets,” 2015 10th International Conference on Communications and Networking in China (ChinaCom), Shanghai, China, 2015, pp. 692-696, doi: 10.1109/CHINACOM.2015.7498026.

[8] H. Li, Y. Cai and B. Champagne, ”SIC aided physical-layer network coding for multi-way relay channels,” 2017 15th Canadian Workshop on Information Theory (CWIT), Quebec City, QC, Canada, 2017, pp. 1-5, doi: 10.1109/CWIT.2017.7994813.

[9] X. Wang and L. Lu, ”Deep Neural Networks Enhanced Phase Asynchronous Physical-Layer Network Coding,” 2022 International Conference on Cyber-Enabled Distributed Computing and Knowledge Discovery (CyberC), Suzhou, China, 2022, pp. 181-184, doi: 10.1109/CyberC55534.2022.00037.

[10] S. Zhang and S. C. Liew, “Physical layer network coding with multiple antennas,” in IEEE Wireless Communication and Networking Conference, Sydney, Australia, Apr. 2010, pp. 1–6.

[11] P. Chen, S. C. Liew and L. Shi, ”Bandwidth-Efficient Coded Modulation Schemes for Physical-Layer Network Coding with High-Order Modulations,” in IEEE Transactions on Communications, vol. 65, no. 1, pp. 147-160, Jan. 2017, doi: 10.1109/TCOMM.2016.2622693.

[12] B. Okyere, L. Musavian and R. Mumtaz, ”Performance Analysis of Physical Layer Network Coding in Massive MIMO Systems With M-QAM Modulations,” in IEEE Transactions on Vehicular Technology, vol. 70, no. 5, pp. 4631-4645, May 2021, doi: 10.1109/TVT.2021.3071372.

[13] J. Liu, P. Chen and F. Chen, ”Performance of Deep Learning for Multiple Antennas Physical Layer Network Coding,” 2021 15th International Symposium on Medical Information and Communication Technology (ISMICT), 2021, pp. 179-183, doi: 10.1109/ISMICT51748.2021.9434923.