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5 Extra Dimensional Metric Reversal Symmetry and its Prospect for Cosmological Constant and Zero-point Energy Problems, Automatic Pauli-Villars-like Regularization, and an Interesting Kaluza-Klein Spectrum

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Abstract. The metric reversal symmetry was introduced in the context of cosmological constant problem. Besides proposing a solution to the cosmological constant problem the metric reversal symmetry has also provided a framework for solution of the zero-point energy problem, an automatic Pauli-Villars-like regularization, and an interesting Kaluza-Klein spectrum with interesting phenomenological implications. In this talk I give a brief overall summary and discussion of these topics with their potential implications.

5.1 Introduction: Metric reversal symmetry and the cosmological constant problem

In this talk I will consider a symmetry that may be called metric reversal symmetry, in particular, the extra dimensional representations of this symmetry. This symmetry, first, was introduce in [1] as a possible solution to cosmological constant (CC) problem [2] in a classical setting in extra dimensions. Below I define the metric reversal symmetry and mention its use for CC problem. In the following sections I review my recent studies on the use of this symmetry at quantum level, namely, zero-point energy problem, an interesting Kaluza-Klein spectrum, an automatic Pauli-Villars-like regularization, and some of their possible implications.

Metric reversal is defined by

$$ds^2 = g_{AB} dx^A dx^B \to -ds^2 \tag{5.1}$$

This transformation has two realizations: The first [1,3,4] is

$$x^A \to i x^A \,, \; g_{AB} \to g_{AB} \tag{5.2}$$

The second [5,6,7] is

$$x^A \to x^A \,, \; g_{AB} \to -g_{AB} \tag{5.3}$$

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Metric reversal symmetry (MRS) may be imposed at the level of the equations of motion (EM) (e.g. Einstein equations) by requiring the equations be covariant under MRS or at the level of the action by requiring the action be invariant under MRS. MRS forbids a CC in any dimension if it is imposed at the level of EM [3,4]. MRS can be only classically viable if it is introduced at the level of EM. On the other hand MRS can be extended to the quantum domain if it is imposed to the action. So I prefer to impose MRS at the level of action functional. The gravitational action

$$S_R = \frac{1}{16\pi G} \int \sqrt{(-1)^S g}\, R\, d^D x \tag{5.4}$$

is invariant under either of (5.2) or (5.3) only in

$$D = 2(2n+1) \quad , \quad n = 0, 1, 2, 3, ... \quad . \tag{5.5}$$

while the CC action

$$S_C = \frac{1}{8\pi G} \int \sqrt{g}\, \Lambda\, d^D x. \tag{5.6}$$

is forbidden in $2(2n+1)$ dimensions.

So if our space is taken to be 2(2n+1) dimensional (or if the gravitation and the CC reside on a 2(2n+1) dimensional subspace of a larger space) then the cosmological constant (CC) is forbidden. In this framework the accelerated expansion of the universe may be attributed to a small breaking of MRS or to an alternative mechanism (such as quintessence, modified gravity etc.) if the symmetry is taken to be exact. Another point to to be mentioned is that two realizations of MRS are not equivalent in matter sector while they are wholly equivalent in the gravitational sector. For example $F_{AB}F^{AB}$ is odd under (5.2) while it is even under (5.3). So two realizations of MRS may be considered to be two different symmetries after the introduction of matter. As we shall see in the next section this point may be used to construct a model that solves zero-point energy problem as well. The details of these points and some other less major points may be found in [1,6,8].

5.2 Metric Reversal symmetry and zero-point energy problem

Quantization results in by-product energies that survive even in the absence of any particle. These energies (i.e. zero-point energies (ZPE)) are some kind of vacuum energy. They emerge as zero modes of harmonic oscillators or fields in quantum theory. The total ZPE associated with a particle is constant, and is found as the sum over the contributions due to different momenta, and is naively infinite. However ZPE is eliminated by subtracting ZPE from total energy. In the quantum field theory (QFT) in flat space this elimination (normal ordering) has no physical effect because changing the energy by a constant does not change the physical results. However in QFT in curved space this naive elimination of ZPE is not well-defined because gravity couples to all energies. So subtraction process affects the physical out-come. Moreover after normal ordering a non-zero vacuum energy remains and it is proportional to particle masses. So even the renormalized zero-point energy of electron gives a vacuum energy density that is 10^{36} times the observed energy density of the universe. This may be called ZPE problem. Renormalized ZPE may be identified by CC or as a different kind of vacuum energy depending on how the infinities are regulzarized in renormalization procedure. Moreover CC may get classical field theoretic contributions such as vacuum expectation of scalar feilds. Therefore it is better to consider ZPE problem separately.

In an attempt for a solution for ZPE one must take the following points into account: CC problem as well should be addressed. Therefore the dimension of the subspace we live in should be D = 2(2n+1). Another point is that It is easier to impose the symmetry so that ZPE vanishes instead of trying to make it small. This requires the symmetry be exact while the scale factor, $a(t)$ in Robertson-Walker metric breaks the symmetry generated by (5.2). Therefore both realizations of MRS should be used so that the realization of MRS generated by (5.3) is kept intact to impose ZPE vanish while the realization of MRS generated by (5.2)is broken. The following is the summary of a model that satisfies these criteria [9].

Consider a space consisting of the sum of 2(2n+1) and 2(2m+1) (e.g 6 and 2) dimensional subspaces with the metric

$$\begin{aligned} ds^2 &= g_{AB}\, dx^A\, dx^B + g_{A'B'} dx^{A'}\, dx^{B'} \\ &= \Omega_z(z)[g_{\mu\nu}(x)\, dx^\mu dx^\nu + \tilde{g}_{ab}(y)\, dy^a dy^b] + \Omega_y(y) \tilde{g}_{A'B'}(z)\, dz^{A'} dz^{B'} \end{aligned} \tag{5.7}$$

$$\Omega_y(y) = \cos k|y|) \ , \quad \Omega_z(z) = \cos k'|z| \tag{5.8}$$

$$\begin{aligned} A, B &= 0, 1, 2, 3, 5, ...N \ , \quad N = 2(2n+1) \\ A', B' &= 1', 2', , ...N' \ , \quad N' = 2(2m+1) \\ \mu\nu &= 0, 1, 2, 3 \ , \quad a, b = 1, 2, ..., N-4 \ , \quad n, m = 0, 1, 2, 3...... \ . \end{aligned}$$

The usual four dimensional space is embedded in the first space $g_{AB}\, dx^A\, dx^B$.

I assume that the gravitational sector is invariant under both realizations of MRS, that is, under

$$\begin{aligned} & ds^2 \rightarrow -ds^2 \quad \text{as} \quad x^A \rightarrow i x^A \, , \; x^{A'} \rightarrow i x^{A'} \\ & g_{AB} \rightarrow g_{AB} \, , \; g_{A'B'} \rightarrow g_{A'B'} \qquad (5.9) \\ & \Rightarrow \; \Omega_z \rightarrow \Omega_z \, , \; \Omega_y \rightarrow \Omega_y \\ & g_{\mu\nu} \rightarrow g_{\mu\nu} \, , \; \tilde{g}_{ab} \rightarrow \tilde{g}_{ab} \, , \; \tilde{g}_{A'B'} \rightarrow \tilde{g}_{A'B'} \qquad (5.10) \end{aligned}$$

and

$$\begin{aligned} & ds^2 \rightarrow -ds^2 \quad \text{as} \quad ky \rightarrow \pi - ky \, , \; k'z \rightarrow \pi - k'z \\ & x^A \rightarrow x^A \, , \; x^{A'} \rightarrow x^{A'} \qquad (5.11) \\ & \Rightarrow \; \Omega_z \rightarrow -\Omega_z \, , \; \Omega_y \rightarrow -\Omega_y \\ & g_{\mu\nu} \rightarrow g_{\mu\nu} \, , \; \tilde{g}_{ab} \rightarrow \tilde{g}_{ab} \, , \; \tilde{g}_{A'B'} \rightarrow \tilde{g}_{A'B'} \ . \qquad (5.12) \end{aligned}$$

Note that the requirements of the homogeneity and isotropy of the 4-dimensional space together with the equations (5.9-5.12) set $g_{\mu\nu}$ to the Minkowski metric $\eta_{\mu\nu} = diag(1, -1, -1, -1)$. Later (5.10) will be broken by a small amount in the matter sector to accommmodate the cosmic expansion.

The gravitational action is taken to be

$$S_R = \frac{1}{16\pi\,\tilde{G}}\int dV\,\tilde{R}^2 \tag{5.13}$$

$$dV = dV_1\,dV_2\,,\quad dV_1 = \sqrt{g(-1)^S}\,d^N x\,,\quad dV_2 = \sqrt{g'(-1)^{S'}}\,d^{N'}x' \tag{5.14}$$

$$\tilde{R} = R(x,x') + R'(x,x') \tag{5.15}$$

where the meaning of the primed and the unprimed quantities is evident from (5.9). After integration over extra dimensions S_R becomes

$$\begin{aligned} S_R &= \frac{M^{N+N'-4}}{16\pi\,\tilde{G}}\int \sqrt{(-1)^S g}\sqrt{(-1)^{S'}g'}\,2\,R(x)\,R'(x')\,d^N x\,d^{N'}x' \\ &= \frac{1}{16\pi\,G}\int \sqrt{(-1)^S g}\,R(x)\;d^N x \end{aligned} \tag{5.16}$$

where

$$\frac{1}{16\pi\,G} = M_{pl}^2(\frac{M}{M_{pl}})^2 M^{N+N'-6}\frac{1}{16\pi\,\tilde{G}}\int \sqrt{(-1)^{S'}g'}\,2\,R'(x')\,d^D x' \tag{5.17}$$

which is the usual Einstein-Hilbert action. The cosmological constant term is still forbidden by either realization of MRS.

Now we consider the subject that is the heart of this section, namely, the zero-point energies induced by the matter sector. Here we consider only the kinetic term of a scalar field here since this is enough to give the essential points of the formulation. The other details and consideration of the other fields can be found in [9]. Consider the kinetic part of the Lagrangian, $\mathcal{L}_{Mk}$ for a scalar field (in the space given in (5.9)

$$\mathcal{L}_{\phi\,k} = \mathcal{L}_{\phi\,k1} + \mathcal{L}_{\phi\,k2} \tag{5.18}$$

$$\mathcal{L}_{\phi\,k1} = \frac{1}{2}g^{AB}\partial_A\phi\partial_B\phi\,,\quad \mathcal{L}_{\phi\,k2} = \frac{1}{2}g^{A'B'}\partial_{A'}\phi\partial_{B'}\phi \tag{5.19}$$

For simplicity I take $g_{\mu\nu} = \eta_{\mu\nu}$. Then the corresponding action is

$$\begin{aligned} S_{Mk} &= \int dV\,\mathcal{L}_{Mk} \\ &= \frac{1}{2}\int \sqrt{(-1)^S g}\sqrt{(-1)^{S'}g'}\,d^D x\,d^D x'[\frac{1}{2}g^{AB}\partial_A\phi\partial_B\phi + \frac{1}{2}g^{A'B'}\partial_{A'}\phi\partial_{B'}\phi] \\ &= \frac{1}{2}\int d^4x\,dy_1\,dy_2\,dz_1\,dz_2\,\Omega_z^3\Omega_y\,\{\Omega_z^{-1}[\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (\frac{\partial\phi}{\partial y_1})^2 - (\frac{\partial\phi}{\partial y_2})^2] \\ &\quad -\Omega_y[(\frac{\partial\phi}{\partial z_1})^2 + (\frac{\partial\phi}{\partial z_2})^2]\} \\ &= \frac{1}{2}LL'\int d^4x\int_0^L\int_0^{L'} dy\,dz\,\cos^3 k'z\cos ky\{\cos^{-1}k'z[\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (\frac{\partial\phi}{\partial y})^2] \\ &\quad - \cos^{-1}ky(\frac{\partial\phi}{\partial z})^2\} \end{aligned} \tag{5.20}$$

where $y = y_2$, $z = z_2$ is adopted.

ϕ may be Fourier decomposed as

$$\phi = \phi_{AA}(x,y,z) + \phi_{AS}(x,y,z) + \phi_{SA}(x,y,z) + \phi_{SS}(x,y,z)$$

$$\phi_{AA}(x,y,z) = \sum_{n,m} \phi^{AA}_{n,m}(x)\, \sin(n\,ky)\, \sin(m\,k'z) \tag{5.21}$$

$$\phi_{AS}(x,y,z) = \sum_{n,m} \phi^{AS}_{n,m}(x)\, \sin(n\,ky)\, \cos(m\,k'z) \tag{5.22}$$

$$\phi_{SA}(x,y,z) = \sum_{n,m} \phi^{SA}_{n,m}(x)\, \cos(n\,ky)\, \sin(m\,k'z) \tag{5.23}$$

$$\phi_{SS}(x,y,z) = \sum_{n,m} \phi^{SS}_{n,m}(x)\, \cos(n\,ky)\, \cos(m\,k'z) \tag{5.24}$$

$$k = \frac{\pi}{L}\,,\ k' = \frac{\pi}{L'}\,,\ \ 0 \le y \le L\,,\ \ 0 \le z \le L'\ ,\quad n, m = 0, 1, 2, ...$$

where we have used $k = \frac{\pi}{L}$, $k' = \frac{\pi}{L'}$ since $0 \le y \le L$, $0 \le z \le L'$. After replacing this expansion in (5.20) and requiring the action be invariant under extra dimensional parity one obtains

$$\begin{aligned} S_{Mk} = \frac{1}{2}LL' \int d^4x \{ & \eta^{\mu\nu} \sum_{n,m,r,s} \partial_\mu(\,\phi_{n,m}(x)\,)\,\partial_\nu(\,\phi_{r,s}(x)\,) \\ & \times \int_0^L dy\ \cos ky\ \sin(n\,k|y|)\sin(r\,k|y|) \\ & \int_0^{L'} dz\ \cos^2 k'z \sin(m\,k'|z|))\ \sin(s\,k'|z|)) \\ -k^2 & \sum_{n,m,r,s} nr\,\phi_{n,m}(x)\,\phi_{r,s}(x) \int_0^L dy\ \cos ky\ \ \cos(n\,k|y|)\cos(r\,k|y|) \\ & \times \int_0^{L'} dz\ \cos^2 k'z \sin(m\,k'|z|))\ \sin(s\,k'|z|))\} \\ -k'^2 & \sum_{n,m,r,s} ms\,\phi_{n,m}(x)\,\phi_{r,s}(x) \int_0^L dy\ \sin(n\,k|y|)\sin(r\,k|y|) \\ & \times \int_0^{L'} dz\ \cos^3 k'z \cos(m\,k'|z|))\ \cos(s\,k'|z|)) = \end{aligned} \tag{5.25}$$

$$
\begin{aligned}
&= \frac{1}{32}(LL')^2 \int d^4x \{\eta^{\mu\nu} \sum_{r,s} \\
&\partial_\mu \big[\phi_{r-1,s-2}(x) + \phi_{r-1,s+2}(x) - \phi_{r-1,-s-2}(x) - \phi_{r-1,2-s}(x) \\
&+ 2\phi_{r-1,s}(x) - 2\,\phi_{r-1,-s}(x) + \phi_{r+1,s-2}(x) \\
&+ (\,\phi_{r+1,s+2}(x) - \phi_{r+1,-s-2}(x) - \phi_{r+1,2-s}(x) \\
&+ 2\phi_{r+1,s}(x) - 2\phi_{r+1,-s}(x) - \phi_{-r-1,s-2}(x)\,) \\
&- \phi_{-r-1,s+2}(x) + \phi_{-r-1,-s-2}(x) + \phi_{-r-1,2-s}(x) - 2\phi_{-r-1,s}(x) \\
&+ 2\phi_{-r-1,-s}(x) - \phi_{1-r,s-2}(x) - \phi_{1-r,s+2}(x) + \phi_{1-r,-s-2}(x) \\
&+ \phi_{1-r,2-s}(x) - 2\phi_{1-r,s}(x) + 2\phi_{1-r,-s}(x)\,\big]\partial_\nu(\,\phi_{r,s}(x)\,) \\
&-k^2 \sum_{r,s} r\big[\,(r-1)(\,\phi_{r-1,s-2}(x) - \phi_{1-r,s-2}(x)\,) \\
&+ (r-1)(\phi_{r-1,s+2}(x) - \phi_{1-r,s+2}(x)\,) \\
&- (r-1)(\,\phi_{r-1,-s-2}(x) - \phi_{1-r,-s-2}(x)\,) \\
&- (r-1)(\,\phi_{r-1,2-s}(x) - \phi_{1-r,2-s}(x)\,) + 2(r-1)(\,\phi_{r-1,s}(x) - \phi_{1-r,s}(x)\,) \\
&- 2(r-1)(\,\phi_{r-1,-s}(x) - \phi_{1-r,-s}(x)\,) \\
&+ (r+1)(\,\phi_{r+1,s-2}(x) - \phi_{-r-1,s-2}(x)\,) \\
&+ (r+1)(\,\phi_{r+1,s+2}(x) - \phi_{-r-1,s+2}(x)\,) \\
&- (r+1)(\,\phi_{r+1,-s-2}(x) - \phi_{-r-1,-s-2}(x)\,) \\
&- (r+1)(\,\phi_{r+1,2-s}(x) - \phi_{-r-1,2-s}(x)\,) + 2(r+1)(\,\phi_{r+1,s}(x) - \phi_{-r-1,s}(x)\,) \\
&- 2(r+1)(\,\phi_{r+1,-s}(x) - \phi_{-r-1,-s}(x)\,)\,\big]\phi_{r,s}(x) \\
&-\frac{1}{2}k'^2 \sum_{r,s} s\,\big[\,(s-3)(\,\phi_{r,s-3}(x) - \phi_{r,3-s}(x)\,) \\
&+ (s+3)(\,\phi_{r,s+3}(x) - \phi_{r,-s-3}(x)\,) + 3(s-1)(\,\phi_{r,s-1}(x) - \phi_{r,1-s}(x)\,) \\
&+ 3(s+1)(\,\phi_{r,s+1}(x) - \phi_{r,-s-1}(x)\,) + (3-s)(\,\phi_{-r,s-3}(x) - \phi_{-r,3-s}(x)\,) \\
&+ (s+3)(\,\phi_{-r,-s-3}(x) - \phi_{-r,s+3}(x)\,) \\
&+ 3(1-s)(\,\phi_{-r,s-1}(x) - \phi_{-r,1-s}(x)\,) \\
&- 3(s+1)(\,\phi_{-r,s+1}(x) - \phi_{-r,-s-}(x)\,)\,\big]\,\phi_{r,s}(x)\}.
\end{aligned} \tag{5.26}
$$

We notice that the odd modes are coupled to even modes and vica versa. In fact this is a result of the invariance under $ky^a \to \pi - ky^a$, $k'z^a \to \pi - k'z^a$ that enforces the coupling of even and odd modes to compensate the minus coming from the volume element in the action.

This makes the energy-momentum tensor $T_{\mu\nu}$ be of the same form in the extra dimensional Fourier modes. The replacement of the expansion of the modes in terms of creation and annihilation operators

$$
\phi_{n,m}(x) = \sum_{\mathbf{k}} [\,a_{n,m}(\mathbf{k})\,\zeta(t)e^{ik.x} + a^\dagger_{n,m}(\mathbf{k})\,\zeta^*(t)e^{-ik.x}\,] \tag{5.27}
$$

in the energy momentum tensor

$$
T^\nu_\mu = \sum_{m,n,r,s} \partial_\mu\phi_{n,m}(x)\,\partial^\nu\phi_{r,s}(x) - g^\nu_\mu\mathcal{L} \tag{5.28}
$$

results in the terms of the form

$$< 0|T_\mu^\nu|0 > \propto < 0|\, a_{n,m} a_{r,s}^\dagger |0 > = 0 \;, \quad < 0|\, a_{r,s}^\dagger a_{r,s} |0 > = 0 \qquad (5.29)$$
$$n \neq r \quad \text{and/or} \quad m \neq s$$

(because $a_{r,s}|0 >= 0$, and $[\, a_{n,m}, a_{r,s}^\dagger \,] = 0$ for $n \neq r$ and/or $m \neq s$) . In other words there is no contribution to vacuum energy density due to zero-point energies in this scheme. This solves the zero point-energy problem.

5.3 Metric reversal symmetry and an interesting Kaluza-Klein spectrum

In this section we shall consider a model where all except a finite number of Kaluza-Klein modes (i.e. the extra dimenional Fourier modes) are screened by the conformal factor in the metric. Note that both the form of the conformal factor and the form of the mixing of the Kaluza-Klein modes are determined by MRS. The details of the analysis given here may be found in [10]. In this scheme it is not enough to produce a mode to in order to detect it have high enough energies to produce the mode but it is also necessary to have them high enough momenta relative to the detector (to expose to the sizes smaller than the extra dimension(s)). Therefore it has interesting phenomenological implications.

Adopt the following 5-dimensional space

$$ds^2 \;=\; \cos kz\, [\eta_{\mu\nu}(x)\, dx^\mu dx^\nu \;-\; dz^2\,] \qquad \mu, \nu \;=\; 0, 1, 2, 3 \qquad (5.30)$$

where the extra dimension is taken to be compact and have the size L, and $k = \frac{2\pi}{L}$. Consider fermions with the action

$$S_f = \int (\cos kz)^{\frac{5}{2}}\, \mathcal{L}_f\, d^4x\, dz$$
$$= \int (\cos kz)^2\; i\bar{\chi}\gamma^a(\, \partial_a \,+\, \frac{1}{16} \tan kz\, [\gamma_a \;,\; \gamma_5]\,)\chi\; d^4x\, dz \;+\; \text{H.C.} \qquad (5.31)$$
$$\{\gamma^a, \gamma^b\} \;=\; 2\eta^{ab} \;\;,\;\; (\eta^{ab}) = diag(1, -1, -1, -1, -1)$$

where H.C. stands for Hermitian conjugate, and the term with the coefficient $\frac{1}{16}$ is the spin connection term.

We impose the following symmetries on the action

$$kz \;\rightarrow\; \pi \,+\, kz. \qquad (5.32)$$
$$x^a \;\rightarrow\; -x^a \quad a = 0, 1, 2, 3, 4 \qquad (5.33)$$
$$\chi_n(x) \;\rightarrow\; \epsilon^{\lambda_n}\, \mathcal{CPT}\, \chi_n(-x) \quad , \quad \lambda_n = \frac{i}{2}(-1)^{\frac{n}{2}} \qquad (5.34)$$

where the superscript a refers to the tangent space, ϵ is some constant, and $\mathcal{CPT}$ denotes the usual 4-dimensional CPT operator (acting on the spinor part of the field). We also impose anti-periodic boundary conditions in the extra dimension i.e. $\chi(x, z) = -\chi(x, z + 2\pi L)$.

The extra dimensional Fourier expansion of χ in the light of invariance under

$$\chi = \chi_{\mathcal{A}} + \chi_{\mathcal{S}} \tag{5.35}$$

$$\chi_{\mathcal{A}}(x,z) = \sum_{|n|=1}^{\infty} \tilde{\chi}^{\mathcal{A}}_{|n|}(x)\ \sin\left(\frac{1}{2}|n|\,kz\right) \tag{5.36}$$

$$\chi_{\mathcal{S}}(x,z) = \chi_0(x) + \sum_{|n|=1}^{\infty} \tilde{\chi}^{\mathcal{S}}_{|n|}(x)\ \cos\left(\frac{1}{2}|n|\,kz\right) \tag{5.37}$$

$$\tilde{\chi}^{\mathcal{A}(\mathcal{S})}_{|n|}(x) = \chi^{\mathcal{A}(\mathcal{S})}_{n}(x) - (+)\,\chi^{\mathcal{A}(\mathcal{S})}_{-n}(x)$$

where n are odd integers (due to anti-boundary conditions in the z-direction), the absolute value signs in $|n|$ is used to emphasize the positiveness of n (due to the symmetry $x^a \to -x^a$).

After replacing (5.35) in (5.31) we obtain

$$\sum_{r,s=0}^{\infty}\int d^4x\, i\bar{\chi}_{(2|r|+1)}\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\chi_{(2|s|+1)} \times 2\int dz\ (\cos kz)^2$$
$$\left[\cos\frac{2|r|+1}{2}kz\,\cos\frac{2|s|+1}{2}kz - \sin\frac{2|r|+1}{2}kz\,\sin\frac{2|s|+1}{2}kz\right] + \text{H.C.}$$
$$= \sum_{r,s=0}^{\infty}\int d^4x\ i\bar{\chi}_{(2|r|+1)}\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\chi_{(2|s|+1)}$$
$$\int_0^L dz\ (\cos 2kz + 1)\ \cos\left(|r|+|s|+1\right)kz\ + \text{H.C.}$$
$$= \frac{1}{2}\sum_{r,s=0}^{\infty}\int d^4x\, i\bar{\chi}_{(2|r|+1)}\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\chi_{(2|s|+1)}\int_0^L dz\ \left[\cos\left(|r|+|s|-1\right)kz\right] + \text{H.C.} \tag{5.38}$$

where $2r+1 = 4l+1$, $2s+1 = 4p+3$ (l,p=0,1,2,....) or vica versa. The non-zero contribution to (5.38) are due to the terms where the arguments of the cosine functions are zero (or multiples of 2π) i.e. the modes that satisfy

$$|r|+|s|-1 = 0 \quad\Rightarrow\quad r=0\ ,\ s=1 \quad\text{or}\quad s=1\ ,\ r=0 \tag{5.39}$$

Therefore the result of the integration in (5.38) is

$$\frac{L}{2}\int d^4x\left[i\bar{\chi}_1\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\chi_3 + i\bar{\chi}_3\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\chi_1\right] + \text{H.C.} \tag{5.40}$$

$$= \frac{1}{2}L\int d^4x\left[i\bar{\psi}\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\psi - i\bar{\tilde{\psi}}\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\tilde{\psi}\right] + \text{H.C.} \tag{5.41}$$

$$\psi = \frac{1}{\sqrt{2}}\left(\chi_1 + \chi_3\right)\ ,\ \tilde{\psi} = \frac{1}{\sqrt{2}}\left(\chi_1 - \chi_3\right) \tag{5.42}$$

This means that at scales larger than the size of the extra dimension (which effectively corresponds to integration over the extra dimension) only one fermion and one ghost fermion is observed. The other modes are only observed at smaller scales while they are screened at large extra dimensional length scales due to the screening because of the (the cosine) form of the conformal factor. These modes can be observed only in interactions with energies higher than the inverse size of the extra dimension. At distances greater than the size of the extra dimension(s) even when they are already excited they seem hidden (unless they have high relative momenta when they interact in the detector). In other words these modes behave like a strange form of dark matter. The experimental predictions of this model are quite different from the usual Kaluza-Klein prescription and need further study. In high energy colliders the signature of these modes would be a sudden increase in the strength of the interactions and a high correlation between the interacting particles. In my opinion the phenomenological implications of this scheme deserves a separate and detailed study by its own.

5.4 Metric reversal symmetry and an automatic Pauli-Villars-like regularization

In the usual Kaluza-Klein scheme, Kaluza-Klein tower is an additional source of infinites that should be regulated. This property of compact extra dimensions is one of the major problems of quantum field theory in extra dimensions. On the other we will see that in the spaces with metric reversal symmetry (MRS) there is the possibility of an automatic, Pauli-Villars-like on contary to the generic extra dimensional spaces. Below I summarize a model of this type. The details of this model may be found in [11].

Consider the following 7-dimensional space ($\mu, \nu = 0, 1, 2, 3$)

$$ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu - \cos^2 k_2 y_2 \,[\, dy_1^2 + \cos^2 k_3 y_3 dy_2^2 + dy_3^2\,] \tag{5.43}$$

where the extra dimensions are compact and have the sizes L_1, L_2, L_3, and $k_1 = \frac{2\pi}{L_1}$, $k_2 = \frac{2\pi}{L_2}$, $k_3 = \frac{2\pi}{L_3}$. Assume the symmetry

$$x^a \to -x^a \ , \quad a = 0, 1, 2, 3, 5 \tag{5.44}$$

$$x^b \to -x^b \ , \quad b = 0, 1, 2, 3, 6 \tag{5.45}$$

where $x^5 = y_1$, $x^6 = y_2$, $x^7 = y_3$; and anti-periodic boundary conditions are adopted for the 5th and 6th directions while periodic boundary condition is adopted for the 7th direction for the field χ i.e. $\chi(x, z) = -\chi(x, z + L)$ for $z = y_1$, $L = L_1$ or $z = y_2$, $L = L_2$ while $\chi(x, y_3) = \chi(x, y_3 + L_3)$. Then the Fourier expansion of a field χ is

$$\begin{aligned}\chi(x, z) = \sum_{n=1}^{\infty} \{\, & f_{|n|}[\cos(\frac{|n|kz}{2}) + \sin(\frac{|n|kz)}{2})] \\ & + g_{|n|}[\cos(\frac{|n|kz)}{2}) - \sin(\frac{|n|kz)}{2})]\}\chi_{|n|}(x)\end{aligned} \tag{5.46}$$

where $z = y_1, y_2$, $k = k_1, k_2$, $a_{|n|}$, $b_{|n|}$, $f_{|n|}$, $g_{|n|}$ are some constants. Even and odd n correspond to periodic and anti-periodic boundary conditions [13], respectively.

The modes χ_n are taken to transform under (5.44) and (5.45) as

$$\varphi_{n,m,r}(x) \rightarrow \xi^{\lambda_n}\mathcal{CPT}\,\varphi_{n,m,r}(-x) \quad \text{as} \quad x^a \rightarrow -x^a \tag{5.47}$$
$$\varphi_{n,m,r}(x) \rightarrow \xi^{\lambda_m}\mathcal{CPT}\,\varphi_{n,m,r}(-x) \quad \text{as} \quad x^b \rightarrow -x^b \tag{5.48}$$
$$\varphi_{n,m,r}(x) \rightarrow \xi^{\lambda_n+\lambda_m}\mathcal{CPT}\,\varphi_{n,m,r}(-x)$$
$$\text{as } x^a \rightarrow -x^a\,,\; x^b \rightarrow -x^b \tag{5.49}$$
$$\lambda_n = \frac{i}{2}(-1)^{\frac{n}{2}} \quad \lambda_m = \frac{i}{2}(-1)^{\frac{m}{2}} \qquad a = 0,1,2,3,5\;;\; b = 0,1,2,3,6$$

(where n, m, r are the modes corresponding to y_1, y_2, y_3 directions, respectively; ξ is some constant *other than 1 or -1*, and $\mathcal{CPT}$ denotes the part of (4-dimensional) CPT transformation acting on the spinor part of the field. I also impose the symmetry

$$k_1y_1 \rightarrow k_1y_1 + \pi \tag{5.50}$$
$$k_2y_2 \rightarrow k_2y_2 + \pi \tag{5.51}$$

In the light of the above observations I consider the following action

$$S_{fk1} = \int d^4x\; d^3y\; \cos^3 k_2y_2\; \cos k_3y_3 \frac{1}{2}[\mathcal{L}_{fk11} + \mathcal{L}_{fk12}] + \text{H.C.} \tag{5.52}$$
$$\mathcal{L}_{fk11} = \frac{i}{4}[(\bar{\chi}_{(1)}\gamma^\mu\,\partial_\mu\chi_{(3)} + \bar{\chi_{(1)}}^P\gamma^\mu\,\partial_\mu\chi^P_{(3)}) + y_1 \rightarrow -y_1] \tag{5.53}$$
$$\mathcal{L}_{fk12} = \frac{i}{4}[(\bar{\chi}\gamma^\mu\,\partial_\mu\chi^P - \bar{\chi}^P\gamma^\mu\,\partial_\mu\chi) + (y_1 \rightarrow -y_1)]. \tag{5.54}$$

After inserting the explicit form of χ and imposing the symmetries (5.44), (5.45), (5.49), (5.51) one finds

$$\mathcal{L}_{fk1} = \frac{1}{2}[\mathcal{L}_{fk11} + \mathcal{L}_{fk12}]$$
$$= \sum_{n_1,m_1=1}^{\infty} A^{(1,3)}_{n1,m1}\, i\bar{\chi}_{n_1}(x,y)\gamma^\mu\partial_\mu\chi_{m_1}(x,y)\,\cos\frac{n_1+m_1}{2}k_1y_1 + \text{H.C.} \tag{5.55}$$

where $y = y_2, y_3$. The spectrum at the scales larger than the size of the extra dimensions may be found by integration of $[\mathcal{L}_{fk11} + \mathcal{L}_{fk12}]$ over the extra dimensions.

$$S_{fk1} = \int d^4x\; d^2y\; \cos^3 k_2y_2\; \cos k_3y_3 \sum_{n_1,m_1=1}^{\infty} A^{(1,3)}_{n1,m1}\, i\bar{\chi}_{n_1}(x,y)\gamma^\mu\partial_\mu\chi_{m_1}(x,y)$$
$$\times \int dy_1\, \cos\frac{n_1+m_1}{2}k_1y_1 + \text{H.C.} = 0 \tag{5.56}$$
$$A^{(1,3)}_{n1,m1} = (f^*_{n1}g_{m1} + g^*_{n1}f_{m1} + f^*_{n1}f_{m1} - g^*_{n1}g_{m1})$$

The upper index $*$ denotes complex conjugate, H.C. stands for Hermitian conjugate, and f_n, g_n's are those given in (5.46). The subscripts (1), (3), and the superscripts $(1,3)$ above refer to the modes with $n = 4p+1$ and $n = 4p+3$, respectively, where $p = 0,1,2,......$ The $y_1 \to -y_1$ terms in the above equations stands for the term obtained from the previous one by replacing y_1's in that term by $-y_1$ and insures the invariance of the Lagrangian $\mathcal{L}_{fk1}$ under (5.44). The values of n_1, m_1 in (5.54,5.56) are fixed by the requirement of invariance under (5.50), (5.47), and are given by

$$n_1 \;=\; 4l_1 \,+\, 1\;,\;\; m_1 \;=\; 4p_1 \,+\, 3 \quad \text{or vica versa} \qquad l_1, p_1 = 0,1,2,....... \tag{5.57}$$

It is evident that (5.56) gives zero because $\int_0^{L_1} \cos \frac{n_1+m_1}{2} k_1 y_1 \, dy_1 = 0$ since $n_1 + m_1 \neq 0$. Hence there are no observable fermions at scales larger than the sizes of the extra dimensions. Therefore an additional action must be introduced to accaount for the usual fermions while S_{fk1} may be used for a Pauli-Villars-like regularization as we shall see. Assume that on the hyper-surface, $y_3 = y_1$ the symmetry (5.44) (and (5.47)) is broken by a small amount while there is an unbroken symmetry under the separate (and simultaneous) applications of (5.36), (5.37), and under the simultaneous application of (5.44) and (5.45) (and (5.47) and (5.48). Consider the following action that obeys these conditions

$$S_{fk2} \;=\epsilon \int \delta(k_3y_3 - k_1y_1) \cos^3 k_2y_2 \, \cos k_3y_3 \frac{1}{2}[\mathcal{L}_{fk21} \,+\, \mathcal{L}_{fk22}] \,+\, \text{H.C.} \tag{5.58}$$

$$\begin{aligned}\mathcal{L}_{fk21} \;=\; & \frac{i}{8}[(\bar{\chi}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi_{(1,3)} + \bar{\chi}^{P1,P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1,P2}_{(1,3)} - \bar{\chi}^{P1}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1}_{(1,3)} - \\ & \bar{\chi}^{P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P2}_{(1,3)}) + \,(y_{1,2} \to -y_{1,2})]\end{aligned} \tag{5.59}$$

$$\begin{aligned}\mathcal{L}_{fk22} \;=\; & \\ & \frac{i}{8}[(\bar{\chi}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1}_{(1,3)} + \bar{\chi}^{P1}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi_{(1,3)} - \bar{\chi}^{P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1,P2}_{(1,3)} - \bar{\chi}^{P1,P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P2}_{(1,3)} \\ & +\bar{\chi}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P2}_{(1,3)} + \bar{\chi}^{P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi_{(1,3)} - \bar{\chi}^{P1}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1,P2}_{(1,3)} - \bar{\chi}^{P1,P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1}_{(1,3)} \\ & +\bar{\chi}^{P1}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P2}_{(1,3)} + \bar{\chi}^{P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1}_{(1,3)} + \bar{\chi}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi^{P1,P2}_{(1,3)} + \bar{\chi}^{P1,P2}_{(1,3)}\gamma^{\mu}\,\partial_{\mu}\chi_{(1,3)}) \\ & +(y_1 \to -y_1)]\end{aligned} \tag{5.60}$$

where $\epsilon << 1$ is some constant that accounts for the breaking of the symmetry (5.44) by a small amount. The superscripts P1, P2 refer to the χ's transformed under (5.36), (5.37), respectively. After replacing the Fourier expansion of χ and integrating over extra dimensions one obtains

$$\begin{aligned}S_{fk2} \;=\; & \frac{\epsilon L_1 L_2 L_3}{4\pi}(f_1^* g_1 + g_1^* f_1 + f_1^* f_1 - g_1^* g_1) \\ & (f_3^* g_3 + g_3^* f_3 + f_3^* f_3 - g_3^* g_3) \int d^4x \, i\bar{\chi}_{13}\gamma^{\mu}\partial_{\mu}\chi_{13}\end{aligned} \tag{5.61}$$

In other words only the mode χ_{13} is observed at large scales. If we take $n_3 = 0$ to be the lowest lying mode in y_3 direction then the usual fermions (i.e. the zero mode) are identified by χ_{130}.

Although only S_{fk2} is relavant on the brane $y_1 = y_3$ and at large scales both of S_{fk1} and S_{fk2} are relavant on the brane. One must consider small patches in extra dimensional space to regulariza the affect of the delta function. Therefore we integrate $\mathcal{L}_{fk1}$ and $[\mathcal{L}_{fk21}+\mathcal{L}_{fk22}]$ on the patch

$$-\Delta \leq u \leq \Delta\,, \quad v \leq v' \leq v+\Delta' \quad , \quad u = k_1y_1 - k_3y_3\,, \quad v = k_1y_1 + k_3y_3 \quad (5.62)$$

The result of the integration may be expressed as

$$\int d^4x\,\mathcal{L}_{eff} \quad (5.63)$$

where

$$\mathcal{L}_{fk2}^{eff} = \frac{i}{2}\lim_{x'\to x}\partial_\mu\ \left(\bar{\chi}_{130}(x'),\bar{\chi}_{310}(x')\bar{\chi}_{330}(x')\right)\,\tilde{\mathbf{M}}\,\gamma^\mu \begin{pmatrix}\chi_{130}(x)\\ \chi_{310}(x)\\ \chi_{330}(x)\end{pmatrix} \quad (5.64)$$

here

$$\tilde{\mathbf{M}} = \begin{pmatrix}\tilde{\mathcal{A}} & \tilde{\mathcal{B}} & \tilde{\mathcal{C}}\\ \tilde{\mathcal{B}} & \tilde{\mathcal{D}} & 0\\ \tilde{\mathcal{C}} & 0 & 0\end{pmatrix} \quad (5.65)$$

where

$$\tilde{\mathcal{A}} \simeq \epsilon\cos^3 k_2y_2 \sum_{p_1,s_1=0}^{\infty}\tilde{A}_{p_1s_1}^{(1,1)}\,\tilde{T}_{p_1,s_1}^{(1,3)}(y_1)\sum_{p_2,s_2=0}^{\infty}\tilde{A}_{p_2s_2}^{(3,3)}\cos\left[2(p_2+s_2)+1\right]k_2y_2 \quad (5.66)$$

$$\tilde{\mathcal{B}} \simeq \cos^3 k_2y_2 \sum_{p,s=0}^{\infty}A_{ps}^{(1)}(y_2)\,T_{p,s}(y_1) \quad (5.67)$$

$$\tilde{\mathcal{C}} \simeq \cos^3 k_2y_2 \sum_{p,s=0}^{\infty}A_{ps}^{(3)}(y_2)\,T_{p,s}(y_1) \quad (5.68)$$

$$\tilde{\mathcal{D}} \simeq \epsilon\cos^3 k_2y_2 \sum_{p_1,s_1=0}^{\infty}\tilde{A}_{p_1s_1}^{(3,3)}\,\tilde{T}_{p_1,s_1}^{(3,1)}(y_1)\sum_{p_2,s_2=0}^{\infty}\tilde{A}_{p_2s_2}^{(1,1)}\cos\left[2(p_2+s_2)+3\right]k_2y_2 \quad (5.69)$$

here

$$\begin{aligned}\tilde{T}_{p_1,s_1}^{(1,3)}(y_1) &= \frac{\Delta'}{2}\{\frac{\cos(p_1+s_1+1)(k_1y_1+k_3y_3)}{p_1+s_1+1}\\ &+\frac{\cos(p_1+s_1)(k_1y_1+k_3y_3)}{p_1+s_1}\} \quad (5.70)\\ \tilde{T}_{p_1,s_1}^{(3,1)}(y_1) &= \frac{\Delta'}{2}\{\frac{\cos(p_1+s_1+2)(k_1y_1+k_3y_3)}{p_1+s_1+2}\\ &+\frac{\cos(p_1+s_1+1)(k_1y_1+k_3y_3)}{p_1+s_1+1}\} \quad (5.71)\\ T_{p,s}(y_1) &= \frac{\Delta'}{(p+s)(p+s+1)}[\sin(p+s)\Delta\,\cos(p+s+1)(k_1y_1+k_3y_3)\\ &+\sin(p+s+1)\Delta\,\cos(p+s)(k_1y_1+k_3y_3) \quad (5.72)\end{aligned}$$

Note that $\Delta' << 2\pi$ is employed in (5.69) and (5.70-5.72) since Δ and Δ' should be taken as small as possible because my aim is to study point-wise as much as possible (while without causing any ambiguity due to the delta function on the brane). Therefore provided that $\epsilon \ll 1$

$$\tilde{\mathbf{M}} \simeq \begin{pmatrix} 0 & \tilde{\mathcal{B}} & \tilde{\mathcal{C}} \\ \tilde{\mathcal{B}} & \tilde{\mathcal{D}} & 0 \\ \tilde{\mathcal{C}} & 0 & 0 \end{pmatrix} \tag{5.73}$$

Hence the conclusions about the spectrum of the fields at the points $k_1y_1 \neq k_3y_3$ essentialy remain the same at the points $k_1y_1 = k_3y_3$ (or at the points $k_1y_1 \simeq k_3y_3$). The diagonalization of $\mathbf{M}$ in (5.73) results in

$$\mathcal{L}_{eff} \simeq iB(y)[\bar{\psi}_1(x)\gamma^\mu \partial_\mu \psi_1(x) - \bar{\psi}_2(x)\gamma^\mu \partial_\mu \psi_2(x)] \tag{5.74}$$

$$\psi_1 = \frac{1}{2\sqrt{2}}[\chi_{130} + (\cos\theta\chi_{310} - \sin\theta\chi_{330})] \tag{5.75}$$

$$\psi_2 = \frac{1}{2\sqrt{2}}[\chi_{130} - (\cos\theta\chi_{310} - \sin\theta\chi_{330})] \tag{5.76}$$

$$\tan\theta = \frac{\mathcal{B}}{\mathcal{C}} \quad , \quad B(y) = \sqrt{(\mathcal{B}^2 + \mathcal{C}^2)} \ , \quad y = y_1, y_2 \tag{5.77}$$

Hence the spectrum at scales smaller than the size of the extra dimension has a fermion and a ghost fermion coupled to each standard model fermion that appears at scales greater than the size of the extra dimensions. There is another state $\psi_3 = \sin\theta\chi_{310} + \cos\theta\chi_{330}$ but this does not contribute to (5.74). So it is an auxiliary field. Although sign of the kinetic term of ψ_2 in (5.74) is opposite of a usual fermion (and so it is a ghost-like field) it does not suffer from the problems of the usual ghosts. ψ_1 or ψ_2 in (5.74) can not be introduced or removed from (5.74) because (5.74) follows from the couplings of χ_{130}, χ_{310}, χ_{330}. So ψ_1, ψ_2 form a single system. For example in this case ψ_1, ψ_2 may be considered as the components of a single field with a 8-component spinor and the gamma matrices given by $\gamma^\mu \odot \tau_3$ where $\odot$ denotes tensor product and τ_3 is the third Pauli matrix. This solves the problem of negative norm for ψ_2 because there is single norm i.e. that of the system composed of ψ_1, ψ_2. Moreover since ψ_1 and ψ_2 have the same internal space properties and they form a single system they may be assigned the same 4-momentum with positive energy, and this solves the negative energy problem of ψ_2. However the extension of this argument to the fields other than the fermions is not straightforward and requires additional study.

Eq.(5.74) implies an automatic regularization. The fermion ghost fermion pair at smaller scales naturally introduces a cut-off for the loop calculations. This may be seen better as follows: At scales larger than the size of the extra dimensions the relevant field is $\chi_{130}(x)$ and its propagator is

$$D(p) = \frac{i}{\not{p} + m} \tag{5.78}$$

(where m is the mass of the field at scales larger than the sizes of extra dimensions (e.g. induced by Higgs mechanism)) while at smaller scales the relevant fields are

ψ_1 and ψ_2 with the effective propagator

$$\begin{aligned} D_{eff}(p) &= D_1(p) + D_2(p) \sim \frac{i}{B'(\not p + m_1)} - \frac{i}{B'(\not p + m_2)} \\ &= i\frac{m_2 - m_1}{B'(\not p + m_1)(\not p + m_2)} \\ B' &= N\,B(y)\cos^3 k_2 y_2 \cos k_3 y_3 \end{aligned} \quad (5.79)$$

where m_1, m_2, in general, may depend on y_1, y_2, and I have assumed for sake of generality that ψ_1, ψ_2 may have two different effective masses at scales smaller than the size of extra dimensions that may be induced by spin connection terms, Higgs mechanism, or some other mechanism. For $m_1 = m_2$ this equivalent to finite renormalization while for $m_1 \neq m_2$ it is equivalent to Pauli-Villars regularization [14] at propagator level.

5.5 Conclusion

We have seen that metric reversal symmetry gives interesting results for a wide range of issues and problems in physics, namely, cosmological constant problem, zero-point energy problem, regularization of extra dimensional quantum field models, and Pauli-Villars regularization. We have also found a an interesting Kaluza- Klein spectrum that may give interesting signatures in accelerator experiments and quite different, non-conventional dark matter-like spectrum. Therefore the next step in this direction may be more realistic models of this form and a detailed study of their phenomenological implications.

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