

# **EFFECTS OF DARK PHOTONS ON PULSAR KICKS**

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**by  
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# ABSTRACT

## EFFECTS OF DARK PHOTONS ON PULSAR KICKS

Several studies that have been carried out on supernova explosions and their remnants (pulsars) so far ended up with two fundamental problems: How are the left-handed neutrinos released out of the star and how do the supernova explosions happen even the inner core region of the star is extremely dense for propagation of the left-handed neutrinos? How can the pulsars attain very high kick velocities after the burst? The main goal of this thesis is to contribute to resolutions of the aforementioned problems by using new particles. The particles under concern are dark-photon which is immune to all the forces in the SM but gravity and interacts with the SM spectrum through only its kinetic mixing with the photon (actually the hypercharge gauge boson) and the right-handed neutrino which possesses dark electric charge (couples to dark photon). By proposing new particles, we find an interval for the photon-dark photon mixing parameter consistent with the values proposed by the string theory and the experiments. We calculate some pulsar kick velocities consistent with the observations for some magnetic field values and mixing parameter values. We show that the right-handed neutrinos may have electric charges which are not quantized and on the order of  $10^{-6}e$ .

# ÖZET

## KARA FOTONLARIN ATARCA İTKİLERİ ÜZERİNDEKİ ETKİLERİ

Süpernova patlamaları ve pulsarlar hakkında yapılagelen çalışmalar iki temel soruyu ortaya çıkartmıştır: Yıldızın içi sol-el nötrinoların yayılmasına izin vermeyecek kadar yoğun olduğu halde sol-el nötrinolar yıldı dışına nasıl atılmaktadır ve süpernova patlamaları nasıl mümkün olabilmektedir? Patlamanın kalıtı olarak ortaya çıkan pulsarlar böylesine yüksek itki hızlarına nasıl ulaşabilmektedir? Bu projenin amacı, yukarıdaki sorular ile ifade edilen problemin çözümüne yeni bir takım parçacıklar önererek katkıda bulunmaktır. Söz konusu olan parçacıklar: çekim hariç temel kuvvetlerin ve parçacıkların standart modeli (SM) içindeki kuvvetlerden etkilenmeyen, foton (daha doğrusu hiperyük ayar bozonu) ile yaptığı kinetik karışım dışında etkileri gözlemlenmeyen kara-foton ile kara elektrik yüküne sahip sağ-el nötrinolardır. Yeni parçacıklar önererek, sicim teorisinin ve deneylerin öngördüğü değerlerle uyuşan foton-kara foton karışım parametresi değer aralığı bulunmuştur. Bazı manyetik alan ve karışım parametresi değerleri için gözlemlerle uyuşan pulsar itki hızları hesaplanmıştır. Sağ-el nötrinoların kuantize olmayan  $10^{-6}e$  civarında elektrik yüküne sahip olabileceği gösterilmiştir.

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# CHAPTER 1

## INTRODUCTION

Supernovae, which are the final stages of the evolutions of the massive stars with masses of  $8 - 25M_{\odot}$ , release extremely huge amount of energy per second ( $\sim 10^{51}$  erg/s) via left-handed neutrinos. Their remnants are pulsars characterized by their high core density (around the nuclear density,  $10^{14}g/cm^3$ ), huge magnetic field ( $\sim 10^{12}G$ ), high kick velocities and high rotation speeds [13]. These events, which are not clearly understood today, are natural laboratories of astrophysics, nuclear physics, condensed matter physics and particle physics. Several studies that have been carried out on supernova events so far ended up with some problems: What is the relation between supernovae and their progenitors? What are the properties of mass, radius, magnetic field of the stars remaining from supernovae? There are lots of such questions about supernova events but the most fundamental and unclear ones:

- **Problem 1 :** Given that interior of the star is too dense to allow for propagation of left-handed neutrinos and shock waves resulting from sudden stop of the imploding matter are not powerful enough, how is it possible that supernovae bursts happen?
- **Problem 2 :** How is it possible that the supernovae remnants, pulsars, attain such very high kick velocities?

To understand the real mechanism underlying behind these magic events, the authors in [5] proposed two fundamental solutions for the last evolutionary stage of the massive stars. To them, supernovae can be result of reignition of nuclear fusion in a degenerate star (e.g. white dwarfs, as Type Ia SN) or a core collapse ( Type II, Type Ib/c) can cause a huge explosion. On the way of understanding the basic prepositions, authors in [62] showed that gravitational energy on the order of  $10^{53}erg$  is turned into energy release via neutrino emission and it gives rise to a huge SN shock wave. However, there are lots of missing points in these kind of propositions. The mechanism proposed in this study is based on cooperation between the right-handed neutrino and dark photon, and can mainly be expressed by two points below:

- Concerning problem 1 above, the right-handed neutrinos are produced from the left-handed ones in the core of the star, then they propagate to the surface of the star, and finally they get converted back to the left-handed ones via neutrino oscillations and coupling of the dark photon to neutrino dark magnetic moment.
- Concerning problem 2 above, the right-handed neutrinos under dark magnetic field, due to quantization, get distributed into Landau levels differently for different spin directions (with respect to the dark magnetic field direction) , and this asymmetry causes right-handed neutrinos of differing spin to propagate into different directions with different fluxes. This anisotropic propagation causes the neutron star to receive a kick to become a high-speed pulsar.

In Chapter 2, evolution of the massive stars are briefly explained, supernova explosions and pulsars are summarized to be get prepared to the particle physics applications in the stellar environment. In Chapter 3 and Chapter 4, we give a brief explanation of the Maxwellian electrodynamics and dark electrodynamics respectively. Answers to the questions of what is dark photon? how does not a particle interact with ordinary matter? Is it possible to see behind a wall? are tried to be given in Chapter 5. The conversion of  $\nu_L \rightarrow \nu_R$  is analysed and the number of right-handed neutrinos per unit time per unit energy from the unit volume is calculated in Chapter 6. Asymmetric right-handed neutrino emission and pulsar kicks are studied in Chapter 7. In Chapter 8, the results found in each chapter are examined for some SN 1987A parameters and compared with the literature.

## CHAPTER 2

# EVOLUTION OF MASSIVE STARS: SUPERNOVA EXPLOSIONS AND PULSARS

Even the luminous stars in the sky seem unchanging and stable, they evolve radically during their life. They can differ in size, color, and brightness. Then, one can ask; why are they different? why do not we see a notable change in a star's appearance during our life? To the astronomers and astrophysicists, one can never see a notable change in a star's color and brightness during his life because a star can live on the order of millions or billions of years. Therefore, stellar evolution is theoretically constructed by the astrophysicists.

Evolution of a star begins with condensation of gas clouds, which consists of predominantly hydrogen, named nebulae. The condensation or contraction occurs due to the gravitational collapse. While the giant molecular cloud collapses, gravitational potential energy is released as heat. Due to the increase in temperature and pressure, further contraction is opposed and the condensation turns into a protostar (hot, rotating gas cloud in hydrostatic equilibrium). As energy releasing, it contracts, the total kinetic energy of the particles inside the core becomes sufficiently high to halt the Coulomb repulsion between the hydrogen nuclei and the nuclear fusion starts. When the hydrogen fuel is totally consumed, pressure due to the nuclear reactions can not balance the gravitational collapse. As the temperature and density increase due to contraction, the collapse is halted by helium burning. These series of reactions continue with burning of carbon, neon, oxygen and silicon respectively. At each step, the new nuclear fuel is the yield of the previous reaction. The stars having masses more than  $11M_{\odot}$  may have a core consisting of Fe and Ni whereas stars with lower masses may end up with a core consisting of O-Ne-Mg. In a star whose bare mass (remaining after the star sheds its outer shell, envelope, into a planetary nebula ) is less than Chandrasekhar limit ( $\sim 1.4M_{\odot}$ ), electron degeneracy pressure (Pauli exclusion principle) is at work and saves it from collapsing. Such a star is known as white dwarf. If the mass is more than this limit then the electron degeneracy can no longer balance the inward force and the star collapses. Temperature and pressure increases again due to former collapse. At this point, mass of the star determines what kind of reactions will take place. If mass of the star takes a value between  $15 - 20M_{\odot}$ , electrons are captured by heavy elements and neutrinos are emitted.

The iron atoms are disintegrated into  $\alpha$  particles in a star with mass which is equal or more than  $20M_{\odot}$  and this gives rise to energy loss. At this stage, electrons are captured by protons and neutrons and neutrinos are formed via inverse  $\beta$  decay; Urca process:



The stability of the star is determined by neutron degeneracy pressure and the star becomes a dense collapsed neutron rich core (nuclear density of  $\sim 10^{14}g/cm^3$ ) so that it becomes incompressible and rebounds the collapse. This collapse results in an extremely luminous explosion called *supernova explosion* [1]. Supernovae are extremely exotic physical events in the universe such that one of them named SN 1987A is the most luminous cosmic object ( $\sim 10^{52}erg/s$ ) visible to naked eye. The energy released in this SN explosion during tens of seconds is ejected via neutrinos with a percentage of 99. These explosions have become more important events after observation of SN 1987A so that their types and thermonuclear processes have been studied by several authors [2,3,4].

The supernovae can be grouped as Type I and Type II according to variety in their light curves (apparent magnitude vs time) and their element spectra. If supernovae show hydrogen lines, they are called as Type II otherwise they are Type I. Type I supernovae show a sharp maximum in their light curve and their luminosity decreases slowly after the sharp peak. There are three subgroups of Type I supernovae: Type Ia, Type Ib, and Type Ic. If supernovae show silicon lines in their spectra they are called Type Ia. If there are helium lines in their spectra they are Type Ib otherwise they are Type Ic. The supernovae which are results of massive stars, Type II, are divided into some subgroups. Most of them showing broad hydrogen lines in their spectra are Type II-P while the ones whose spectral lines are narrow are called Type II-N supernovae. Type II-L shows a steady decrease in the light curve; L denotes linear even the curve is not a real linear. The other one is Type II-b which shows helium lines even it shows hydrogen lines at early stages. Because of similarities with Type I-b, it is called Type II-b. [5]. Although SNI and SNII types differ from each other in terms of several properties, they are all the results of death of the stars having mass more than . The supernovae where the neutrinos take place are the core collapse ones; SNIb, SNIc and SNII. The SN 1987A discovered in February 1987 is Type II-P supernova.

To some authors [6,7,8] the rebound generates a shock wave and it gives rise to spreading of outer layers of the star, called supernova explosion. However, this explosion mechanism is not confirmed by the computer simulations [9]. On the other hand, there are some other attempts [10,11,12] trying to explain SN explosions by proposing new particles. However, supernovae are still mystery.

The supernovae collapsing into a neutron star can form a highly magnetized and rotating neutron star called *pulsar*. The first pulsar was discovered by Hewish et al. in 1968. Pulsars are treated as the light houses of the universe because of the fact that they emit radiation with regular time intervals. There is a difference in alignment of the magnetic field axis and the rotational axis of a pulsar and the radiation is emitted through the magnetic axis. This is because the pulsars seem like light houses. There are more than 1000 pulsars known with low rotational periods change from 1.5 ms to 8.5 ms. While some pulsars like Crab pulsar emit radio waves, some of them called x-ray pulsars emit x-rays. The x-ray pulsars are the members of the binary systems [13]. They are generally characterized by their extremely huge  $10^{12}G$  magnetic field and their space velocities changing from  $400km/s$  to more than  $1000km/s$  [14]. Their extreme properties make them our particle physics, condensed matter physics, nuclear physics, and astrophysics laboratories in the space.

## CHAPTER 3

### CLASSICAL ELECTRODYNAMICS: A BRIEF REVIEW

It is known that the universe is originated from a singularity hence it is natural to think that from sub-atomic scale to the large scale of stars and galaxies, the whole universe must be governed by a single law of physics. This law have to explain not only everyday of life, but also being of protons and neutrons together, radioactive decay, and the dynamics of the galaxies. Today, even there is not a unified theory which combines the physics laws explaining the events at each scale, what kind of physics underlying at each scale is almost known. The nucleons are hold together by a short range force called strong force while the radioactive decays are governed by the weak force. The planets in the solar system move around the sun in a fixed trajectory due to the gravitational force. The classical electrodynamics defines the other force called the electromagnetic force which is long range and second in order of decreasing strength. As understood from its name, it is associated with electric and magnetic fields and responsible for holding atoms and molecules together.

The classical electrodynamics were constructed by Charles-Augustin de Coulomb, Andre Marie Ampere, Michael Faraday, and the other scientists, but all the laws of electrodynamics were put in a complete and compact form by James Clerk Maxwell. The dynamics of electric and magnetic fields are described by a total number of 8-coupled equations called Maxwell's equations. The differential form of the Maxwell's equations in free space (absence of polarizable or magnetic media )with SI unit system are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (3.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3.3)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (3.4)$$

where  $\epsilon_0$  is permittivity of free space,  $\mu_0$  is permeability of free space,  $\mathbf{E}$  is the electric field vector,  $\mathbf{B}$  is the magnetic field vector,  $\rho$  is charge density, and  $\mathbf{J}$  is current density.

The first equation called Gauss' Law for electricity states that the total electric flux through any closed surface is proportional to the total electric charge enclosed within the closed surface. The second equation is known as Gauss' Law for magnetism and it states that there is no magnetic flux through an closed surface. The third equation named as Faraday's Law of Induction describes that time-varying magnetic fields creates an electric field. The last equation which is known as Ampere's Law gives a relation between magnetic field and electric current. It implies that electric field flowing through a loop creates a magnetic field and the line integral of this magnetic field around the closed loop is proportional to that of electric field.

In static electricity and magnetism, Coulomb's Law and Biot-Savart law are enough to give what one needs. However, there should be generalization of the Maxwell equations to provide an answer to the question of what are the electric and magnetic fields due to the time dependent charge and current density. In this case, the fields should be represented by scalar potential of  $\Phi$  and vector potential of  $\mathbf{A}$ . Since it is known that divergence of curl is zero, one can write the magnetic field in terms of the vector potential  $\mathbf{A}$ :

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (3.5)$$

In electrostatics curl of electric field is zero so the electric field  $\mathbf{E}$  can be written as the gradient of the scalar potential  $\Phi$  because curl of gradient is zero.

$$\mathbf{E} = -\nabla\Phi \quad (3.6)$$

using two equations above one can write the second homogenous Maxwell equation (Faraday's law) as

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{A}) \quad (3.7)$$

then the electric field in terms of scalar and vector potential can be written as

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} \quad (3.8)$$

two homogenous Maxwell equations are satisfied by the electric and magnetic potentials written in terms of scalar and vector potentials. In order to obtain the inhomogenous Maxwell equations in terms of the potentials, one can use the electric and the magnetic field vectors written in terms of the potentials. If the electric field expression is put into the first Maxwell equation, the equation below is obtained

$$\nabla^2\Phi + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\frac{\rho}{\epsilon_0} \quad (3.9)$$

and using both of the field written in terms of the potentials and utilizing from the identity  $\nabla \times (\nabla \times \mathbf{V}) = \nabla(\nabla \cdot \mathbf{V}) - \nabla^2\mathbf{V}$ , one can get the fourth Maxwell equation

$$\nabla^2\mathbf{A} - \frac{1}{c^2} \frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla \left( \nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial\Phi}{\partial t} \right) = -\mu_0\mathbf{J} \quad (3.10)$$

where  $c = 1/\sqrt{\epsilon_0\mu_0}$  speed of light. All the Maxwell equations have been expressed in terms of the electromagnetic potentials so far. However, the inhomogenous ones are coupled, namely, both depend on scalar and vector potentials. To get the wave equations of the uncoupled form

$$\nabla^2\Phi - \frac{1}{c^2} \frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho}{\epsilon_0} \quad \nabla^2\mathbf{A} - \frac{1}{c^2} \frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J} \quad (3.11)$$

the scalar and vector potential should be transformed in a way that the equation above can be obtained. The significant point here is that the transformation must leave  $\mathbf{E}$  and  $\mathbf{B}$  unchanged. This transformation of the scalar and vector potentials is defined as

$$\mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \nabla\chi \quad , \quad \Phi \rightarrow \Phi' = \Phi - \frac{\partial\chi}{\partial t} \quad (3.12)$$

these transformations are exactly what one needs to get the same electric and magnetic fields and they are called *gauge transformations*. If the transformed fields are put into the equations then the condition which makes the equations uncoupled is obtained

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial\Phi}{\partial t} = 0 \quad (3.13)$$



this gauge is named as *Lorentz gauge* and it is commonly used in terms of not only its property of making the wave equations uncoupled but also not being a concept dependent of the coordinate system. Another gauge is the *Coulomb gauge* which is used in electrostatics.

$$\nabla \cdot \mathbf{A} = 0 \quad (3.14)$$

This gauge states that the scalar potential due to a charge density is constant for the observers in the earth and in the moon at the same time. In fact, two observers can not feel the same potential because of the fact that scalar potential decreases with increasing distance and there must be a retardation between the source and observer [15,16].

While the concept of the gauge transformations mentioned so far seems more mathematical, choosing these gauges comes from some physical needs. It is known from the special relativity that space and time phenomena can change for the observers in the inertial reference frames. The sources of the fields,  $\rho$  and  $\mathbf{J}$  transform like space and time by Lorentz transformations. In this chapter, we will not mention about the transformed fields under Lorentz transformations but follow a simpler way. As seen from the uncoupled inhomogenous Maxwell equations, the charge density and the current density are components of a four vector  $J^\mu = (\rho, \mathbf{J})$ , the scalar and the vector potentials are components of the photon field  $A^\mu = (\Phi, \mathbf{A})$ . Using the Lorentz-Heaviside units where  $\epsilon_0 = \mu_0 = 1$ , we can write the Lagrangian density:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu \quad (3.15)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the field strength tensor of the photon field. The first part of the Lagrangian density is the kinetic term of the electromagnetic field  $A_\mu$  and the second term stands for the fermion field interaction with the photon field. To find Maxwell's equations from the Lagrangian density, we use Euler-Lagrange equation [17]:

$$\frac{\partial \mathcal{L}}{\partial A_\beta} - \partial_\alpha \left[ \frac{\partial \mathcal{L}}{\partial (\partial_\alpha A_\beta)} \right] = 0 \quad (3.16)$$

Varying the first term of the Lagrangian density with respect to  $A_\beta$  we get

$$\begin{aligned}
\frac{1}{4}\partial_\alpha \left[ \frac{\partial (F_{\mu\nu}F^{\mu\nu})}{\partial (\partial_\alpha A_\beta)} \right] &= \frac{1}{4}\partial_\alpha \left\{ \frac{\partial F_{\mu\nu}}{\partial (\partial_\alpha A_\beta)} F^{\mu\nu} + F_{\mu\nu} \frac{\partial (g^{\mu\lambda}F_{\lambda\rho}g^{\nu\rho})}{\partial (\partial_\alpha A_\beta)} \right\} \\
&= \frac{1}{4}\partial_\alpha \left\{ \frac{\partial F_{\mu\nu}}{\partial (\partial_\alpha A_\beta)} F^{\mu\nu} + g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu} \frac{\partial F_{\lambda\rho}}{\partial (\partial_\alpha A_\beta)} \right\} \\
&= \frac{1}{4}\partial_\alpha \left\{ \frac{\partial F_{\mu\nu}}{\partial (\partial_\alpha A_\beta)} F^{\mu\nu} + F^{\lambda\rho} \frac{\partial F_{\lambda\rho}}{\partial (\partial_\alpha A_\beta)} \right\} \\
&= \partial_\alpha F^{\alpha\beta}
\end{aligned} \tag{3.17}$$

where antisymmetric property of the field strength tensor is used. Variation of the second term gives

$$J^\mu \frac{\partial A_\mu}{\partial A_\beta} = J^\mu \delta_\mu^\beta = J^\beta \tag{3.18}$$

The homogenous Maxwell's equations are obtained by the variation of dual tensor  $G^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\rho\sigma}F_{\rho\sigma}$ . Finally, eight set of equations can be put in a compact form of

$$\partial_\alpha F^{\alpha\beta} = J^\beta \quad \partial_\alpha G^{\alpha\beta} = 0 \tag{3.19}$$

## CHAPTER 4

### DARK ELECTRODYNAMICS

In the previous Chapter, the classical electrodynamics was reviewed briefly. It was showed that Maxwell's equations can be written in a covariant form and they can be derived from the electromagnetic Lagrangian density. We firstly reviewed the Maxwellian electrodynamics because we are interested in the role of the magnetic field in the supernova core. It is known that the supernova explosions and rotation of the pulsars are mainly related with the magnetic field inside of the star. In this chapter, however, we introduce (even similar studies exist under the title of extra-photon or hidden-photon, see the next chapter) another type of electrodynamics which is identical to the Maxwellian electrodynamics in terms of some properties. This is such an electrodynamics that there are charge densities, current densities, electric and magnetic fields as in the Maxwellian electrodynamics.

#### Maxwellian Electrodynamics

$$J^\mu = (\rho, \mathbf{J})$$

$$A^\mu = (\Phi, \mathbf{A})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

#### Dark Electrodynamics

$$J'^\mu = (\rho', \mathbf{J}')$$

$$A'^\mu = (\Phi', \mathbf{A}')$$

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$$

where  $J^\mu$  is the current four vector of the Maxwellian electrodynamics,  $J'^\mu$  is the current four vector of another electrodynamics.  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the field strength tensor of the gauge field of Maxwellian electrodynamics while  $F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$  is the field strength tensor of the gauge field field of the second electrodynamics.

As a matter of fact, the main goal of introducing another type of electrodynamics is to find an answer to the question of how do the neutrinos escape from the dense supernova core. As it was mentioned in the introductory chapter, there are several attempts to solve this problem but this is still a mystery. To overcome this problem, we can focus on the effect of magnetic field on neutrino escape. However, it is a fact that neutrinos have no electric charge so they do not interact with the magnetic field inside the dense media. Therefore, escape of neutrinos can not be related to any electromagnetic interaction of neutrinos. Nevertheless, the electrodynamics which is introduced as identical to the Maxwellian electrodynamics might be a solution candidate. We shall define this electrodynamics such that all the charged particles (particles charged under symmetries of the standard model of particles SM) protons and electrons, for example, can not interact with the fields of this electrodynamics except for kinetic mixing of the gauge fields of the two electrodynamics. For this reason, we call this electrodynamics as "dark" electrodynamics. The mixing term means that if there exists electromagnetic field then there also exists the dark electromagnetic field. Therefore, it is natural to write the ordinary magnetic field in terms of the dark-magnetic field. In addition to this, if we also introduce a particle called right-handed neutrino, which is charged under the magnetic field and can turn into the left-handed neutrino due to magnetic moment interaction, we may find a way to explain the mechanism underlying behind the supernova explosion. The relation between the magnetic field and the dark-magnetic field can be obtained by adding a mixing term which mixes  $A_\mu$  and  $A'_\mu$  to the electromagnetic Lagrangian:

$$\frac{1}{2}\kappa F^{\mu\nu} F'_{\mu\nu} \quad (4.1)$$

where  $\kappa$  is a mixing parameter. The the full Lagrangian becomes

$$\mathcal{L}_{Kinetic} = -\frac{1}{4}F^{\mu\nu} F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu} F'_{\mu\nu} - \frac{1}{2}\kappa F^{\mu\nu} F'_{\mu\nu} + J^\mu A_\mu + J'^\mu A'_\mu \quad (4.2)$$

where the first two terms are the kinetic terms of  $A_\mu$  and  $A'_\mu$  respectively. The third term is the kinetic mixing term while the last two terms are the interaction terms of the particles live in Maxwellian sector and the dark-sector.

As in the previous chapter, varying the Lagrangian with respect to  $A_\mu$  and  $A'_\mu$  gives these two set of equations

$$\partial_\mu F'^{\mu\nu} + \kappa \partial_\mu F^{\mu\nu} + J^\nu = 0 \quad (4.3)$$

$$\partial_\mu F'^{\mu\nu} + \kappa \partial_\mu F^{\mu\nu} + J'^\nu = 0 \quad (4.4)$$

where  $E^i = -F^{0i}$  and  $\epsilon^{ijk} B^k = -F^{ij}$ . As we are interested in magnetic field-dark magnetic field relation, we ignore the electric field. These two equations can be written as

$$\nabla \times (\mathbf{B} + \kappa \mathbf{B}' + \nabla \phi) + \mathbf{J} = 0 \quad (4.5)$$

$$\nabla \times (\mathbf{B}' + \kappa \mathbf{B} + \nabla \varphi) + \mathbf{J}' = 0 \quad (4.6)$$

where  $\phi$  and  $\varphi$  are scalars. Since the right-handed neutrinos are produced from the left-handed ones, we can ignore the current density of the particles charged under the dark magnetic field for the beginning of the Urca process. As curl of gradient is zero, then we obtain the relation between the magnetic field and the dark magnetic field

$$\mathbf{B}' = -\kappa \mathbf{B} \quad (4.7)$$

this result says that both magnitude and direction of the dark magnetic field are different from the magnitude and the direction of the magnetic field. The value of the dark magnetic field depends only on the  $\kappa$  parameter. In the next chapter, the new field strength tensor  $F'_{\mu\nu}$  and parameter  $\kappa$  will be  $X_{\mu\nu}$  and  $\chi$  respectively.

## CHAPTER 5

### DARK PHOTONS

The four vector  $A^\mu$  consisting of the scalar and the vector potentials is called photon field. Photon  $\gamma$  is a massless elementary particle which is responsible for shining and heating our universe. It can be treated as a form of electromagnetic radiation and it is the force carrier of the electromagnetic force. As pointed out in the previous chapter, there are also weak and strong forces rather than the electromagnetic force and they have their own force carrier particles. These forces are unified by a theory called the standard model of the particles (SM). The SM which is based on the gauge symmetry of  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  explains most of phenomena in today's known particle physics, where C stands for color, L stands for isospin, and Y stands for hypercharge. However, this model is not enough to understand the nature totally. One of the problems is related to the gravitational interactions which is aimed to solve by quantum gravity and the string theory. Another problem is the flavour problem which asks the question why are there three generations. In addition to these, some observational phenomena also can not be explained by the SM. To the astrophysicists and cosmologists, nearly 22% of the universe should consist of non-baryonic dark matter which is not identified in the SM while approximately 74% of the universe should be made of dark energy which has not been understood yet. There should be an extension of the SM to deal with this problem so that the sources of the dark matter and the dark energy should be understood. However, extension of the SM requires introduction of some additional particles which are not included by the SM. Nevertheless, the properties of the new particles should be known; Are they very massive or light? What kind of charge properties do they have? Can their effects be visible or invisible to our particle detectors? The new particles can either have big masses so that they can be traced in huge accelerators by sensitive detectors or they can be light. To some particle physics models, if light particles exist then they should interact weakly with the SM particles via radiative corrections with massive particles. The light particles can live at low energies and their life space is generally called as "hidden sector". They might be good dark matter candidates and they are called as WISPs (weakly interacting sub-ev particles). There are also massive dark matter candidates named WIMPs (weakly interacting massive particles)[18]. These kind of particles are very important in terms of widening our understanding about the astrophysical events. For example, there

are no well defined models explaining the supernova explosion mechanism and the reality underlying behind escape of neutrinos from the dense stellar media. To solve this novel, these particles can be used in stellar media thanks to their interaction properties. To some authors, WISPs play important roles in stellar evolution [19,10,20,21,22,23,11]. The WISPs can couple to the standard photon via interaction vertex which is called mixing term. The mixing term indicates that when photon propagates, there is a possibility of conversion into WISPs. The possibility of conversion can be treated as oscillation which is called quantum flavour oscillations. Today, there are several "light-shining-through-walls" experiments searching for the photon-WISP oscillations [24,25,26,27]. The main idea in these kind of experiments is to shine a beam of laser onto a wall and detect the amount of photons after the wall. One may think how does it possible to see behind a wall even the wall is opaque to the light. However, in the context of this model there is a possibility of detecting photons after the wall. To the experimental approaches, when the photons hit the wall, they convert into the WISPs and they can transverse through the wall easily because of the fact that they can interact weakly with the SM particles. And then, WISPs can again convert into the ordinary photons via oscillations so we can detect them. There are some other experiments aiming to wide oscillation region of the photons. "Any-Light-Particle-Search" (ALPS) experiment at DESY Hamburg [28] is one of the attempts and it aims to search for WISPs with large masses up to MeV scale. The key idea in this experiment is that the external magnetic field which is perpendicular to the photon propagation polarizes the photons and the resulting WISP oscillation probability becomes different. There are also PAVLAS and Q&A collaborations [29,30] other than ALPS using the magnetic field in their experiments and searching for WISPs.

There are several theoretical studies [31] on the extension of the SM . One of the WISPs coming from the extensions of the SM is a gauge boson of new Abelian group  $U(1)_X$  which is called "paraphoton" or "hidden photon" [32]. These two photons can be treated as the photons of two distinct worlds. The connection between world of the ordinary photon and the hidden photon is maintained via kinetic mixing of two world's photons thanks to gauge invariance so that this effect appears in the low-energy Lagrangian :

$$\mathcal{L}_{mix} = \frac{1}{2}\chi F_{\mu\nu} X^{\mu\nu} \quad (5.1)$$

The field strength tensor of the ordinary photon field  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  can couple to the field strength tensor  $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$  of a "dark photon"  $X_\mu$ (we state as  $F'_{\mu\nu}$

in the previous Chapter)- the gauge field of an Abelian group  $U(1)_X$  via dimensionless parameter  $\chi$  (the  $\kappa$  in the previous Chapter) . The gauge bosons of the hidden sector can be massive (up to 1TeV), light or massless. The upper limit on the mixing parameter is  $\sim 10^{-2}$  for the massive hidden photons [33,34]. The massless hidden sector photons can be found in the string theory and their effects can be shown by using the conformal field theory and supergravity. If the HPs are massless so that the gauge symmetry of the hidden sector is unbroken, then there is no limit on the value of the mixing parameter [35]. If there are light hidden sector fermions  $f_h$  which are charged under the hidden

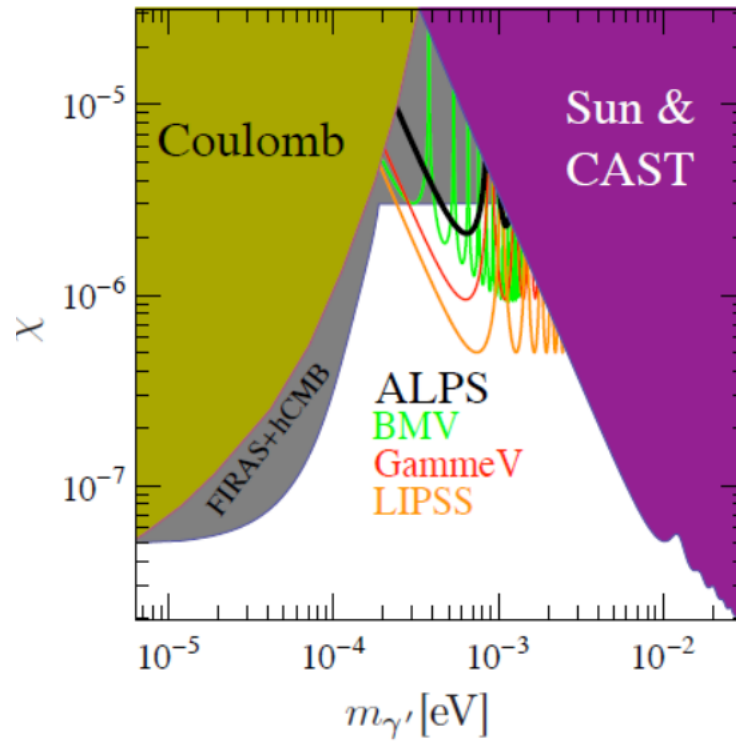


Figure 5.1. The experimental limits on the dark photon-photon mixing angle  $\chi$  and the dark photon mass  $m'_{\gamma}$ . (for more details see ref. [35])

sector gauge symmetry, there can be a way to limit on the value of the mixing parameter because these particles called minicharged or milicharged particles (MCPs) depend on the mixing parameter  $\chi$ . Their electric charges are not a multiple of the electric charge  $q_h = \chi g_h \equiv \epsilon e$  and where  $g_h$  is gauge coupling constant of the hidden sector [36,37].



There are many experiments searching for the MCPs. The laser polarisation experiments [38,39] aim to limit on the MCPs with small masses. In these kind of experiments is that a number of photons are being retarded or lost after linearly polarised light is sent through a magnetic field. The retardation is thought to be caused by the interaction of the hidden sector particles with laser beam while being lost is related with pair production to the hidden sector matter particles. Another type of laser experiments [40] use the

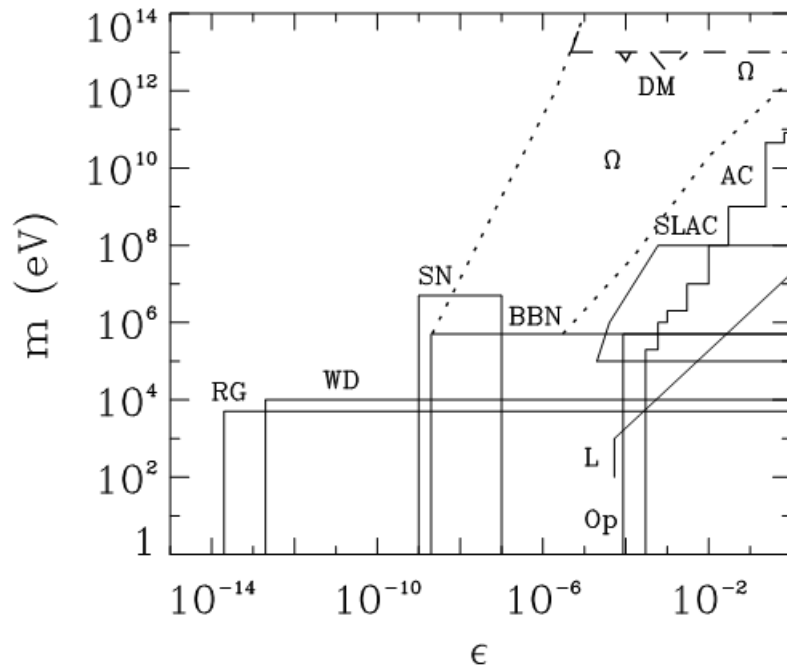


Figure 5.2. "Regions of mass-charge space ruled out for milli-charged particles. The solid and dashed lines apply to the model with a paraphoton; solid and dotted lines apply in the absence of a paraphoton. The bounds arise from the following constraints: (AC) accelerator experiments, (Op) the Tokyo search for the invisible decay of ortho-positronium, (SLAC) the SLAC milli-charged particle search, (L) the Lamb shift, (BBN) nucleosynthesis, (RG) plasmon decay in red giants, (WD) plasmon decay in white dwarfs, (DM) dark matter searches (SN) Supernova 1987A". [37]

idea that a thick wall is shone with a laser beam which can convert into the hidden sector photons even the hidden photons can be massless. This oscillation is induced by hidden sector matters in the magnetic field. To the authors, if photons are sent to a magnetic field then they can pair produce the MCPs. Moreover, the MCP loops can induce ordinary photon-hidden sector photon oscillations even if the hidden photons are massless.

## CHAPTER 6

### RIGHT HANDED NEUTRINO PRODUCTION

It is known that neutrinos (left-handed) are produced in the core of the supernovae via inverse beta decay. The supernova explosion, SN 1987A is famous because it was the first time that interactions of neutrinos from a supernova explosion were observed by the Kamiokande and IMB water Cherenkov detectors. This explosion encouraged the theoretical physicists to focus on the open questions about the supernovae which were studied before. Escape of the neutrinos from extremely dense supernova core is one of the most puzzling questions in astrophysics. There are several studies trying to understand the real mechanism underlying behind the supernovae by using the idea of neutrino magnetic moment–core plasma interaction. In the minimal extension of the SM, the magnetic moment of the neutrinos can be given as [41,42]

$$\mu_i = 3.2 \times 10^{-19} \left( \frac{m_i}{1eV} \right) \mu_B \quad (6.1)$$

where  $i = 1, 2, 3$  stands for the neutrino flavour. Nevertheless, one of the several extensions of SM [43] showed that neutrinos can have magnetic moments larger than this value by orders of magnitude. Authors in [44,45,46] proposed that Solar neutrino problem can be solved by spin precession of neutrinos having larger magnetic moments in the magnetic field of the Sun. In addition to Solar neutrino problem, supernovae were realized to be another astrophysical event in which the neutrinos interacting with the environment via their magnetic moments. A solution to the SN 1987A neutrino problem was firstly proposed by A. Dar [47]. According to A. Dar, huge number of left-handed neutrinos produced via inverse beta decay in the supernova core turn into the right-handed ones due to magnetic moment–plasma (electrons and protons) interaction. This process is followed by back conversion of the right-handed neutrinos to the left-handed neutrinos in the supernova envelope due to spin precession in the magnetic field. After this idea, the process chirality flip was studied by several authors. The authors in [48] proposed the process of  $\nu_{Le^-} \rightarrow \nu_{Re^-}$  and  $\nu_{Lp} \rightarrow \nu_{Rp}$  in the core of supernova while the core collapses. They put an upper limit on the neutrino magnetic moment as  $\mu_\nu < (0.2 - 0.8) \times 10^{-11} \mu_B$ .

An upper limit greater than the value in [48] was put by the authors in [49] by using the methods of thermal field theory. More detailed analyses were performed by the authors in [50,51] by contributing proton effects to the photon propagator. Several studies which treat the right-handed neutrinos as sterile to the other SM particles were performed. However, these authors assumed that the back conversion ( $\nu_R \rightarrow \nu_L$ ) occurs in the interstellar magnetic field. Neutrinos take important role not only in supernova bursts but also in the cooling of stars [52,53]. There are also experiments searching for the neutrino magnetic moment. The GEMMA [54] is one of them and the upper limit put on the neutrino magnetic moment by the experiment is  $\mu_\nu < 5.8 \times 10^{-11} \mu_B$ .

In this chapter, we will focus on a different scenario. We propose that the left-handed neutrinos possess dark magnetic moment which is identical to the ordinary magnetic moment and they convert into the right-handed neutrinos due to magnetic moment–dark magnetic field interaction.

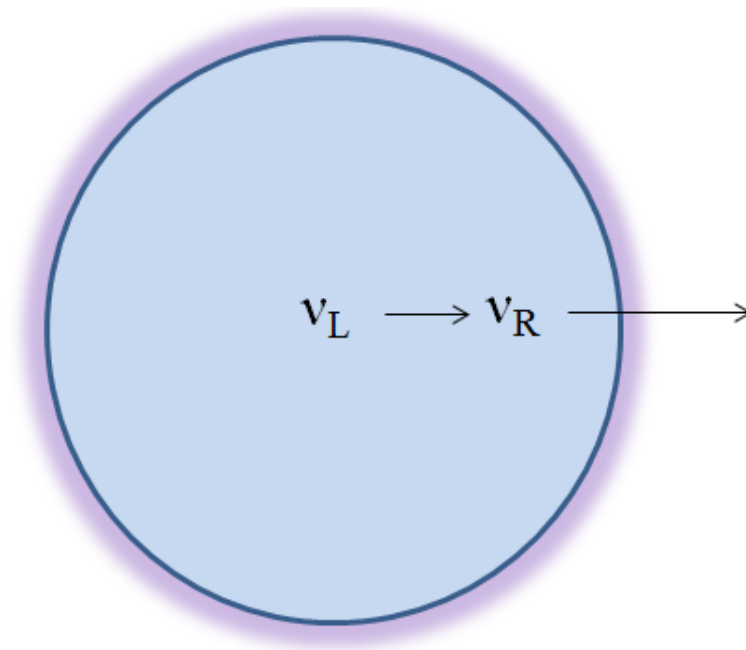


Figure 6.1. Conversion of the left-handed neutrinos into the right-handed neutrinos due to magnetic moment–dark magnetic field interaction.

A fermion  $\psi$  interacts with the electromagnetic potential  $A_\mu$  via the Lagrangian

$$\mathcal{L} = \bar{\psi}i\gamma^\mu\mathcal{D}_\mu\psi + \rho\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} \quad (6.2)$$

where the first term stands for the kinetic part, and the second for the dipole interaction. A given fermion field can always be decomposed into two chiralities: the left-handed and right-handed ones (where the decomposition is Lorentz invariant and sustainable for massless fermions):

$$\psi = \psi_L + \psi_R \quad (6.3)$$

where  $\psi_L = P_L\psi$  and  $\psi_R = P_R\psi$  with  $P_{L,R} = (1 \mp \gamma_5)/2$ , when acting on  $\psi$ . If we put the decomposed wave function into the equation (6.2) and ignore the kinetic term, we write the Lagrangian density:

$$\mathcal{L} = \rho [\bar{\psi}_L\sigma^{\mu\nu}\psi_R F_{\mu\nu} + \text{h.c.}] \quad (6.4)$$

The transition matrix for the process  $\nu_L \rightarrow \nu_R$  can be written

$$S_{fi} = -\frac{i\mu'_\nu}{2} \int d^4x \bar{\psi}_R(\sigma^{\mu\nu} F'_{\mu\nu})\psi_L \quad (6.5)$$

where  $\mu'_\nu$  is the neutrino dark magnetic moment,  $F'_{\mu\nu}$  is the field strength tensor of the dark photon,  $\psi_L$  is incident plane wave describing a left handed neutrino of momentum  $p$  spin  $s_i$  and  $\psi_R$  is outgoing plane wave describing a right-handed neutrino of momentum  $k$  and  $s_f$

$$\psi_L = \sqrt{\frac{m_L}{E_L V}} \nu_L(p, s_i) \exp(-ipx) \quad (6.6)$$

where we normalize  $\psi$  to unit probability in a box of volume  $V$ . In the same way

$$\bar{\psi}_R = \sqrt{\frac{m_R}{E_R V}} \bar{\nu}_R(k, s_f) \exp(ikx) \quad (6.7)$$

then the transition matrix

$$S_{fi} = -\frac{i\mu'_\nu}{2V} \sqrt{\frac{m_L m_R}{E_L E_R}} [\bar{\nu}_R(k, s_f) (\sigma^{\mu\nu}) \nu_L(p, s_i)] \int d^4x e^{i(k-p)x} F'_{\mu\nu}(x) \quad (6.8)$$

$$\tilde{F}'_{\mu\nu}(k-p) = \int d^4x e^{i(k-p)x} F'_{\mu\nu}(x) \quad (6.9)$$

where

$$F'_{\mu\nu}(x) = \partial_\mu A'_\nu(x) - \partial_\nu A'_\mu(x) \quad (6.10)$$

$$\tilde{F}'_{0\nu}(k-p) = \int d^4x e^{i(k-p)x} (\partial_0 A'_\nu(x) - \partial_\nu A'_0(x)) \quad (6.11)$$

$$\tilde{F}_{0\nu}(k-p) = \int d^3x e^{-i(\vec{k}-\vec{p})\cdot\vec{x}} \int e^{i(E_R-E_L)t} \frac{\partial A'_\nu}{\partial t} dt - \int d^4x e^{i(k-p)x} \partial_\nu A_0(x) \quad (6.12)$$

using integration by parts we can write the electric field components of the tensor as

$$\begin{aligned} \tilde{F}_{0i}(k-p) &= \partial_0 \tilde{A}_i - \partial_i \tilde{A}_0 \\ &= -i(\epsilon \tilde{A}_i - q_i \tilde{A}_0) \end{aligned} \quad (6.13)$$

where  $\epsilon = (E_R - E_L)$  and  $\vec{q} = (\vec{k} - \vec{p})$

$$\tilde{F}_{ij}(k - p) = -i(q_i \tilde{A}_j - q_j \tilde{A}_i) \quad (6.14)$$

$$\tilde{F}'_{\mu\nu}(k - p) = -i(q_\mu \tilde{A}'_\nu - q_\nu \tilde{A}'_\mu) = -i\tilde{F}'_{\mu\nu}(k - p) \quad (6.15)$$

and as a result, the scattering matrix is written:

$$S_{fi} = -\frac{\mu'_\nu}{2V} \sqrt{\frac{m_L m_R}{E_L E_R}} [\bar{\nu}_R(k, s_f) (\sigma^{\mu\nu} \tilde{F}'_{\mu\nu}) \nu_L(p, s_i)] \quad (6.16)$$

Since we are working in a finite volume  $V$ , the probability  $dw$  for a production in which in the final state the particle has momentum between  $p$  and  $p + dp$  and takes place at any time between  $-T/2$  and  $T/2$  can be written as

$$dw = \frac{\mu'^2_\nu}{4V^2} \frac{m_L m_R}{E_L E_R} |M_{fi}|^2 \frac{V d^3 k}{(2\pi)^3} \quad (6.17)$$

The production rate  $d\Gamma$ , which is the probability per unit time can be obtained by dividing the probability by  $T$

$$d\Gamma = \frac{\mu'^2_\nu}{4VT} \frac{m_L m_R}{E_L E_R} |M_{fi}|^2 \frac{d^3 k}{(2\pi)^3} \quad (6.18)$$

$$d\Gamma = \frac{\mu'^2_\nu}{4VT} \frac{m_L m_R}{E_L} |\bar{\nu}_R(k, s_f) (\sigma^{\mu\nu} \tilde{F}'_{\mu\nu}) \nu_L(p, s_i)|^2 \frac{d^3 k}{E_R (2\pi)^3} \quad (6.19)$$

The  $|M|^2$  is the quantity we actually need to determine the crosssections and lifetimes. In this case the relevant crosssection and rate is related to average over all the initial spin configurations  $i$ , and the sum over all the final spin configurations,  $f$  [55].  $\langle |M|^2 \rangle \equiv$  average over initial spins , sum over final spins of  $|M_{if}|^2$

$$[\bar{\nu}_L (\sigma^{ab} \check{F}'_{ab}) \nu_R] [\bar{\nu}_L (\sigma^{cd} \check{F}'_{cd}) \nu_R]^* = [\bar{\nu}_L (\sigma^{ab} \check{F}'_{ab}) \nu_R] [\bar{\nu}_R (\sigma^{cd} \check{F}'_{cd}) \nu_L] \quad (6.20)$$

The cross section is a sum of equation above over final spin states and an average of over initial states. The spin sum can be written as follows:

$$\begin{aligned} |\bar{M}|^2 &= \frac{1}{2} \left\{ \sum_{s=1,2} \nu_L^{(s)}(p) \bar{\nu}_L^{(s)}(p) \right\} (\sigma^{ab} \check{F}'_{ab}) \left\{ \sum_{s=1,2} \nu_R^{(s)}(k) \bar{\nu}_R^{(s)}(k) \right\} (\sigma^{cd} \check{F}'_{cd}) \\ &= \frac{1}{2} [\Lambda_+(p_i)]_{\theta\alpha} (\sigma^{ab} \check{F}'_{ab})_{\alpha\beta} [\Lambda_+(p_f)]_{\beta\gamma} (\sigma^{cd} \check{F}'_{cd})_{\gamma\theta} \\ &= \frac{1}{8m_L m_R} \text{Tr}[(\not{\mathbf{p}} + m_L)(\sigma^{ab} \check{F}'_{ab})(\not{\mathbf{k}} + m_R)(\sigma^{cd} \check{F}'_{cd})] \end{aligned} \quad (6.21)$$

where  $\not{\mathbf{p}} = \gamma^\mu p_\mu$ ,  $\not{\mathbf{k}} = \gamma^\rho k_\rho$ ,  $\sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$ .

$$\begin{aligned} \text{Tr}[(\not{\mathbf{p}} + m_L)(\sigma^{ab} \check{F}'_{ab})(\not{\mathbf{k}} + m_R)(\sigma^{cd} \check{F}'_{cd})] &= -\frac{1}{4} \check{F}'_{ab} \check{F}'_{cd} p_\mu k_\rho [\text{Tr}(\gamma^\mu \gamma^a \gamma^b \gamma^\rho \gamma^c \gamma^d)] \\ &\quad - \text{Tr}(\gamma^\mu \gamma^a \gamma^b \gamma^\rho \gamma^d \gamma^c) \\ &\quad + \text{Tr}(\gamma^\mu \gamma^b \gamma^a \gamma^\rho \gamma^d \gamma^c) \\ &\quad - \text{Tr}(\gamma^\mu \gamma^b \gamma^a \gamma^\rho \gamma^c \gamma^d) \\ &\quad - \frac{1}{4} \check{F}'_{ab} \check{F}'_{cd} m_L m_R [\text{Tr}(\gamma^a \gamma^b \gamma^c \gamma^d)] \\ &\quad - \text{Tr}(\gamma^a \gamma^b \gamma^d \gamma^c) \\ &\quad + \text{Tr}(\gamma^b \gamma^a \gamma^d \gamma^c) \\ &\quad - \text{Tr}(\gamma^b \gamma^a \gamma^c \gamma^d) \end{aligned} \quad (6.22)$$

$$\begin{aligned}
Tr [(\not{p} + m_L)(\sigma^{ab}\check{F}'_{ab})(\not{k} + m_R)(\sigma^{cd}\check{F}'_{cd})] &= 8\check{F}'^{\mu d}\check{F}'_{dc}p_\mu k^c \\
&+ 8\check{F}'^{\mu c}\check{F}'_{cd}p_\mu k^d \\
&+ 8\check{F}'_{ba}\check{F}'^{a\mu}p_\mu k^b \\
&+ 8\check{F}'_{ab}\check{F}'^{b\mu}p_\mu k^a \\
&+ 8\check{F}'_{ab}\check{F}'^{ab}(p_\mu k^\mu + m_L m_R)(6.23)
\end{aligned}$$

each term has momentum of  $q = k - p$ .

$$\begin{aligned}
8\check{F}'^{\mu d}(k-p)\check{F}'_{dc}(k-p)(k_\mu - q_\mu)k^c &= 8\check{F}'^{\mu d}(k-p)\check{F}'_{dc}(k-p)k_\mu k^c \\
&- 8\check{F}'^{\mu d}(k-p)\check{F}'_{dc}(k-p)q_\mu k^c \quad (6.24)
\end{aligned}$$

the second term in Equation (6.24) is zero

$$\check{F}'^{\mu d}(q)\check{F}'_{dc}(q)q_\mu k^c = 0 \quad (6.25)$$

since we do not take electric field into account, we ignore  $\check{F}'^{0\mu}$  and  $\check{F}'^{\mu 0}$  and we take only  $\check{F}'^{ij}$  components. Then first term in Equation (6.23) can be written as

$$\begin{aligned}
8k_i\check{F}'^{ij}\check{F}'_{jr}k^r &= 8k_i\epsilon^{ijk}\check{B}_k\epsilon_{jrm}\check{B}^mk^r \\
&- 8k_ik^r\epsilon_{jik}\epsilon_{jrm}\check{B}_k\check{B}^m \\
&- 8k_ik^r(\delta_{ir}\delta_{km} - \delta_{im}\delta_{kr})\check{B}_k\check{B}^m \\
&- 8(\vec{k} \cdot \vec{k})(\vec{B} \cdot \vec{B}) + 8(\vec{k} \cdot \vec{B})^2 \quad (6.26)
\end{aligned}$$

since we have four same terms, first four terms in Equation (6.24) can be written as

$$-32(\vec{k} \cdot \vec{k})(\vec{B} \cdot \vec{B}) + 32(\vec{k} \cdot \vec{B})^2 \quad (6.27)$$



and the last term

$$\begin{aligned} 8\check{F}_{ab}(q)\check{F}^{ab}(q)(k_\mu k^\mu + m_L m_R) &= \epsilon_{ijk}\check{B}^k \epsilon^{ij\rho}\check{B}_\rho(m_R^2 + m_R m_L) \\ &= 16\check{B} \cdot \check{B}(m_R^2 + m_R m_L) \end{aligned} \quad (6.28)$$

then total trace is written as

$$\begin{aligned} Tr[(\not{p} + m_L)(\sigma^{ab}F_{ab})(\not{k} + m_R)(\sigma^{cd}F_{cd})] &= -32(\vec{k} \cdot \vec{k})(\check{B} \cdot \check{B}) + 32(\vec{k} \cdot \check{B})^2 \\ &+ 16\check{B} \cdot \check{B}(m_R^2 + m_R m_L) \end{aligned} \quad (6.29)$$

Put the scattering amplitude into the production rate equation

$$d\Gamma = \frac{\mu_\nu^2}{4VT} \frac{m_L m_R}{E_L} |M|^2 \frac{d^3k}{E_R(2\pi)^3} \quad (6.30)$$

$$d\Gamma = \frac{\mu_\nu^2}{32VTE_L} (Tr[(\not{p} + m_L)(\sigma^{ab}F_{ab})(\not{k} + m_R)(\sigma^{cd}F_{cd})]) \frac{d^3k}{E_R(2\pi)^3} \quad (6.31)$$

$$d\Gamma = \frac{\mu_\nu'^2}{2VTE_L} \left( 2(\vec{k} \cdot \check{B}')^2 + (\check{B}' \cdot \check{B}')(m_R^2 + m_R m_L - 2(\vec{k} \cdot \vec{k})) \right) \frac{d^3k}{E_R(2\pi)^3} \quad (6.32)$$

If the magnetic field is constant then the differential production rate is found

$$d\Gamma = \frac{\mu_\nu'^2 |\check{B}'|^2}{E_L} (2\pi)\delta^4(k - p) \left[ m_R^2 + m_R m_L - 2|\vec{k}|^2 \sin^2 \theta \right] \frac{d^3k}{2E_R} \quad (6.33)$$

The number of neutrinos in the energy interval is given by

$$\frac{dN(E_L)}{dE_L} = \int \frac{d^3p}{(2\pi)^3} \frac{V}{E_L} f_F(p) \quad (6.34)$$

where  $f_F(p)$  is the Fermi-Dirac distribution function for neutrinos. If  $n = N/V$  then the density of neutrinos ( the number of neutrinos per unit volume unit energy);

$$\frac{dn(E_L)}{dE_L} = \int \frac{d^3p}{E_L(2\pi)^3} f_F(p) \quad (6.35)$$

The number of right-handed neutrinos produced per unit time per energy from unit volume;

$$\frac{dn_{\nu_R}}{dE_R} = \frac{dn(E_L)}{dE_L} \Gamma(E_L) \quad (6.36)$$

also the probability per unit time that any left-handed neutrino will turns into right-handed neutrino can be written as

$$\frac{\frac{dn_{\nu_R}}{dE_R}}{\frac{dn(E_L)}{dE_L}} = \Gamma(E_L) \quad (6.37)$$

$$\frac{dn_{\nu_R}}{dE_R} = \frac{1}{2\pi^2} \int p^2 dp \frac{f_F(p)}{E_L} \Gamma(E_L) \quad (6.38)$$

$$\begin{aligned} \frac{dn_{\nu_R}}{dE_R} &= \frac{\mu_\nu'^2 |\vec{B}'|^2}{2\pi^2} \int_0^\infty \frac{2\pi p^2 dp}{E_L^2} f_F(p) \int_{-\infty}^\infty d^4k \delta(k^2 - m_R^2) \theta(E_R) \delta^{(4)}(k - p) \\ &\times [m_R^2 + m_R m_L - 2|\vec{k}|^2 \sin^2 \theta] \end{aligned} \quad (6.39)$$

$$\begin{aligned} \frac{dn_{\nu_R}}{dE_R} &= \frac{\mu_\nu'^2 |\vec{B}'|^2}{2\pi^2} \int_0^\infty \frac{2\pi p^2 dp}{E_L^2} f_F(p) \delta(E_L^2 - |\vec{p}|^2 - m_R^2) \theta(E_L) \\ &\times [m_R^2 + m_R m_L - 2|\vec{p}|^2 \sin^2 \theta] \end{aligned} \quad (6.40)$$

$$\begin{aligned} \frac{dn_{\nu R}}{dE_R} &= \frac{\mu_\nu'^2 |\vec{B}'|^2}{2\pi^2} \int_0^\infty f_F(p) \frac{\pi p dp}{E_L^2} \delta(|\vec{p}'| - \sqrt{E_L^2 - m_R^2}) \theta(E_L) \\ &\times [m_R^2 + m_R m_L - 2|\vec{p}'|^2 \sin^2 \theta] \end{aligned} \quad (6.41)$$

To take this integral and get a physical meaning  $m_R$  must be equal to  $m_L$  (in terms of conservation of energy). Therefore the number of neutrinos produced per unit time per unit energy from unit volume

$$\frac{dn_{\nu R}}{dE_R} = \frac{\mu_\nu'^2 |\vec{B}'|^2}{2\pi^2 E_R^2} 2\pi f_F(p) [m_R^2 (E_R^2 - m_R^2)^{1/2} - (E_R^2 - m_R^2)^{3/2} \sin^2 \theta] \theta(E_R)$$

$$\frac{dn_{\nu R}}{dE_R} = \frac{\mu_\nu'^2 B'^2}{2\pi^2} f(E) \theta(E_R) \quad (6.42)$$

## CHAPTER 7

### ASYMMETRY IN RIGHT-HANDED NEUTRINO EMISSION AND PULSAR KICKS

There are several propositions to explain the origin of the pulsar kicks. There is no exception in these explanations that pulsars should attain a kick in a step of their evolution. The authors in [56] proposed that there should be an asymmetric supernova explosion to accelerate the pulsars with such high velocities. One of the well accepted explanations on the origin of pulsar kick velocities is based on the idea of asymmetric neutrino emission [57]. An asymmetry of  $\sim 3\%$  can give a kick velocity of  $\sim 1000\text{km/s}$  to a neutron star with  $1.4M_{\odot}$ . Asymmetry in neutrino distribution due to parity violation in weak interactions in high magnetic fields was discussed in [58]. Another explanation but with a different aspect of thinking came from the authors in [59]. They argued that neutrino oscillations ( $\nu_{\tau} \rightarrow \nu_{e-}$ ) deforms the shape of the neutrinosphere in proto-neutron stars. This change in shape results in anisotropy in the momentum of neutrinos escaping from the star and can be reason of pulsar kick velocities. Sterile neutrinos interacting very weakly with the other SM particles were used to understand the kick mechanism [60]. A pulsar kick mechanism based on the idea of symmetric neutrino distribution from direct quark Urca process is argued in [61]. To them, electrons which move perpendicular to the magnetic field inside the star and they sit in the Landau levels asymmetrically due to their spin orientation. Neutrinos from the direct quark Urca process escape from the star asymmetrically because of parity violation. The same authors [62] calculate a kick velocity of  $\sim 1000\text{km/s}$  due to momentum conservation by using the neutrino luminosity and taking the temperature inside the star  $T \sim 5\text{MeV}$ , radius of quark phase  $R \sim 10\text{km}$ .

$$dv = \frac{A}{M_{ns}} L dt \quad (7.1)$$

where  $A$  is asymmetry in neutrino distribution,  $M_{ns}$  is mass of the neutron star,  $L$  is neutrino luminosity and  $t$  is the time elapsed by neutrino emission.

In this chapter, we perform the same calculations in [62] but not for the electrons only for the right-handed neutrinos. We argue that the right-handed neutrinos charged under the dark magnetic field are situated in Landau energy levels. An asymmetry due to their different spin orientation can be generated. The equation of motion of a relativistic particle in the dark electromagnetic field:

$$\{\gamma^\mu(\hat{p}_\mu - qA'_\mu) - m\} \Psi = 0 \quad (7.2)$$

Since we are interested in only the magnetic field interaction, we take the dark scalar potential  $A'_0 = 0$  and we get the equation of the form

$$i\hbar \frac{\partial}{\partial t} \Psi = \vec{\alpha} \cdot (\hat{\vec{p}} - q\vec{A}') \Psi + m\gamma^0 \Psi \quad (7.3)$$

the spinor  $\Psi$  can be represented as two parts (components with positive and negative energy) and it is put into the equation:

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \Phi \\ \chi \end{pmatrix} = \left[ \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} (\hat{\vec{p}} - q\vec{A}') + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} m \right] \begin{pmatrix} \Phi \\ \chi \end{pmatrix} \quad (7.4)$$

Each decoupled equations are written as

$$(E - m) \Phi = \vec{\sigma} \cdot (\hat{\vec{p}} - q\vec{A}') \chi \quad (7.5)$$

$$(E + m) \chi = \vec{\sigma} \cdot (\hat{\vec{p}} - q\vec{A}') \Phi \quad (7.6)$$

where  $\vec{\alpha} = \gamma^0 \gamma^i$  and the Dirac basis the gamma matrices are of the form

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad (7.7)$$

from two decoupled equations we obtain the equation in terms of nonrelativistic part of the spinor:

$$(E^2 - m^2) \Phi = \vec{\sigma} \cdot (\hat{\vec{p}} - q\vec{A}') \vec{\sigma} \cdot (\hat{\vec{p}} - q\vec{A}') \Phi \quad (7.8)$$

using the relation  $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = \vec{a} \cdot \vec{b} + i\vec{\sigma} \cdot (\vec{a} \times \vec{b})$  where  $\vec{a}$  and  $\vec{b}$  are any two vectors and  $\hat{\vec{p}} = -i\nabla$ ;  $\hbar = c = 1$

$$\begin{aligned} (E^2 - m^2) \Phi &= \left( \hat{\vec{p}}^2 - 2q\vec{A}' \cdot \hat{\vec{p}} + q^2 \vec{A}'^2 - q\vec{\sigma} \cdot \vec{B}' \right) \Phi \\ &= \left( \hat{\vec{p}}^2 - 2qA'_y p_y + q^2 A_y'^2 - q\sigma_z B'_z \right) \Phi \\ &= \left( p_x^2 + p_y^2 + p_z^2 - q^2 B'^2 x^2 - qB'(\sigma_z + 2xp_y) \right) \Phi \end{aligned} \quad (7.9)$$

the vector potential has been chosen to be  $\vec{A}' = (0, B'x, 0)$  to obtain the z component of the dark-magnetic field. Since the right-hand side of the equation commutes with the components of the momentum operator  $\hat{p}_y$  and  $\hat{p}_z$ , we can introduce the ansatz [63]

$$\Phi_\sigma(x) = e^{(p_y y + p_z z)} f(x) \chi_\sigma \quad (7.10)$$

where  $\chi_\sigma$  is the unit spinor. Insertion into the equation yields

$$(E^2 - m^2) f(x) = \left( -\frac{d^2}{dx^2} + p_y^2 + p_z^2 + q^2 B'^2 x^2 - 2qB'xp_y - qB'\sigma_z \right) f(x) \quad (7.11)$$

which can be written as

$$\left[ -\frac{d^2}{dx^2} + q^2 B'^2 \left( x - \frac{p_y}{qB'} \right)^2 \right] f(x) = (E^2 - m^2 - p_z^2 + qB'\sigma) f(x) \quad (7.12)$$

This is the Schrödinger equation of the harmonic oscillator in the variable  $\zeta = x - p_y/qB'$ . The oscillator energy is  $\hbar\omega = 2|q|B'$  and the eigenvalues thus are  $\lambda_n = (n + 1/2)\hbar\omega = (2n + 1)|q|B'$ . Therefore, the relativistic generalization of the *Landau levels* of a particle in magnetic field is given by

$$E_{P\sigma} = \pm\sqrt{m^2 + P_z^2 + |e|B(2n + 1 + \sigma)} \quad (7.13)$$

As the z-component of the spin is  $s_z = \frac{\hbar}{2}\sigma_z$  whose value is  $1/2$  or  $-1/2$ , we can re-write the equation:

$$E_{p\sigma}^2 = m^2 + p_z^2 + 2|q|B'\eta \quad (7.14)$$

The magnetic field  $B'$  is pointing in the positive z-direction. The Landau quantum number  $\eta$  is defined by its quantum number  $\nu$  and the electron spin  $s$ :

$$\eta = \nu + \frac{1}{2} + s \quad \text{and} \quad s = \begin{cases} +\frac{1}{2} & \text{for } n_+ \\ -\frac{1}{2} & \text{for } n_- \end{cases} \quad (7.15)$$

The neutrino number densities  $n_+$  and  $n_-$  denote neutrinos with spin parallel or anti-parallel to the magnetic field direction, respectively. As it is known that fermion number density can be written as

$$N = g \int \frac{d^3p}{(2\pi)^3} f(E) \quad (7.16)$$

where  $f(E)$  is the Fermi distribution function

$$f(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1} \quad (7.17)$$

where the Boltzman constant  $k_B = 1$ . We can re-write the number density by changing

the variables  $(p_x, p_y) \rightarrow (p, \varphi)$  where  $p_x = p \cos \varphi$  and  $p_y = p \sin \varphi$

$$N = \frac{g}{(2\pi)^3} \int dp_x dp_y dp_z f(E) \quad (7.18)$$

$$dp_x dp_y = \left| \frac{\partial(p_x, p_y)}{\partial(p, \varphi)} \right| dp d\varphi = \begin{vmatrix} \partial p_x / \partial p & \partial p_x / \partial \varphi \\ \partial p_y / \partial p & \partial p_y / \partial \varphi \end{vmatrix} dp d\varphi = p dp d\varphi \quad (7.19)$$

$$p^2 = p_x^2 + p_y^2 = 2qB'\eta \Rightarrow p dp = qB' d\eta \quad (7.20)$$

$$n_{\mp} = \frac{gqB'}{(2\pi)^2} \int_{\eta_{min\mp}}^{\eta_{max}} \int_0^{\infty} f(E) d\eta dp_z \quad (7.21)$$

here, the  $\varphi$  integral gives us  $2\pi$  and the integration over  $d\eta$  is replaced by a summation over  $\eta$  for a small number of Landau levels

$$n_{\mp} = \frac{gqB'}{(2\pi)^2} \sum_{\eta_{min\mp}}^{\eta_{max}} \int_0^{\infty} f(E) dp_z \quad (7.22)$$

Our purpose is to show the particle asymmetry depending on spin orientation, namely, spin polarisation of the neutrinos. The particle asymmetry can be given by

$$\chi = \frac{n_- - n_+}{n_- + n_+} \quad (7.23)$$

Since the particles (actually right-handed neutrinos) are sterile to any thermal excitations, it is natural to consider zero temperature limit. Therefore, the number density is given by

$$n_{\mp} = \frac{gqB'}{(2\pi)^2} \sum_{\eta} \int_0^{\sqrt{\tilde{\mu}^2 - m^2 - 2qB'\eta}} f(E) dp_z \quad (7.24)$$



where the quantised energy was given by

$$E_{P\sigma} = \pm \sqrt{m^2 + P_z^2 + |e|B(2n + 1 + \sigma)} \quad (7.25)$$

$$p = \sqrt{E^2 - m^2 - 2qB'\eta}$$

and it is known that at  $T = 0$  the Fermi energy is equal to the chemical potential  $E = \tilde{\mu}$  so

$$p = \sqrt{\tilde{\mu}^2 - m^2 - 2qB'\eta}$$

The number of Landau levels is limited to  $\eta_{max} = \frac{\tilde{\mu}^2 - m^2}{2qB'}$  where  $\tilde{\mu}$  is the chemical potential. For  $E = \tilde{\mu}$ , the neutrino number densities  $n_{\mp}$  become:

$$n_+ = \frac{qB'}{(2\pi)^2} \sum_{\nu=1}^{\frac{\tilde{\mu}^2 - m^2}{2qB'}} \sqrt{\tilde{\mu}^2 - m^2 - 2\nu qB'} \quad (7.26)$$

$$n_- = \frac{qB'}{(2\pi)^2} \left( \sum_{\nu=1}^{\frac{\tilde{\mu}^2 - m^2}{2qB'}} \sqrt{\tilde{\mu}^2 - m^2 - 2\nu qB'} + \sqrt{\tilde{\mu}^2 - m^2} \right) \quad (7.27)$$

The polarisation is consequently :

$$\chi = \frac{n_- - n_+}{n_- + n_+} = \frac{1}{2 \left( \sum_{\nu=1}^{\frac{\tilde{\mu}^2 - m^2}{2qB'}} \sqrt{1 - \frac{2\nu qB'}{\tilde{\mu}^2 - m^2}} \right) + 1} \quad (7.28)$$

For the case  $(\tilde{\mu}^2 - m^2) \gg 2qB'$  the number of occupied Landau levels is large and the

sum can be transformed back to an integration over  $\nu$  :

$$\int_1^{\frac{\tilde{\mu}^2 - m^2}{2qB'}} \sqrt{1 - \frac{2\nu qB'}{\tilde{\mu}^2 - m^2}} d\nu = \frac{2}{3} \left(1 - \frac{2qB'}{\tilde{\mu}^2 - m^2}\right)^{3/2} \frac{\tilde{\mu}^2 - m^2}{2qB'} \sim \frac{2}{3} \frac{\tilde{\mu}^2 - m^2}{2qB'} \quad (7.29)$$

which simplifies the polarisation to :

$$\chi = \frac{n_- - n_+}{n_- + n_+} \simeq \frac{3}{2} \frac{qB'}{\tilde{\mu}^2 - m^2} \quad (7.30)$$

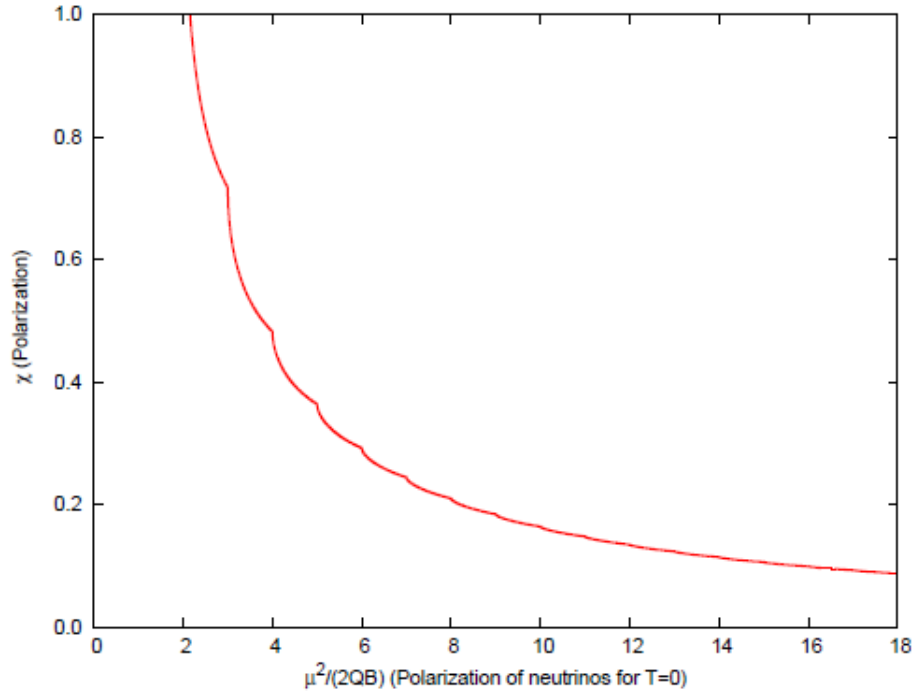


Figure 7.1. Landau levels

## CHAPTER 8

### CONCLUSION AND OUTLOOK

In this Chapter, the results found in each Chapter will be examined for some SN 1987A parameters. We will test the efficiency of dark magnetic moment-dark magnetic field interaction in terms of energy release out of the SN core. On the way of doing these, right-handed neutrino luminosity spectral density and luminosity will be calculated and compared with the results in the other studies in the literature. Because of the fact that the right-handed neutrino luminosity depends on the dark magnetic field which depends on the photon-dark photon mixing parameter, dark magnetic moment, we will put some lower and upper bounds on these parameters. Using the asymmetry formula found in the chapter 7, we will find an interval for the values of dark electric charge of which the right-handed neutrinos can have. Lastly, we will examine the result found for pulsar kick velocities in the chapter 7. Some pulsar kick velocities will be calculated for the photon-dark photon mixing parameter, magnetic field inside the pulsar and the dark magnetic moment of the right-handed neutrinos. The values of these parameters will be examined if they are consistent with the right-handed neutrino luminosity calculations and observations.

#### **Test of Luminosity in SN 1987A :**

The number of right-handed neutrinos per unit time per unit energy from unit volume has been found as

$$\frac{dn}{dE} = \frac{\mu'^2 B'^2}{2\pi^2} f(E)\theta(E_R) \quad (8.1)$$

where  $f(E)$  is a function of energy of the right-handed neutrinos. As it can be understood from the formula, right-handed neutrinos from the left-handed ones are produced via dark magnetic moment-dark magnetic field interaction. This equation gives us the spectral density of the right-handed neutrino luminosity of the supernova core:

$$\frac{dL}{dE} = V \frac{dn}{dE} E \quad (8.2)$$

The function  $dL/dE$  can be sketched for the typical supernova core parameter values:  $T \simeq 30\text{MeV}$ , the neutrino chemical potential  $\tilde{\mu} \simeq 170\text{MeV}$ , the volume  $V \simeq 4 \times 10^{18}\text{cm}^3 = 5 \times 10^{51}\text{MeV}^{-3}$ , the magnetic field  $10^{16}\text{G}$  and the right-handed neutrino dark magnetic moment  $\mu'_\nu = 10^{-11}\mu_B$ . To find the luminosity value in SN 1987A, we integrate

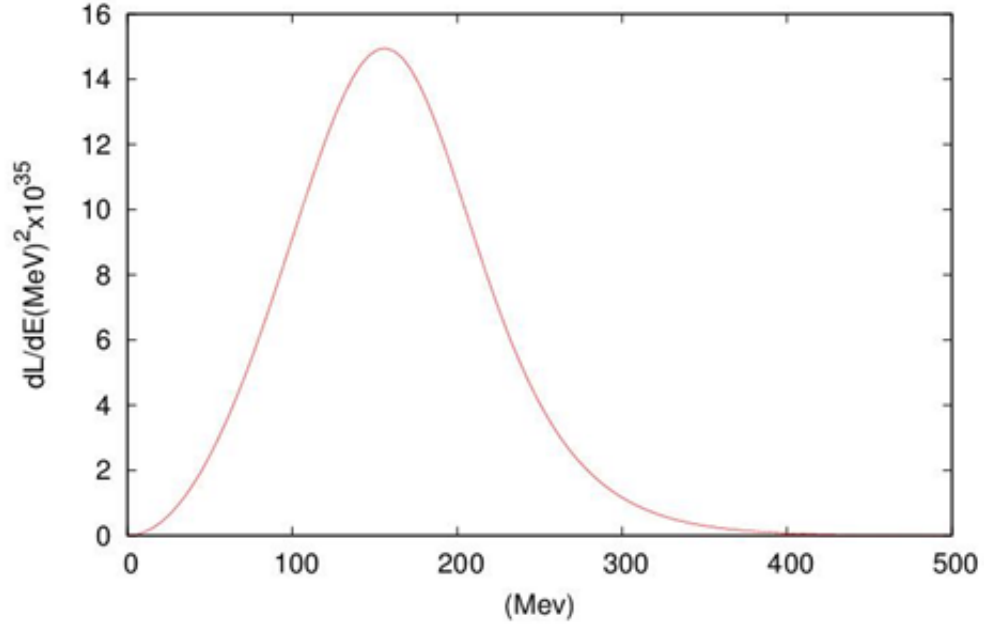


Figure 8.1. The energy spectrum of the right-handed neutrino luminosity for the supernova core parameters.

the spectrum function over energy

$$L = \int V \frac{dn}{dE} E dE \quad (8.3)$$

Even it was observed that energy released per second from SN 1987A is nearly  $10^{51}\text{erg}$ , some theoretical calculations [62] show that iron core ( $M_{core} = 1.4M_\odot$ ,  $R_{core} = 10\text{km}$ ) collapse of SN 1987A can release  $10^{53}\text{erg/s}$ . Since the luminosity which has been calculated depends on the magnetic field and mixing parameter, we can show how does the luminosity change with these parameters for a fixed dark magnetic moment. Since power

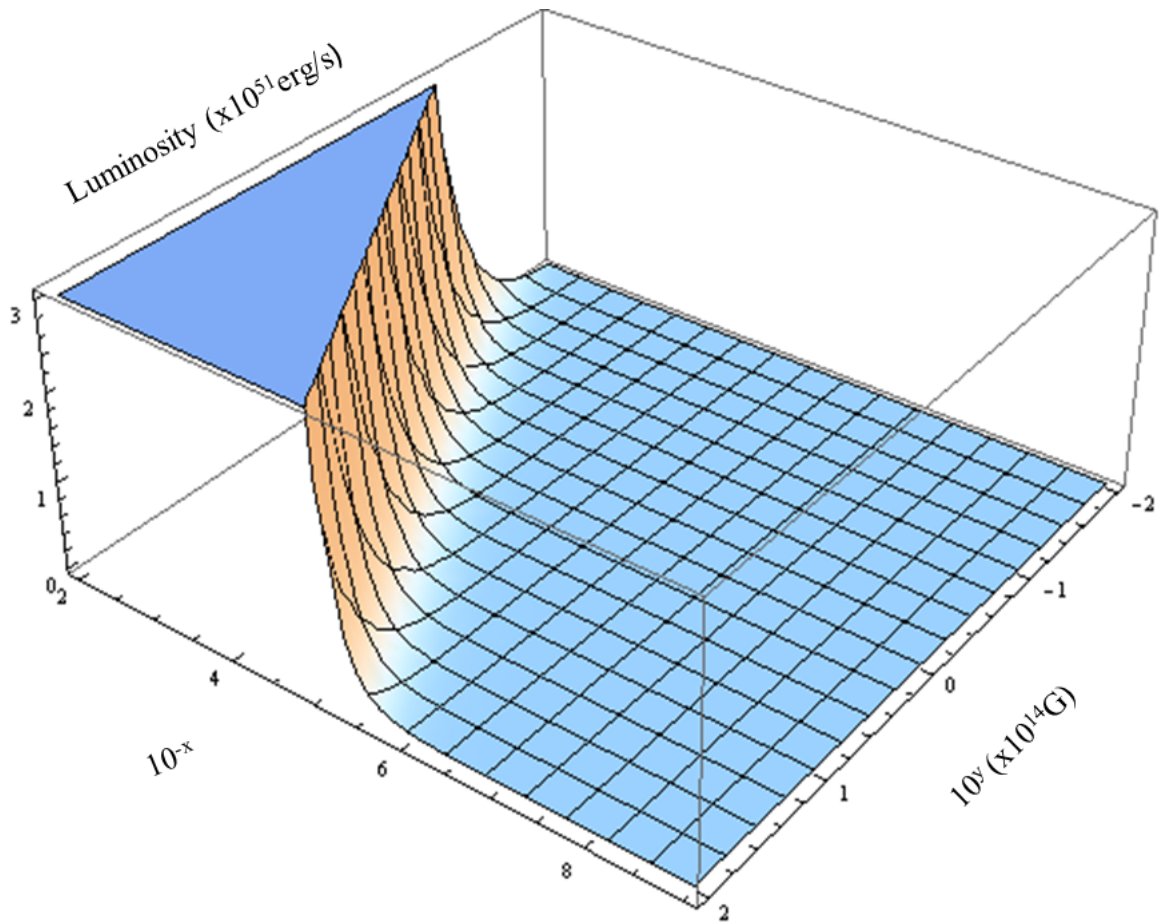


Figure 8.2. Right-handed neutrino Luminosity for the magnetic field values between  $10^{12}\text{G} - 10^{16}\text{G}$  and photon-dark photon mixing parameter ranging from  $10^{-9}$  to  $10^{-2}$ , and the dark magnetic moment is  $10^{-7}\mu_B$

Table 8.1. Neutrino magnetic moment values for different magnetic field values and mixing parameter values

	$10^{-9} \leq \chi \leq 10^{-2}$
$B = 10^{12}G$	$10^{-7}\mu_B \leq \mu' \leq 10^0\mu_B$
$B = 10^{13}G$	$10^{-8}\mu_B \leq \mu' \leq 10^{-1}\mu_B$
$B = 10^{14}G$	$10^{-9}\mu_B \leq \mu' \leq 10^{-2}\mu_B$
$B = 10^{15}G$	$10^{-10}\mu_B \leq \mu' \leq 10^{-3}\mu_B$
$B = 10^{16}G$	$10^{-11}\mu_B \leq \mu' \leq 10^{-4}\mu_B$

radiated from NS 1987A is  $10^{51} \text{erg/s}$ , we can find the values that satisfy this energy release per second.

### **Test of Right-Handed Neutrino Charge :**

The proposition on "milli-charge" of which the right-handed neutrinos may have is that like electrons which are moving in magnetic field larger than the critical magnetic field  $B_{cr} \sim 4.4 \times 10^{13}G$  and situated in the Landau levels, the right-handed neutrinos are situated in the Landau levels in the critical magnetic field. Therefore, right-handed neutrino charge can be found as:

$$q = e \left( \frac{m_\nu}{m_e} \right) \simeq 2 \times 10^{-6} \left( \frac{m_\nu}{eV} \right) \quad (8.4)$$

If mass of the right-handed neutrino  $m_\nu = 1eV$  then its charge is  $q = 2 \times 10^{-6}e$ . This result is exactly consistent with the experimental results in [37] and it means that the mixing parameter  $\chi$  has a value of  $\simeq 10^{-6}$  which is in the experimental interval.

### **Test of Pulsar Kick Velocities :**

As it can be seen from the pulsar kick velocity formula, it depends on several parameters : luminosity, asymmetry, bare mass of the star and the burst duration. However, the most important one is the explosion luminosity which depends on right-handed neutrino dark magnetic moment, photon-dark photon mixing parameter, the magnetic field, temperature, right-handed neutrino mass and neutrino chemical potential. Since change in temperature, neutrino mass and the chemical potential does not effect the kick velocity significantly, we focus on change in the kick velocity while dark magnetic moment, magnetic field and the mixing parameter change. The main goal of this part is to

Table 8.2. Some pulsar kick velocity values for different magnetic field, magnetic moment and mixing parameter values

	$\chi = 10^{-2}$	$\chi = 10^{-3}$	$\chi = 10^{-4}$	$\chi = 10^{-5}$
$10^{12}G, 10^{-6}\mu_B$	$10000km/s$	$100km/s$	—	—
$10^{13}G, 10^{-7}\mu_B$	$10000km/s$	$100km/s$	—	—
$10^{14}G, 10^{-5}\mu_B$	—	—	—	$10000km/s$

find kick velocities on the order of  $1000km/s$  with luminosity of  $10^{51}erg/s$ , the magnetic field ranging from  $10^{12}G$  to  $10^{14}G$  and the mixing parameter taking values in the interval of  $10^{-5} \leq \chi \leq 10^{-2}$ . It is seen from the table that the possible kick velocities which are consistent with the literature can be obtained when the magnetic field takes the values  $10^{12}G \leq B \leq 10^{14}G$ , the dark magnetic moment stays in the interval  $10^{-7}\mu_B \leq \mu' \leq 10^{-5}\mu_B$  and the mixing parameter varies between  $10^{-5}$  and  $10^{-2}$ .

### Discussion :

In this thesis, some new particles (dark photon and right-handed neutrino) are proposed to understand the mechanism underlying behind the supernova events. Due to their very weak interaction with the ordinary matters, they are successful to explain the unsolved problems introduced at the beginning of this thesis because of the tests above. However, this thesis is a preliminary study because we need to examine the other steps related to this study in detail. These steps can be summarized as:

- Properties of dark photons should be studied in detail by the help of Fermi telescope measuring gamma background. Is dark photon massive? if it is, how much does it have? Such questions should be answered under light of the observations.
- The current experiments searching for the particles having "milli-charge" should be followed and an interval for charge and mass of the right-handed neutrinos should be found.
- All the processes determining the production of the left-handed neutrinos should be analysed and the rate of  $\nu_R \rightarrow \nu_L$  conversion until the right-handed ones coming to earth should be studied.
- A similar study for Gamma Ray Bursts (GRBs) should be carried out to test the efficiency of our model and put constraints on the model parameters.

We believe that carrying out these items makes our study more comprehensive and consistent.



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# APPENDIX A

## NOTATION, CONVENTIONS AND TRACE THEOREMS

Relativistic Notation In this thesis study, we use relativistic units ( $\hbar = c = 1$ ) and notations. A general covariant and contravariant four vectors are written as

$$A^\mu = (A^0; A^1, A^2, A^3) = (A^0; \mathbf{A}) \quad (\text{A.1})$$

$$A_\mu = (A_0; \mathbf{A}) \quad (\text{A.2})$$

We use the "Feynman slash" notation

$$\not{p} = \gamma^\mu p_\mu \quad (\text{A.3})$$

The metric tensor we choose

$$g^{\mu\nu} = \text{diag}(1, -1, -1, -1) \quad (\text{A.4})$$

Gamma Matrices Anticommutation relations:

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad (\text{A.5})$$

$$\gamma^5 \equiv g_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \{\gamma^5, \gamma^\mu\} = 0 \quad (\text{A.6})$$

Hermitian conjugates and squares:

$$\gamma^{0\dagger} = \gamma^0, \gamma^{\rho\dagger} = -\gamma^\rho, \gamma^{5\dagger} = \gamma^5, \gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0 \quad (\text{A.7})$$

$$(\gamma^0)^2 = -(\gamma^\rho)^2 = (\gamma^5)^2 = I \quad (\text{A.8})$$

The gamma matrices of Dirac representation:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad (\text{A.9})$$

$I$  is  $2 \times 2$  identity matrix, and the  $2 \times 2$  Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (\text{A.10})$$

which satisfy

$$[\sigma_i, \sigma_j] = 2i\epsilon^{ijk}\sigma_k, \quad \{\sigma_i, \sigma_j\} = 2\delta_{ij}, \quad Tr(\sigma_i\sigma_j) = 2\delta_{ij} \quad (\text{A.11})$$

where  $\epsilon^{ijk}$  is totally antisymmetric;  $\epsilon^{ijk} = \epsilon_{ijk} = 1$  for an even permutation of 1, 2, 3.

## APPENDIX B

### TRACE THEOREMS AND TENSOR CONTRACTIONS

$$\text{Tr}(I) = 4 \quad , \quad \text{Tr}(\gamma_\mu) = 0 \quad , \quad \text{Tr}(\text{odd number of } \gamma) = 0 \quad (\text{B.1})$$

$$\text{Tr}(\gamma_\mu \gamma_\nu) = 4g_{\mu\nu} \quad , \quad \text{Tr}(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma) = 4[g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}] \quad (\text{B.2})$$

Dirac Spinors Positive energy spinor  $u(p)$  and its adjoint spinor  $\bar{u}(p) = u^\dagger(p)\gamma^0$  satisfy the equations

$$(\not{p} - m)u(p) = 0 \quad , \quad \bar{u}(p)(\not{p} - m) = 0 \quad (\text{B.3})$$

Negative energy spinor  $v(p)$  and its adjoint spinor  $\bar{v}(p) = v^\dagger(p)\gamma^0$  satisfy the equations

$$(\not{p} + m)v(p) = 0 \quad , \quad \bar{v}(p)(\not{p} + m) = 0 \quad (\text{B.4})$$

Projection operators

$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p} + m \quad (\text{B.5})$$

$$\sum_\lambda v_\lambda(p)\bar{v}_\lambda(p) = \not{p} - m \quad (\text{B.6})$$