A METHOD TO DESIGN KINETIC PLANAR SURFACE WITH MATHEMATICAL TESSELLATION TECHNIQUES

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Aylin GAZİ

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We approve the thesis of **Aylin GAZİ**

Assist. Prof. Dr. Koray KORKMAZ
Supervisor

Prof. Dr. Rasim ALİZADE
Committee Member

Prof. Dr. Oktay PASHAEV
Committee Member

16 December 2010

Assoc.Prof. Dr. Serdar KALE
Head of the Department of Architecture

Prof. Dr. Sedat AKKURT
Dean of the Graduate School of Engineering of Sciences

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ABSTRACT

A METHOD TO DESIGN KINETIC PLANAR SURFACES WITH MATHEMATICAL TESSELLATION TECHNIQUES

Due to rapid change in activities on modern society in XX^{th} century, need of adaptation has emerged, which was the necessary precondition for the rise of the concept of motion or kinetic in architecture. Kinetic architecture is a controversial interdisciplinary area between architecture and mechanisms science. Many kinetic designers and researchers usually reach transformable, deployable or foldable structures by using mechanical knowledge. However, there are not many researches that focus on the surfaces between kinetic structures. Those surfaces generally covered with flexible or flat materials. Kinetic architects, who usually deal with a particular type of the mechanism, can easily control the design of mechanism. Therefore, a method is necessary to construct a network with planar mechanisms for variable building surfaces due to the fact that it can be a problem of studying during the design process of kinetic building parts. Many questions might be a problem such as how many links should be used, what kind of joints and platform should be chosen and finally the mobility of the whole kinetic system.

To design a surface has been one of the major problems for architects. Through the history, architecture has benefited from mathematics such as golden ratio, fractal geometry and tessellation. Tessellation is a kind of mathematical technique that was usually used to cover a plane without any gaps or overlaps, because of this properties, it uses to design surfaces. So, the main purpose of this study is to develop a methodology to design kinetic planar surfaces with mathematical regular tessellation technique in the light of architectural, mechanical and mathematical interdisciplinary approach.

ÖZET

HAREKETLİ DÜZLEM YÜZEYLERİN TASARIMI İÇİN MATEMATİKSEL TESSELLATİON TEKNİKLERİ İLE BİR METOD

Mimarlık ve sanat tarih boyunca matematiği tasarım aracı olarak kullanmış, tasarım süresince estetik ve sağlamlığı matematiksel dil ile oluşturduğu metot ve tekniklerle sağlamıştır. Antik dönemde geliştirilen altın oran, dönemin estetik ve oran kaygısına cevap aramaya çalışmış, modern mimarlıkta ise matematik daha sağlam ve karmaşık strüktürlerin bir araya getirilmesi için bir araç olmuştur. Tessellation mimarlık ve sanatta kullanılan matematiksel tekniklerden biridir. En temel anlamıyla tessellation bir yüzeyin hiç boşluk bırakılmadan belli bir doku ve ritim ile kaplanması halidir. Antik dönem mimarlığı, estetik ve oran kaygısı içinde yer alırken, 20. yüzyıl mimarlığı ise değişen yaşam koşullarıyla başka arayışları içine girmek zorunda kalmıştır. 20. Yüzyılın modern dünyası kendini yaşamın hızına ve değişen iklimsel ve çevre koşullarına adapte etmek zorunluluğu içinde bulmuş, mimarlık ise bu süreç içinde kendine yeni bir dil geliştirme ihtiyacı edinmiştir, bu yeni dil kinetik mimarlıktır. Kinetik mimarlık her ne kadar uygulama aşamasında ve akademik çerçevede yeni olsa da, hızla gelişmekte olan mimariyi ve mekanizma bilimini birleştiren disiplinler arası bir alandır.. Kinetik tasarımcılar ve araştırmacılar bugüne kadar genellikle yapıların ana strüktürlerini hareketli hale getirmeye çalışmış bu yüzden belli sayıda mekanizmalarla tasarımlarını kolaylıkla kontrol altına alabilmişlerdir. Peki, tasarlanmak istenilen kinetik bir strüktür değil de, büyüklüğü ve formu her tasarıma göre değişen bir yüzey olursa kinetik mimarlık buna nasıl bir cevap arayabilir? Tasarlanması düşünülen bu yüzeyde kaç tane ve hangi formda link ve platformların ve nasıl bir araya geleceği en önemli sorun haline dönüşür ve bu aşamada bitmiş bir tasarımdansa, kinetik mimarlık bir metot ihtiyacı duyar. Bu tezin amacı antik dönemden beri kurulan matematik ve mimarlık ilişkini, kinetik mimaride tekrar irdeleyerek matematik, mimarlık ve mekanizma disiplinlerini birleştiren, matematiksel tessellation tekniği ile oluşturulmuş düzlemsel yüzeylerin, hareketli haline getiren bir metot geliştirmektir.

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CHAPTER 1

INTRODUCTION

1.1. Definition of the Study

Geometry plays a central role in both the design and the analysis of buildings with architectural and engineering aspects at the same time. Throughout the history, many architects and artists have been benefited from some mathematical and geometrical knowledge such as the golden ratio, fractal geometry and tessellation. As it can be seen from artworks in the history, Leonardo Vinci, Albert Dürer, M.S. Escher, Luca Pacioli created a lot of art with the mathematical precision. Liapi (2001) claims that complex and sophisticated knowledge and use of geometry, underlines the conception, design and construction of the most significant achievements in the history of buildings. In the ancient periods, proportioning systems were the most essential elements of visual order and aesthetics, thus architects and artists benefited from the golden ratio in their works. Moreover, in the XXth century, many architects such as Renzo Piona, and Santioga Calatrava have developed their designs by using the relationship between geometry and structure.

Designing the surface has been one of the major problems throughout the history in architecture. For instance, facades of the buildings are always important because they emphasize the identity of the building. Due to this, all architectural styles create their own concept on the facade of the building, by benefiting from the mathematical knowledge. Renaissance uses proportional system on the façade of the building while baroque use façade as a decoration tool. And Islamic architecture develops ornamental style on the facade. This research deals with the surface design and restricted by planar surfaces, so it chooses the tessellation mathematical technique.

Tessellation is a common mathematical technique that is used to cover a plane without any gaps or overlaps. Due to this property of the technique, it has been usually used on the planar surfaces such as facade of buildings. In the literature, tessellation can also be referred as “tiling”. Generally, a tiling is just a way of fitting the pattern together without any gaps or overlaps. However tessellation is a complex mathematical technique; in every day life tessellation can be frequently seen in its simplest forms (Figure 1.1, 1.2, 1.3, 1.4). Tessellation technique has been used in ceiling, wall, floor or facade of the buildings beginning from the first primitive areas, but it reached the highest level with Islamic architecture.

Figure 1.1. Farm fields
(Source: Cornell, 2010)

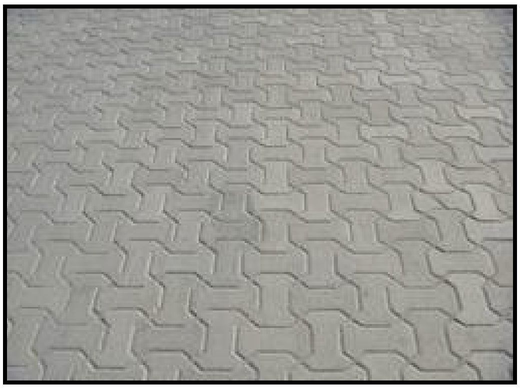


Figure 1.2. Cement Road
(Source: Cornell, 2010)

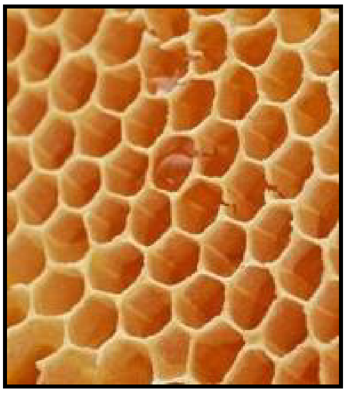


Figure 1.3 Honeycomb
(Source: Cornell, 2010)

Figure 1.4. Pavement
(Source: Cornell, 2010)

When architecture has constituted a relationship between mathematics, it has traditionally been regarded to permanence. All methods and techniques are considered in static state. Zuk and Clark (1970) point out in their book that architecture has usually been perceived as enduring, permanent structure. To achieve this permanence, architecture has searched materials and structural system to reach more stable buildings. On the contrary to this idea, modern life has been affected by the rapidly changing life conditions, and architecture has to find a solution to adapt this living style. For instance, in today's life many buildings give negative impact to the economy and environment due to consuming much more natural and human resources. Meanwhile, due to the rapid change in the activities of modern society and the development in building technology, building parts are in need of change in terms of shape and form to adapt different functionalities and weather conditions to reach positive impact on the economy and environment. As a result, architecture has developed a new aspect that is called kinetic architecture.

Kinetic architecture is an interdisciplinary study between architecture and mechanism science. Adaptability and responsibility are the most important key parameters for kinetic architecture. Kinetic aspect of the architecture mainly suggests a new concept of form with the kinetic parts that are capable of being transformable, deployable or foldable and with mechanical components that can be set in motion.

In kinetic architecture, many researchers deal with a particular type of mechanisms (Gazi and Korkmaz 2010). For instance, when kinetic shelter is designed, the part of the main structure, type of mechanism and number of joints and links can be seen easily. And after this design processes, if the surfaces among the kinetic structures want to be covered, because of the environmental or esthetic problems, usually membranes or elastic materials are used. On the other hand, if the facade of the building is kinetic, the surfaces among the structure should be kinetic too. Thus, kinetic planar surfaces gain importance.

As mentioned above, many methods and techniques that are related with mathematics are restricted with static architecture. This relationship should be reconsidered by using kinetic aspect. In this step the process of mechanism design have to be examined too. Mechanism science considers the design processes in mainly two parts as, analysis and synthesis of mechanism, and uses the mathematical knowledge especially analysis part.

During the design process of kinetic building parts (such as kinetic surface) many questions might be a problem of studying such as how many links should be used, what kind of joints and platforms should be chosen and finally the mobility of the whole kinetic system. In fact those questions are tried to be answered by structural synthesis of mechanism on the design processes of mechanism In the cases of answering these questions, architects need a method or a tool that is currently lacking in the literature when designing kinetic systems.

In the light of this introduction, the present investigation will focus on developing a methodology to design planar surfaces by using mathematical tessellation methods and considering kinetic aspects.

1.2. Aim of the Study

The aim of this study is to develop a method to design kinetic regular and irregular planar surfaces with mathematical tessellation techniques in the light of architectural, mechanical and mathematical interdisciplinary approach.

1.3. Method of the Study

As stated before, the main purpose of this study is to develop a method by considering mathematical tessellation technique to design kinetic planar surfaces. Within the scope of this aim, this study examines the kinetic tessellation in the following order that consists of four steps successively building upon each other.

1-Literature review of the usage of tessellation on planar surfaces in architecture

2-Basic principles and classification of tessellation technique

3- Basic principle of planar mechanisms

4-Devoloping a method for the design of kinetic planar surfaces

In the first step this study displays the usage of tessellation in the three different periods of architecture; as, an ancient period, Islamic architecture period and contemporary architecture period by considering the literature. The purpose of the first step is to display and discuss the usage of tessellation on planar surfaces in architecture. So that it will be easy to understand the way the architects benefited by tessellation to improve their design ability.

In connection with presenting the usage of tessellation on the planar surfaces in architecture in chapter 2, the research focuses on mathematical aspects of tessellation in chapter 3. In this frame, this chapter firstly presents brief information about the meaning of tessellation then will be focused on importance of the symmetry to the tessellation technique. In the light of this, the background of tessellation will be seen by the fundamental properties of tessellation. This chapter has been followed by the classification of the tessellation. The classification will constitute a basis for the iteration way and polygonal shape. This part of the chapter is important to understand the selection of the platform that can be used and the way of their iteration.

The study carried out so far focuses on developing an understanding and description of the kinetic motion. This research has been restricted to the planar surfaces, thus chapter 4 firstly presents brief information about planar mechanisms and carried out by description of the motion of linkages. The discussion has then been followed by processes of mechanisms design. In this step the concept of design of mechanisms will be tried to be understood and will be focus on structural synthesis of mechanism.

The forth step comprises the presentation phase of the case study. Chapter 5 kinetic regular tessellation has been developed. This chapter has been divided into two parts. The first part of the chapter deals with the method that considers regular surfaces and benefited from dual of tessellation. Second step of chapter 5, tries to find a way to adopt this method application to irregular surfaces. In this frame this chapter deals with the general concept of the fractal geometry to cover irregular planar surfaces with regular platforms.

In the processes of developing a method, some computer programs are used in simulations of kinetic tessellation. The form of the kinetic tessellation has been designed by the Mechanical Desktop program. Moreover, the performance of the three regular tessellations is analyzed by Visual Nastran Desktop 4D which is capable of kinematic analysis. Lastly, the mobility of the mechanisms is calculated by using various mobility criterions.

CHAPTER 2

TESSELLATION ON PLANER SURFACE IN ARCHITECTURE

Designing the surface has been one of the major problems for both architects and artists. Throughout the history, architects and artists have benefited from some mathematical methods or techniques in their works such as tessellation. Tessellation technique has reached its top level with the ornamental art and it has been developed in many different areas. This chapter tries to answer the question of how the architects have contributed to the tessellation technique in their designs of planar surfaces, by displaying its usage in the ancient period architecture, Islamic architecture and contemporary architecture.

The art of tessellation have originated early in the history of civilization. When people began to build their houses they tried to fill spaces or planes. Architecture has used the tessellation as a technique on planar surfaces. For example, "Mediterranean peoples concerned with the portraying human beings and natural scenes in intricate mosaics" (Grünbaum and Shephard, 1986). In addition to usage of tessellation technique in mosaics, over the centuries, architects have utilized it in many different areas; for instance, they have decorated floors, walls and ceilings or glazed clay tiles.

2.1. Ancient Period and Tessellation

Over the centuries, architecture has been using tessellation in many different areas. The first houses, churches and castles were built by using broken stones which construct random tessellations. Later, arrangements of prismatic stones and bricks possessed a large regularity.

Through the history, every society has used the tessellation in various forms. However, some of the tessellation examples are universal however; many of them are characteristic to particular people and tribes. Grünbaum and Shephard (1986) point out that various cultures seem to have emphasized different aspects; Mediterranean and

Roman society concerned with portraying human beings and natural sciences; however, Arabs and Moors reflect the complex geometrical patterns on their tessellation. Tessellation technique has been used in variety of different areas on plane. One of the early usages of the tessellation was seen in Iran, Nishapur as a tile panel of the 10th century. Moreover, in the 13-14th centuries the tile panel with the star-cross pattern was another popular tessellation that uses eight-pointed-stars, many of which include a calligraphic border of Persian poetry. Also in that century ceramic tiles provided a perfect material for creating tessellated patterns that could cover entire walls or even buildings which is called molded tile panel. At the Ottoman period (mid-16th century), tessellation technique has been used on the glazed tile panels. In this case, the tiles have been individually painted rather than molded with a design (The Metropolitan Museum of Art, 2004).

Jones Owen (1910) points out in his book that “the formation of patterns by the equal division of similar lines, as by weaving, would give to a rising people the first notions of symmetry, arrangement, disposition, and the distribution of masses”. Moreover, the Egyptian tiling uses as principal three main colours; red, blue and yellow with black and white to define them.

Some of the tessellations from different societies at the ancient period are illustrated in Figure 2.1-2.6.

Figure 2.1. Egyptian tessellation
(Source: SPSU, 2009)

Figure 2.2. Byzantine tessellation
(Source: SPSU, 2009)

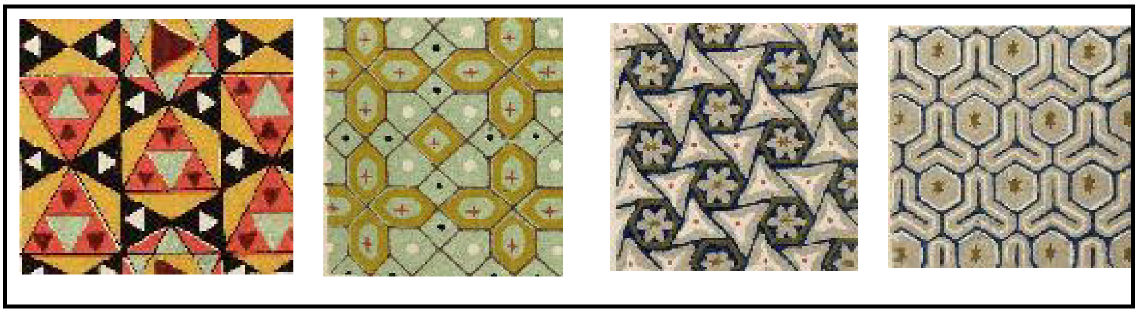


Figure 2.3. Persian tessellation
(Source: SPSU, 2009)

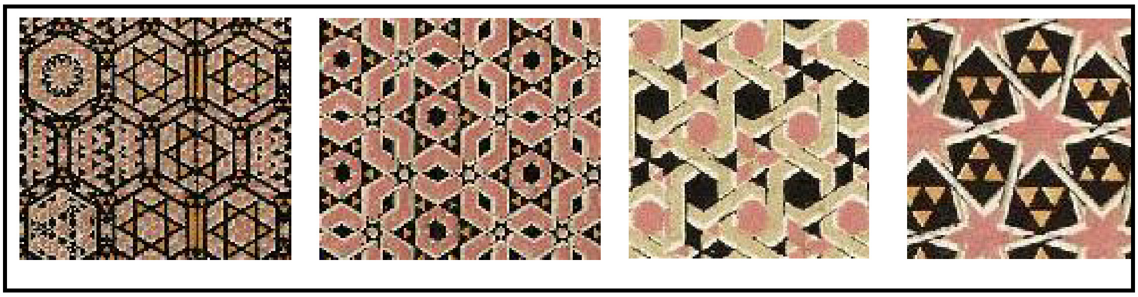


Figure 2.4. Arabian tessellation
(Source: SPSU, 2009)

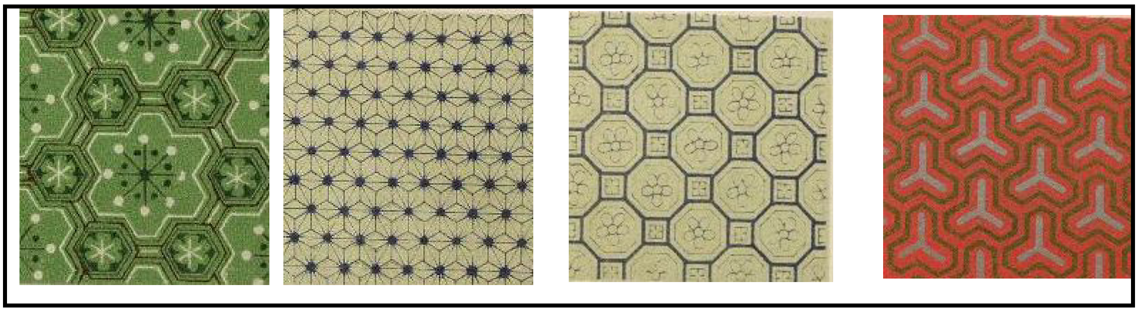


Figure 2.5. Chinese tessellation
(Source: SPSU, 2009)

Figure 2.6. Renaissance tessellation
(Source: SPSU, 2009)

And finally, Morosque tessellation is be displayed in Figure 2.7. Tessellation technique reached its best level with the Islamic architecture, and this tessellation type will be examined deeply in the second part of this chapter.

Figure 2.7. Morosque tessellation
(Source: SPSU, 2009)

As it can be seen above, although there are many tessellations from different societies which are widely separated in time and places, they are often similar to each other. From this point out view, many researchers try to find the main source of this relationship. Carlini and Conversana are one of the researchers who deal with planar tessellation from the ancient period. In their study, they reconstruct the original aspect of the floors of the tabernae in hemicycle of the Trajan Markets in Rome, Italy (Figure 2.8). To reconstruct something, the systematic or typology must be known.

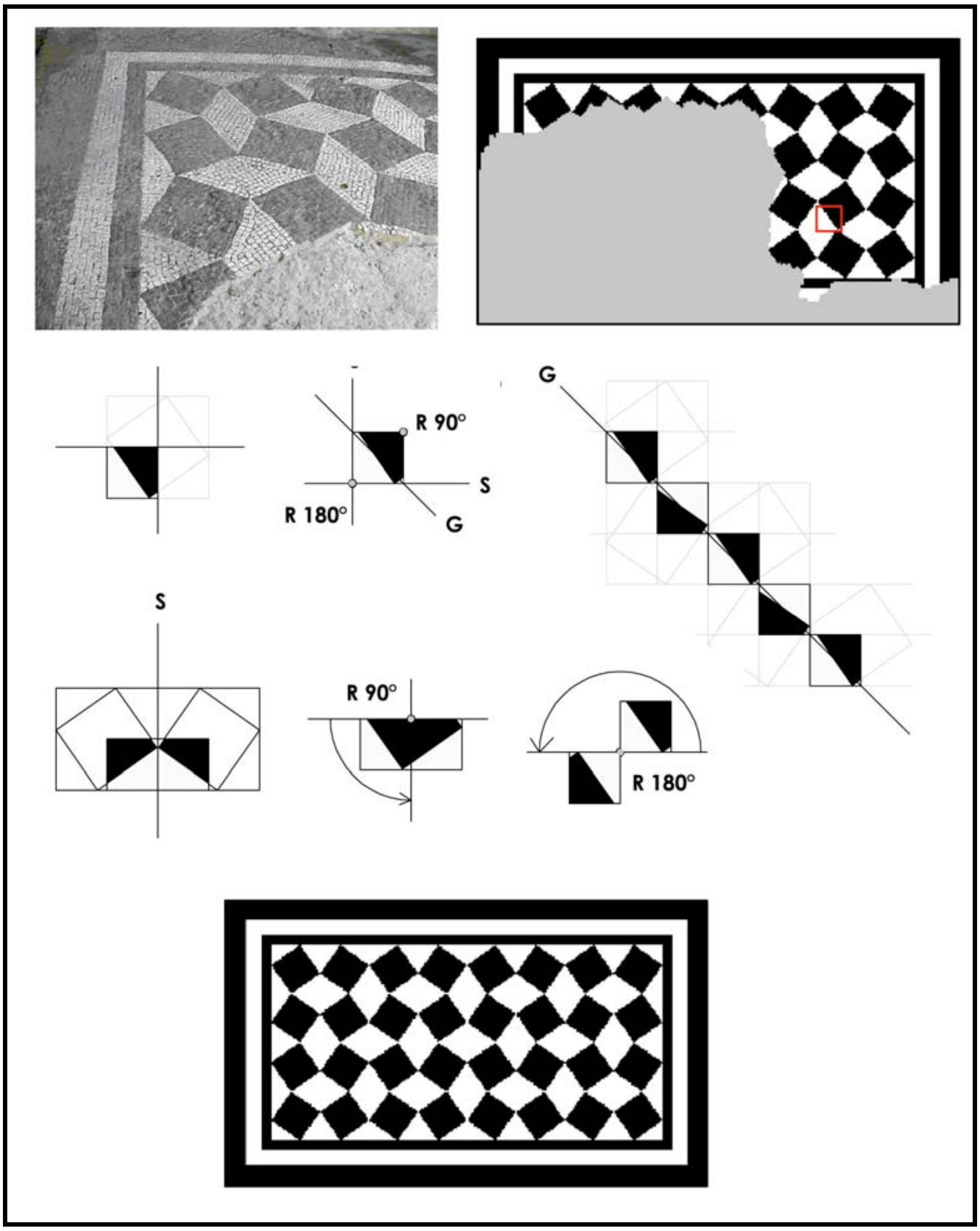


Figure 2.8. Reconstruction of Taberna
(Source: Carlini et al., 2008)

As it can be seen from this example, many tessellations from different societies use geometrical properties, especially symmetry. For this reason many of them are similar to each other. To understand that relationship, symmetry will be examined deeply with the mathematical properties on Chapter 3.

Sure as death, the most important usage of the tessellation on the plane has been seen in the art of the ornament. Medieval Islamic artisans developed intricate geometric tessellation to decorate their mosques, mausoleums, and shrines.

2.2. Islamic Architecture and Tessellation

Islamic artisans have developed very rich system of intricate ornamentation that followed the spread of Islamic culture into Africa, Europe, and Asia to adorn architectural surfaces with geometric patterns (Figure 2.9, 2.10).

Figure 2.9. Alhambra Tessellation
(Source: Flickr, 2010)

Figure 2.10. Taj Mahal Tessellation
(Source: Gettyimages, 2010)

The Islamic world has a rich heritage of incorporating geometry in the construction of intricate designs that appear in architecture. The most well-known usage of the tessellation can be seen in the Alhambra at Granada.

Islamic tessellation from the Alhambra falls into two major categories in a mathematical manner. These are radial patterns (Finite Symmetry Groups) (Figure 2.11) and periodic patterns (Infinite symmetry group) (Figure 2.12). Radial patterns may have rotations about the central point with the total number of symmetries being finite. These types of patterns do not have any translations or glide reflections (Tennant, 2004). On the contrary to this, periodic patterns contain translational symmetries in two independent directions (Tennant, 2004).

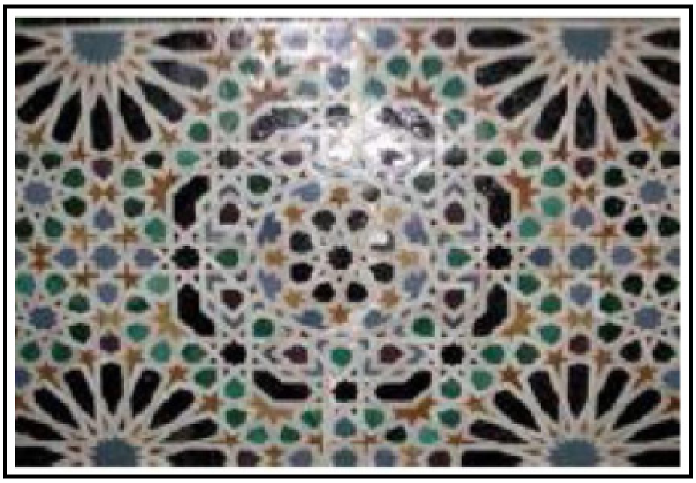


Figure 2.11. Radial tiling from the Alhambra
(Source: Tennant, 2004)

Figure 2.12. Periodic tiling from the Alhambra
(Source: Tennant, 2004)

Ornamental arts in the Islamic architecture has developed themselves and improved some methodology and restrict patterns by considering many mathematic tools which will be mentioned in next chapter. Girih patterns and star patterns are two of the important landmarks for ornamental art in the Islamic architecture.

2.2.1. Girih Patterns

Girih patterns constitute a decorative idiom through Islamic architecture. Girih tiles are a set of five tiles (Figure 2.13) that are used in the creation of tiling patterns for decoration of buildings. The five shapes of the tiles are regular decagon, an elongated, a bow tie, a rhombus and a pentagon. “The five girih tiles share several geometric features because every edge of each polygon has the same length, and two decorating lines intersect the midpoint of every edge at 72° and 108° angles. This ensures that when the edges of two tiles are aligned in a tessellation, decorating lines will continue across the common boundary without changing direction” (Lu and Steinhardt, 2007).

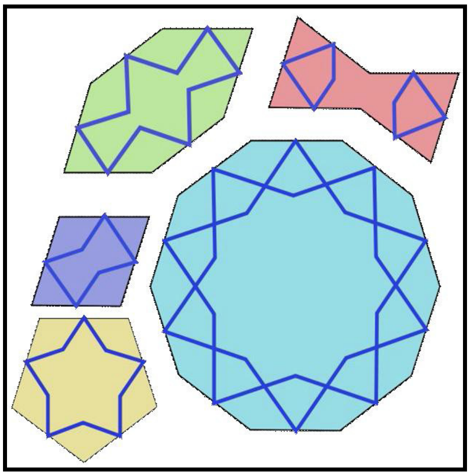


Figure 2.13. Girih Tiles
(Source: Wikipedia, 2009)

According to Tennants (2008) study “The girih tiles themselves are not part of the final pattern but rather the line decoration on the girih tiles determine the design”. A reconstruction of the process of transformation from the girih tiles to the architectural design is illustrated on the 15th Century Timurid Shrine (Figure 2.14). The spandrel tiling from 13th Century Iraq (Figure 2.15) is shown along side the associated girih tile pattern.

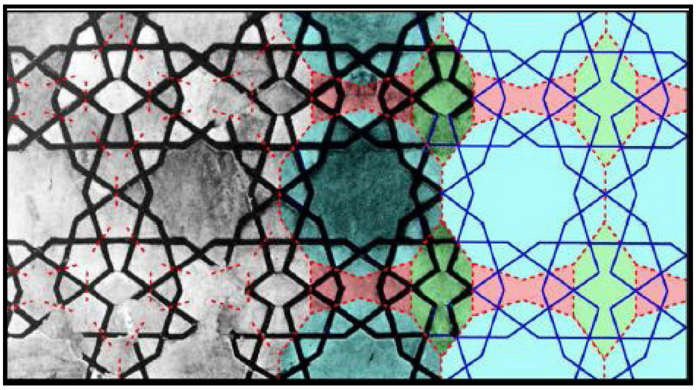


Figure 2.14. Periodic tiling with actual tiling (left) transformed to girih tiling (right), Timurid shrine of Khwaja Abdullah Ansari at Gazargah in Herat, Afghanistan (1425-1429 AD) (Source: Tennant, 2008)

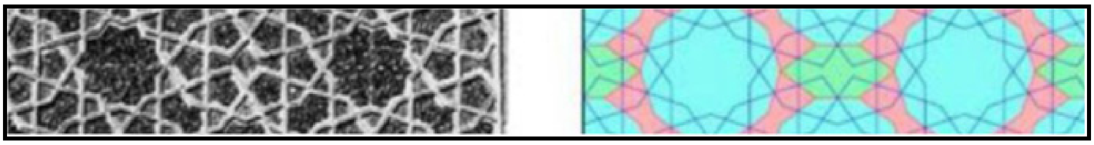


Figure 2.15. Spandrel from the Abbasid Al-Mustansiriyya Madrasa, Baghdad, Iraq (1227-34 AD, (left), and with Girih Tiles (right) (Source: Tennant, 2008)

Girih patterns were constructed by drafting directly a network of zigzagging lines that is also called strapwork, with the use of compass and straightedge. As it can be seen in the examples of the Alhambra Palace, strapwork method circles and squares are transformed into stars and overlapping lattices to form a more intricate symmetric pattern (Figure 2.15) (Tennant, 2008).

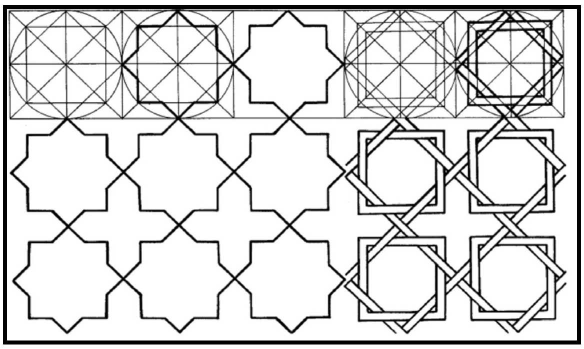


Figure 2.15. Strapwork method showing construction from circles to lines to stars to overlapping lattices (Source: Tenant,2008)

Most of the usage of girih tiles in Islamic architecture was periodic; they had unit cells that were repeated in the same orientation within a lattice. "These tiles enabled the creation of increasingly complex periodic girih patterns, and by the 15th century, the tessellation approach was combined with self-similar transformations to construct nearly perfect quasi-crystalline Penrose patterns, five centuries before their discovery in the West" (Lu and Steinhardt, 2007). The most important example for this type of ornament is the Darb-i Imam Shrine (1453 AD) in Isfahan. As it can be seen in Figure 2.15 and 2.16, the large, thick black line pattern consisting of a handful of decagons and bowties is subdivided in to smaller pattern that can also be generated by a tessellation.

Figure 2.15. Right half of the spandrel
(Source: Lu and Steinhardt, 2007)

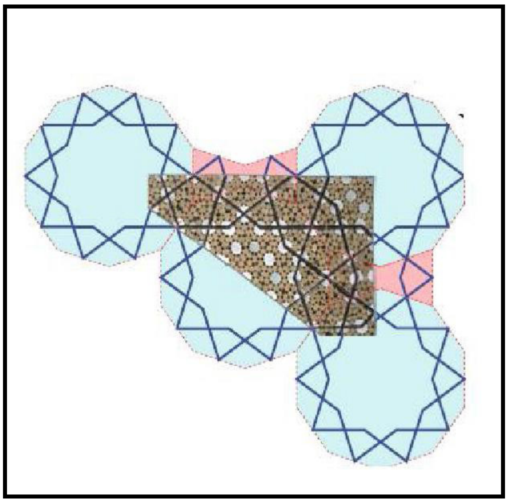


Figure 2.16. Reconstruction of the larger-scale thick line pattern with larger girih tiles, overlaid on the building photograph (Source: Lu and Steinhardt, 2007)

"Tessellating these girih tiles provides several practical advantages over the direct strapwork method, allowing simpler, faster, and more accurate execution by artisans that are unfamiliar with their mathematical properties. A few full-size girih tiles could serve as templates to help position decorating lines on a building surface, allowing rapid, exact pattern generation" (Lu and Steinhardt, 2007).

2.2.1. Star Patterns

Star Patterns are one of the most advanced and specialized features of Islamic Architecture (Figure 2.17, 2.18). These are very complex patterns, but there is very little information about how Islamic patterns were originally devised. In the literature there are few studies and methods try to understand star patterns, but before explaining some of them, psychology behind the symmetric star-shaped should be explained.

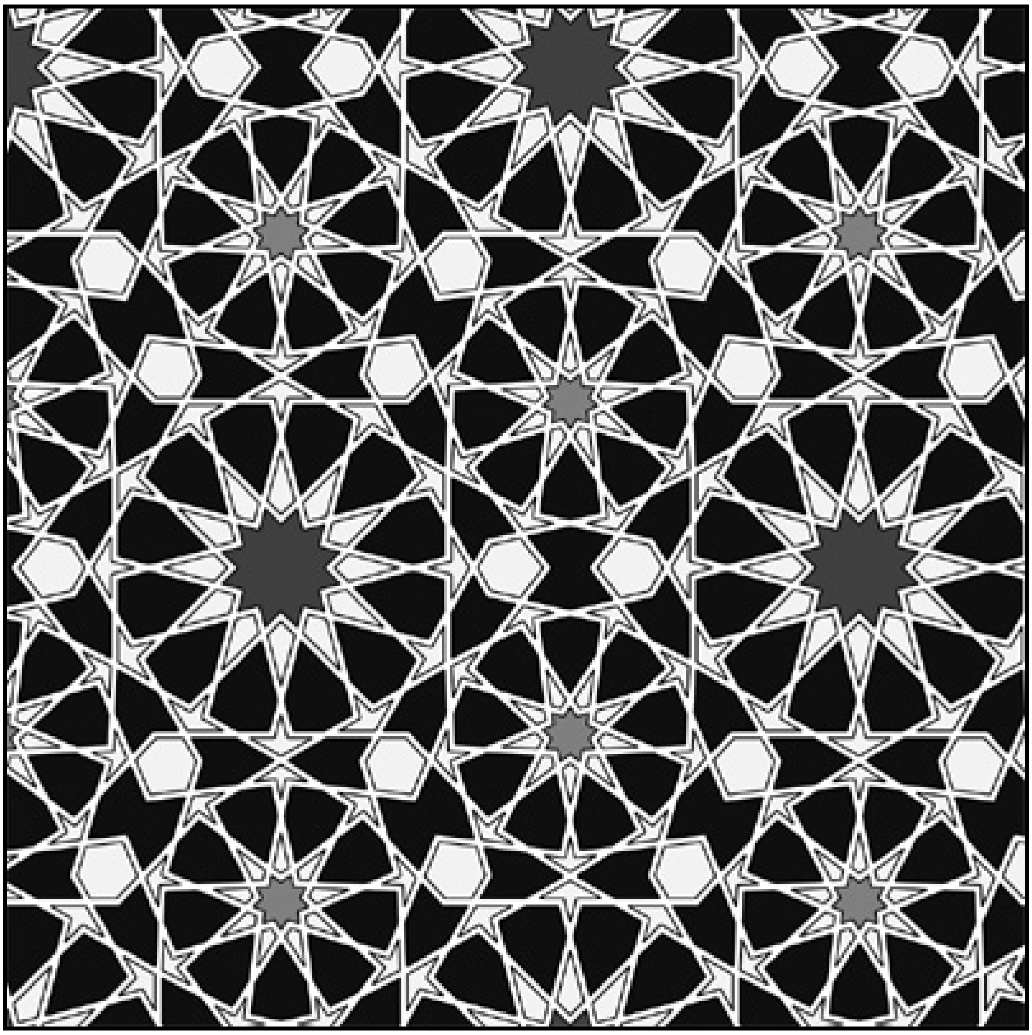


Figure 2.17. Star patterns
(Source: Modelab, 2010)

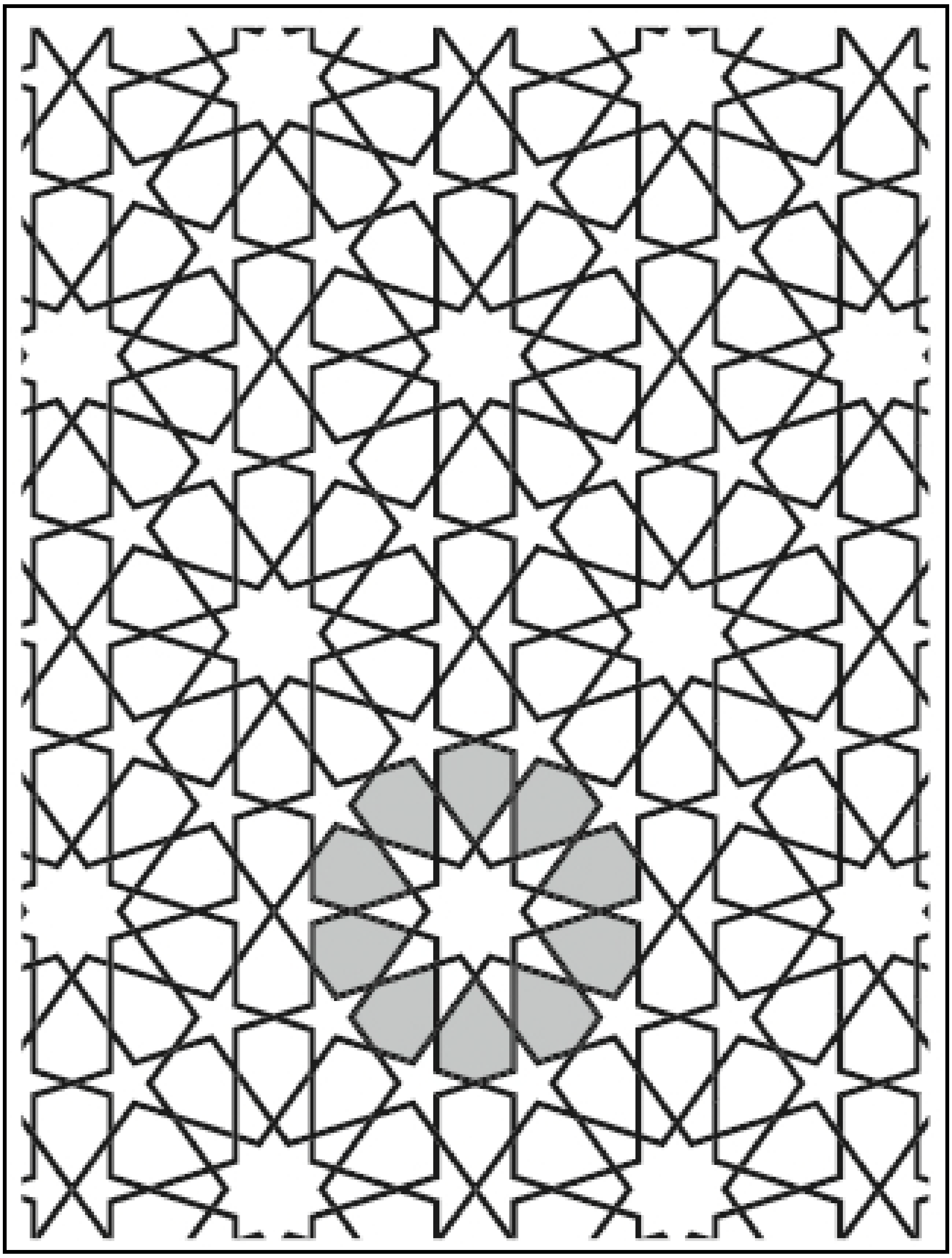


Figure 2.18. Star pattern
(Source: UCR, 2010)

There are many assumptions about why the star patterns were created in Islamic architecture. But the most common one is explained by Abas: he said that “The first thing that needs to be said to challenge this simplistic repetition is that there is a vast body of figurative work by Muslim artists. Everyone has seen examples of Persian miniatures, but apart from these, there exist a large number of realistic life-like pictures of humans as well as animals executed with great virtuosity and naturalism by Muslim artists” (Abas, 2001).

It is still a mystery how the star polygons were constructed. Many researchers developed some methods to construct and reach the star polygon. Kaplan use a method that based on Hankin's polygon in contact technique. In addition to this Aljamali and Banissi (2003) based on their method that minimum number of grids and lowest geometric shape (Figure 2.19, 2.20).

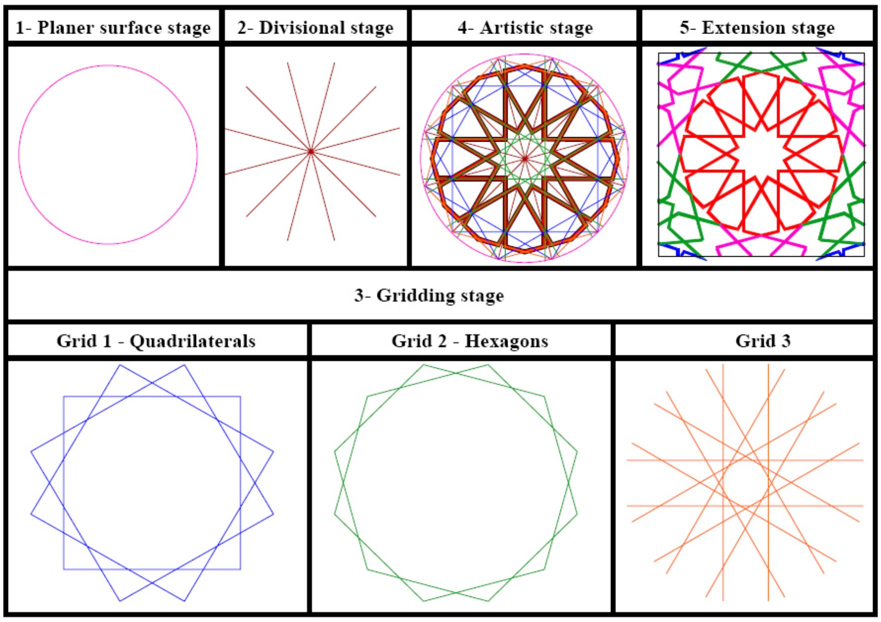


Figure 2.19. 12-rayed Star pattern within the given unit pattern
(Source: Aljamali and Banissi, 2003)

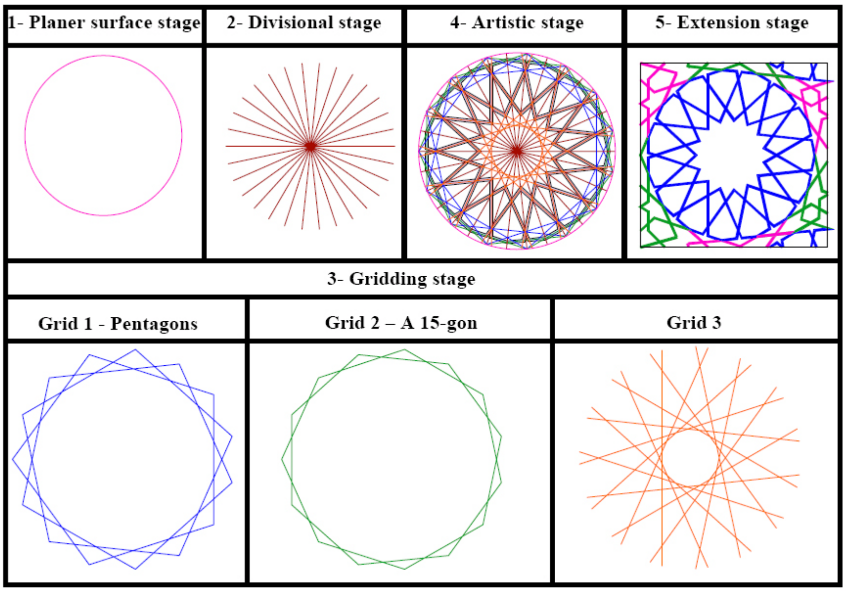


Figure 2.20. 15-rayed Star pattern within the given unit pattern
(Source: Aljamali and Banissi, 2003)

However, tessellation techniques reach the highest level with the ornamental art in Islamic architecture, this technique haven’t been use extensively after this period. Yet, it can rarely be seen in XX. century architecture that tries to use this technique on the planar surfaces especially facade of the building.

2.3. Contemporary Applications of Tessellation

Form and function are the fundamental key parameters for modern architecture. In 20th century architects and artists usually ignore the mathematical method when they consider those parameters. Nevertheless, it can be seen on a few new examples which deal with the tessellation technique.

Through the history both the ancient tessellation and Islamic tessellation examples, generally were periodic which were based on filling of space with regular polygons and patterns. These early attempts are usually considered with 2-D dimensional surfaces and have special meanings to form the culture. Contrary to this, contemporary attempts of tessellation try to constitute parts to whole relationship and it makes the tessellation technique effective today in terms of technical properties (Farshid, 2010)

This part of the chapter aims to examine the usage of tessellation in contemporary architecture with two basic categories of two dimensional and three dimensional applications.

2.3.1. 2-D Dimensional Applications of Tessellation

2-D dimensional application of tessellation has been generally seen on floor, wall or ceiling of the buildings. As it can be seen in Figure 2.21 and 2.22 many designs use regular tessellation and iterate them with periodic way. In addition to utilization of tessellation on floors or walls, architects and urban planners benefited from tessellation technique while designing buildings even in cities.

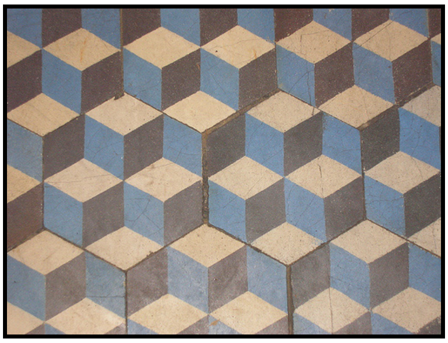


Figure 2.21. Tessellation on the floor
(Source: Flickriver, 2009)

Figure 2.23 display the a hexagonal plan for the New York City that was prepared by Charles Lamp in 1904 and Figure 2.24 shows hexagonal villa plan, designed by Wilhelm Ulrich in 1927. Both of them benefited from regular tessellation as a concept of their design process. (Joseph and Gordon, 2000).

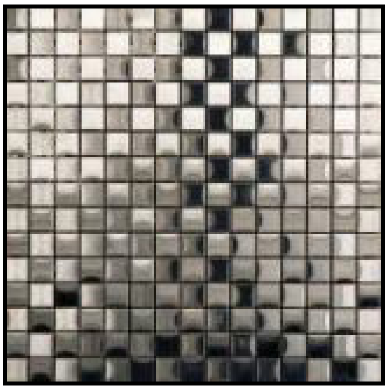


Figure 2.22. Metal wall tessellation
(Source: Bizrate, 2010)

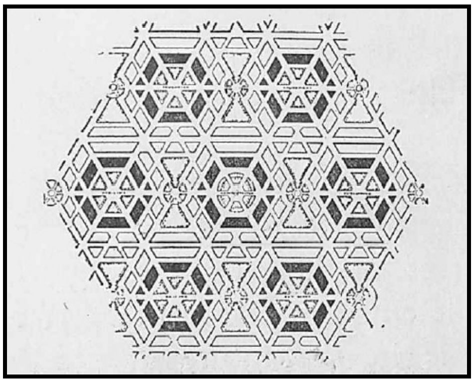


Figure 2.23. Hexagonal city plan
(Source: Joseph and Gordon, 2010)

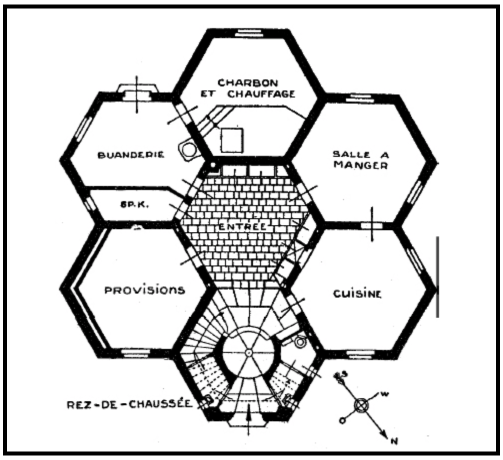


Figure 2.24. Hexagonal villa plan
(Source: Joseph and Gordon, 2010)

Moreover there are some rarely usages of tessellation in architecture, in the 20th century. Tessellation again began to gain importance in monumental buildings. One of the famous examples of 2-D applications of tessellation has been seen in Yale Art Gallery Building ceiling (Figure 2.25). This building was designed by one of the important modern architects Louis Kahn at 1953. As it can be seen in figure, the ceiling of the building has benefited from regular triangular tessellation in terms of horizontal modulations.

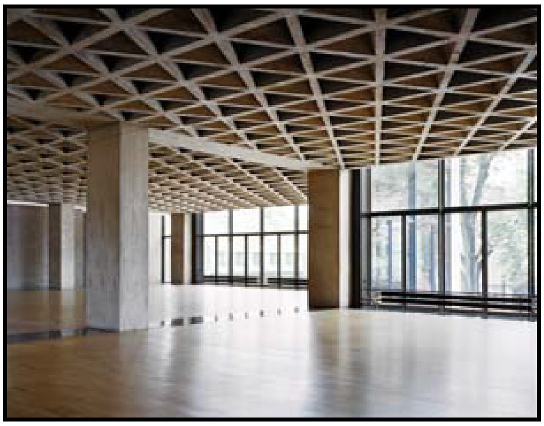


Figure 2.25. Yale art gallery
(Source: Aiany, 2010)

London Swiss Re Building (Figure 2.26) is another example of 2-D applications of tessellation. This building is designed by Norman Foster in 2004. The construction system of the building is glass cladding that benefited from vertical triangular tessellation.

Figure 2.26. London swiss re building
(Source: Greatbuilding, 2010)

Another example for the 2-D vertical tessellation application is Federation Square (Figure 2.27, 2.28). This building is constructed by Lab Architecture Studio in Melbourne. Designers cover the plane in a non-repetitive manner. The main unit of the system triangles which are organized by five into panels, while five panels from the main construction module (Joye, 2007). This construction system is made from very complex structure on the facade of the building and is based on pinwheel aperiodic tessellation.

Figure 2.27. Interior of federation square building
(Source: Knowledgerush, 2010)

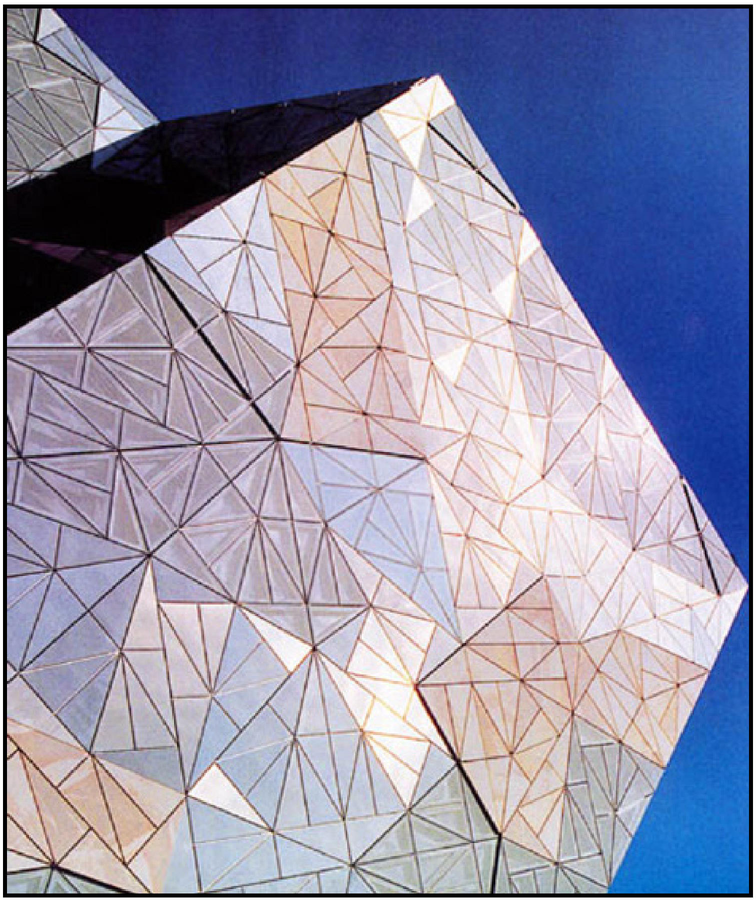


Figure 2.28. Federation square building
(Source: Vyzantiadou et al., 2006)

After some example of the applications of tessellation on the 2d surface, this chapter will focus on very specific type of tessellation and its application. The type of tessellation is called Penrose Tiling. In the next chapter, the main properties of Penrose tiling will be examined in details; in this chapter, its usage in contemporary architecture will be examined.

Storey Hall, RMIT, was designed by Melbourne Architects, Ashton Raggatt McDougall. Storey Hall (Figure 2.29-2.32) at the Royal Melbourne Institute of Technology (RMIT) was constructed in the 1990's, utilizing symmetry, based on Penrose's aperiodic rhombs. The innovative design and creative blending of the hall into the surrounding 19th Century Melbourne neighborhood has won several architectural awards for this modern structure (Tennant, 2008). The most striking feature of the architecture is the pattern of rhombuses creeping up the facade of the building and over the walls and ceiling inside (Nincent, 2010).

Figure 2.29. Storey hall, Melbourne
(Source: Wordpress, 2009)

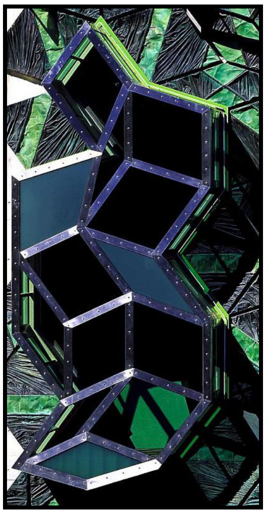


Figure 2.30. Storey hall facade
(Source: Myarchn, 2009)

Figure 2.31. Penrose rhombuses at storey hall
(Source: VIC, 2009)

Figure 2.32. Interior of storey hall
(Source: Myarchn, 2009)

Another example for tessellation in contemporary architecture is Ravensbourne College (Figure 2.33, 2.36) that was designed by Foreign Office Architects. The building inspired by the complex, multi-directional floral patterns of William Morris and others, FOA sought to create its own pattern by breaking the facade into a system of tessellating tiles. This college is clad in a skin of tessellating aluminum tiles. Designers create pattern on the facade by breaking the facade into a system of non-periodic tessellation and they use two irregular pentagons and equilateral triangles. Moreover, to further fragment the rhythm, the tiles contain rotational symmetry of the module. Alejandro Zaera-Polo who is the designer of FOA points out that "The tessellation system also enables us to change the diameter of the window openings by changing the position of certain tiles. Due to the very contingent history of the development of the pattern in the office, we originally thought that the pattern was a Penrose pattern. But after some communication with Roger made possible by Charles Jencks – we found that this was not true, and Penrose – that there were other precedents to the pattern, although the scheme we had was unique. This pattern has now been patented to protect its use" (Architecturetoday, 2010). As it can be seen from the designer's explanation, they improved three types of tiling that were inspired by Penrose tessellation.

Figure 2.33. Rovensbourne college
(Source: Bdonline, 2009)

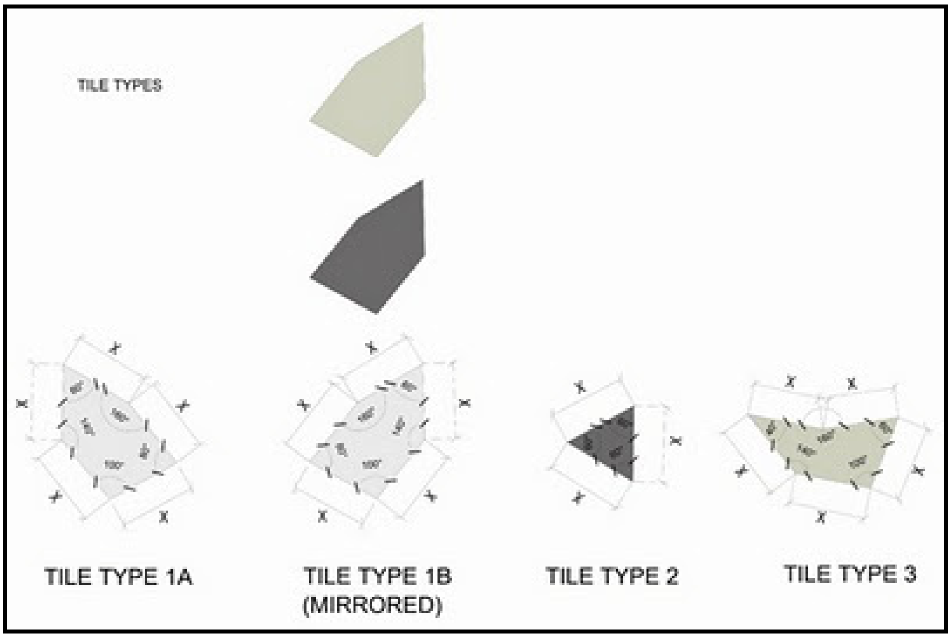


Figure 2.34. Tile types of Ravensbourne college
(Source: Humanscribbles, 2009)

Figure 2.35. Construction of Revensbourne college
(Source: Humanscribbles, 2010)

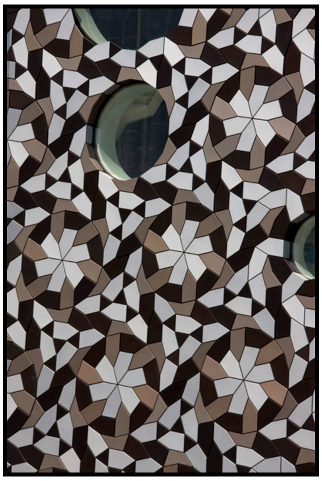


Figure 2.36. Detailed view of the tiling, Revensbourne college
(Source: Architecturetoday, 2010)

2.3.2. 3-D Dimensional Applications of Tessellation

This study also analyzes the applications of tessellation in a 3-d dimension that means a space where the module encloses it. After the digital age, computer systems have began to a design tool for the architect. This development improves the structure system, designer have been much free to their design. 3-d dimensional application of tessellation has been seen by helping this development especially ceiling of building.

One of the famous examples is Biritsh Museum in London (Figure 2.37) designed by Foster and Partners. Foster and Partners tessellate the roof by triangular tessellation. Khairas point out that, this is very complex structure because it manages a three-dimensional object from two-dimensional tessellation (Khaira, 2009). In addition to this Kolerevic added that, the form of the roof obtained by triangulated frame network, all of them different from each other and preparing by helping computer program (Kolorevic, 2005).

Figure 2.37. British museum
(Source: London-Architecture, 2010)

Figure 2.38. DG BANK, Berlin
(Source: Thecityreview, 2010)

Figure 2.39. Louvre museum
(Source: Piercemattiepublicrelations, 2010)

CHAPTER 3

TESSELLATION TECHNIQUE IN MATHEMATICS

The main purpose of this chapter is to analyze the main mathematical properties of the tessellation technique to find an answer to the following questions;

- What kind of shapes can tessellate the plane?
- What kind of symmetrical properties does tessellation have?
- Which techniques could be used to generate intricate design?

3.1. Meaning of Tessellation

Tessellation is a kind of mathematical technique which is used in science, art and architecture (Figure 3.1). Basically, tessellation means to cover a plane without any gaps or overlaps by considering some methods. The origin of the tessellation comes from the Latin word tessella that was the square stone or tile used in ancient Roman mosaic. The word of tiling and mosaics are common synonyms for tessellation (Seymour and Britton, 1989). In addition to this definition Grünbau and Shephard (1986) emphasized in their book that "In mathematical approach, tiling means that accountability condition excludes families in which every tile has zero area (such as point or line segments) but nevertheless the definition admits tilings in which some tiles have bizarre shapes and properties".

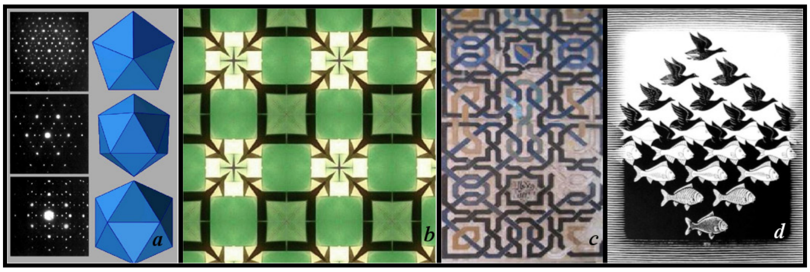


Figure 3.1. Tessellations in art, architecture and science a) Relationship between tessellations and x-ray crystallography, b) Origami tessellation, c) Tesselation surface, d) Echer "sky" (Source: Thinkquest, ISU, 5cense, Flickr, 2009)

It is clear that the art of designing tessellation is extremely old and well developed. By contrast, the science of tessellation and patterns, which means the study of their mathematical properties, is comparatively recent and many parts of the subject have yet to be explored in depth. Although tessellations have been traced back to ancient human cultures (Figure 3.2) and can also be found in the natural world, they have had a relatively short history as a topic for serious mathematical and scientific studies.

Figure 3.2. Some usages of tessellations in various cultures a) Arabic, b) Indian, c) Byzantine (Source: SPSU, 2009)

In mathematical approach, one of the first remarkable studies of tessellation was conducted by Johannes Kepler. Kepler wrote a book named Harmonice Mundi in 1619 that described the regular and semiregular tessellations, which cover the plane with regular polygons. Another important study was done by the Russian crystallographer E.S.Fedorov in 1891; he proved that every tessellation of the plane is constructed in accordance to one of the seventeen different groups of isometries (Thinkquest, 2010). Moroever, Grünbaum and Shephard (1986) explained in their book that in the past there have been many attempts that try to describe and systematize the notation of tessellation. For instance "Bourgoin (1873, 1880, 1883, 1901), Day (1903), Dresser (1862), Edwards (1932), Meyer (1888), Schauermann (1892), Wersin (1953) is noteworthy mainly for the extraordinary extent, which it plagiarizes Bourgouin (1883).

Tessellation technique is mainly governed by rules of geometric repetition, continuity and symmetry. According to this strict adherence to conformity, a relatively simple motif may be used to produce an increasing pattern, which may continue up to infinity (Tennant and Dhabi, 2004). Nevertheless, symmetry has got a major role for creating tessellation. The most famous usage of the tessellation has been seen in the

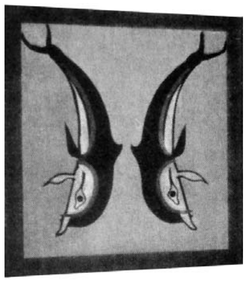
Alhambra at Granada in Spain. As it can be seen in many examples of the ornamental art, many important properties of the tessellation methods depend upon the idea of symmetry (Grünbau and Shephard, 1986).

3.2. Importance of Symmetry

In geometry, symmetry means the property by which the sides of a figure or object reflect each other across to a line or surface (Britannica, 2010). Architects and artists inspired from the symmetry of the nature. Many visual structures that arranged according to the laws of symmetry have been presented from the earliest times. However, the meaning of symmetry evokes two different definitions. In fine art symmetry means Greek aesthetics; which comes to mind harmony, accord, regularity, while in the narrow sense, symmetry identified with mirror symmetry in a vertical reflection line (Japlan, 1994).

Symmetry emerged at the early stages of the human development. In the literature, many examples that are near to each can be shown from different countries. György Darvas (2007) said that “without the conditions required for transportation, the cultures of these distant parts of the globe cannot have influenced one another, though there are suggestions that even in the preceding couple of millennia, common motifs can have spread from Mesopotamia in various directions”. One of the most important examples is the dolphin motif. It is almost the same two dolphins biting into each other’s tails that appear many different areas of the world (Table 3.1)

Table 3.1. Various dolphin motifs from the different areas of the world.
(Source: Darvas, 2007)

		
Neolithic bowl. Yang-shao culture, *c. 2000 BC*	*Fresco*, Megar´on,Tiryns *o, c. 1200 BC*	*The appearance of the dolphin motif, the Tripolis culture in North Africa, 4000–3500 BC*

(Continued on next page)

Table 3.1. (cont.)

		
Ancient Chinese dish, with two fish biting each other's tails	*Design of dolphins biting each other's tails, as it appears on the decoration of Cretan ceramic dishes from the 17th–14th centuries BC, (K. Czernohaus) and in a stylized version (G. Walberg, right)*	
		
Ceramic plate with two fish biting each other's tails. Basin of Mexico, 10th–6th century BC	*Some stages in the development, according to Chinese sources of the fishes biting each other's tails into the yin-yang*	

"Mathematician take as the basis of every geometry the set of undefined elements (point, line, plane) which constitute space, the set of undefined relations and the set of basic apriori assertions" (Japlan, 1994). So symmetry consists of point symmetry, line symmetry, and plane symmetry. This work is restricted to the plane symmetry to analyze tessellation method on the planar surfaces.

Plane symmetry can also be called as wallpaper patterns. Wallpaper patterns generated by the four planer isometries: translations, rotations, reflections and glide reflections.

Translation: A translation of a plane figure means to move it without rotating or reflecting it in a given direction.

Reflection: A reflection is a mapping of all points of the original figure onto the other side of a "mirror that the distance between the image and the mirror line is the same as that between the original figure and the mirror line".

Rotation: To move the motif through a given angle considering to a center point.

Moreover, n-fold rotational symmetry implies that rotation about a point by an angle of 360°/n generates an image indistinguishable from the original.

Glide Reflection: A glide reflection is a combination of two transformations: a reflection and a translation in a same line.

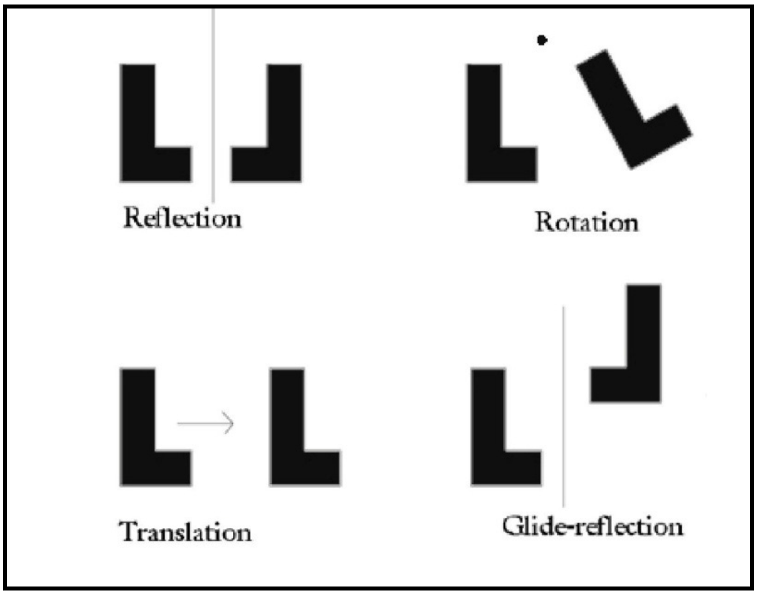


Figure 3.3. Planar Isometries

Kinsey and Moore (2002) emphasized that, all wallpaper patterns are created by the translation of main unit which may or may not have the symmetry of its own by rotation, reflection or glide reflection, moreover they note that, it does not really important where the translation starts, only its direction and distance is important. This means that, if one point is translated on any axis, then all points get translated with the same axis and measured. It should be again noted that all unit cells have translational symmetry in relation with the other unit cells so they can be combined with rotation, reflection and glide reflection to generate additional symmetry operation.

In mathematical literature, wallpaper patterns have characterized with notations. The purpose of this part of the thesis is not analyzing it in detail, but to understand and analyze the historical background of the tessellation and try to see how tessellation technique is benefited from mathematics. In the light of this after some definition of the wallpaper patterns, this part of the chapter aims to show it in a tessellation example.

m: mirroring in an axis

g: glide reflection (translation + reflection)
1: translation by one unit
2, 3, 4, 6: 2-, 3-, 4-, 6-fold rotation
p: primitive cell (which only has objects on the vertices of the lattice)
c: centred lattice (which contains an object in the centre of one of its lattice element) (Darvas, 2007).

To generate the wallpaper pattern notation, it should be started with either p or c for a primitive cell or a face-centered cell. As a second step, the highest order of rotational symmetry 1-fold (none), 2-fold, 3-fold, 4-fold or 6-fold should be indicated to the notation. This is followed by indicating the symmetries relative to the main translation axis of the pattern. After explaining the notation of wallpaper patterns, now let's see a few examples of the notation and Isometry combinations (Table 3.2).
p31m: Primitive cell, 3-fold rotation, mirror axis at 60°
p2mm: primitive cell, 2-fold rotation, mirror axis at 90°

Table 3.2. Combinations of the Isometries

Symmetry Group Number	**International Union of Crystallography symbol**	**Lattice Type**	**Rotation order**	**Axis of Reflection**
1	**p1**	parallelogrammatic	none	none
2	**p2**	parallelogrammatic	2	none
3	**pm**	rectangle	none	parallel
4	**pg**	rectangle	none	none
5	**cm**	rectangle	none	parallel
6	**pmm**	rectangle	2	90
7	**pmg**	rectangle	2	parallel
8	**pgg**	rectangle	2	none
9	**cmm**	rectangle	2	90
10	**p4**	square	4	none
11	**p4m**	square	4*	45°
12	**p4g**	square	4**	90°

(Continued on next page)

Table 3.2. (cont.)

13	**p3**	hexagon	3	none
14	**p3m1**	hexagon	3*	30°
15	**p31m**	hexagon	3**	60°
16	**p6**	hexagon	6	none
17	**p6m**	hexagon	6	30°

All wallpaper patterns consist of a shape or motifs which are iterated at a regular sequence in more than one direction. At this iteration step all wallpaper patterns follow a framework, this framework is called net of symmetry group and the net represents the made of repetition. Also net is named as lattice. Every wallpaper pattern has an associated lattice.

There are only seventeen wallpaper patterns that represent the schema. Now at this stage of the part of the chapter all of the wallpaper patterns will be examined according to their type of net. Mackey defines on his study that, there are only five type of net or lattice possible for the wallpaper patterns. These are; oblique, rectangular, square, centered rectangular and rhombic or hexagonal.

3.2.1. Oblique (Parallegrom) Lattice

There are only two kinds of wallpaper patterns that have oblique lattice (Figure 3.4). These are p1 (Figure 3.5, 3.6) and p2 (Figure 3.7, 3.8).

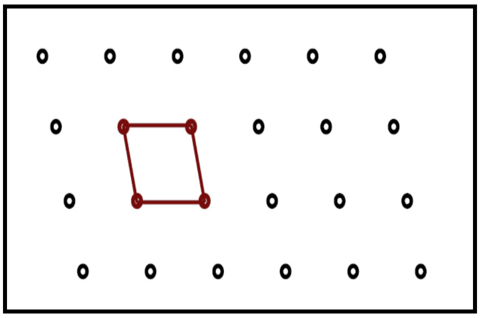


Figure 3.4. Oblique Lattice

* All centers of rotation lie on reflection axes. **Not all centers of rotation lie on reflection axes.(Darvas, 2007).

p1 is the simplest symmetry group because p1 group consists of only translation isometrics. There are not any reflections, glide-reflections or rotations. The two translation axes may be inclined at any angle to each other. However, the only difference of p2 from p1 is the fact that it contains 180° rotations, that is, rotations of order 2. As in all symmetry groups there are translations, but there are neither reflections nor glide reflections. The two translations axes may be inclined at any angle to each other.

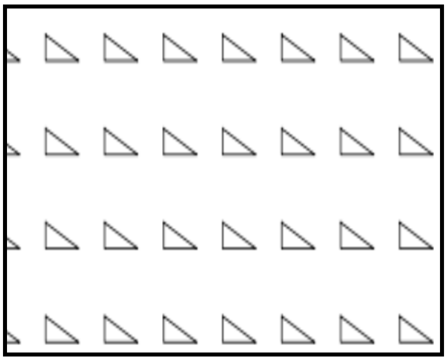


Figure 3.5. p1 group

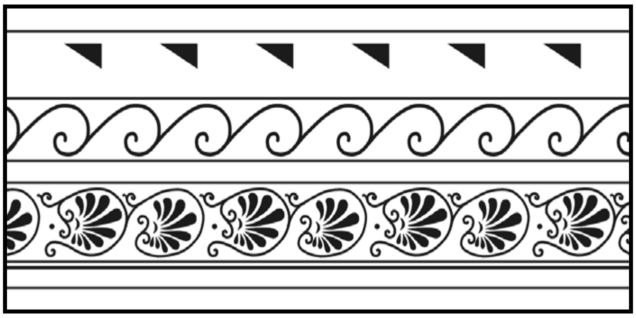


Figure 3.6. Usage of p1
(Source: Darvas, 2007)

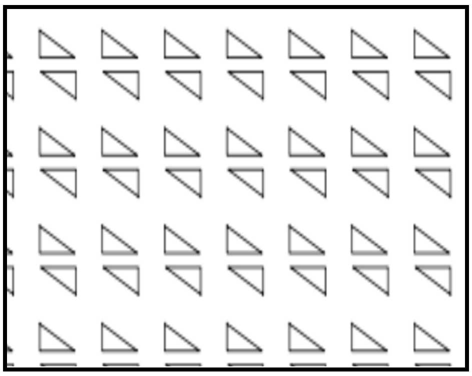


Figure 3.7. p2 group

Figure 3.8. Usage of p2
(Source: SLU, 2010)

3.2.2. Rectangular Lattice

Groups of pm, pg, pmg, pmm and pgg have the rectangular lattice (Figure 3.9).

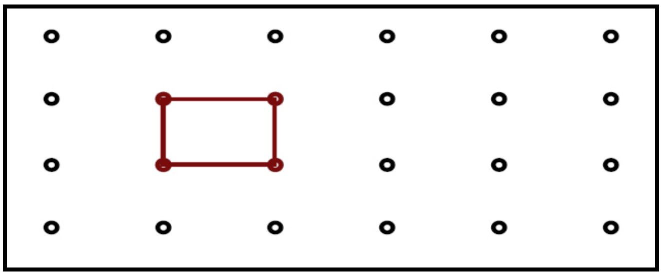


Figure 3.9. Rectangular Lattice

pg (Figure 3.10, 3.11) contains glide reflections. pg group does not include any rotations or reflections. The direction of the glide reflection is parallel to one axis of translation and perpendicular to the other axis of translation. Moreover, pm (Figure 3.12, 3.13) is the first group, which contains reflections. There are not any rotations or glide reflections. The axes of reflections are parallel to one axis of translation and perpendicular to the other axis of translation. In addition to this, pmg (Figure 3.14, 3.15) group contains reflections, and glide reflections which are perpendicular to the reflection axes. It has rotations of order 2 on the glide axes, halfway between the reflection axes. pmm (Figure 3.16, 3.17) contains perpendicular axes of reflection, with 180° rotations where the axes intersect. Finally, the last group of rectangular lattice is pgg (Figure 3.18, 3.19) which contains no reflections, but it has glide-reflections and 180° rotations. There are perpendicular axes for the glide reflections, and the rotation centers do not lie on the axes.

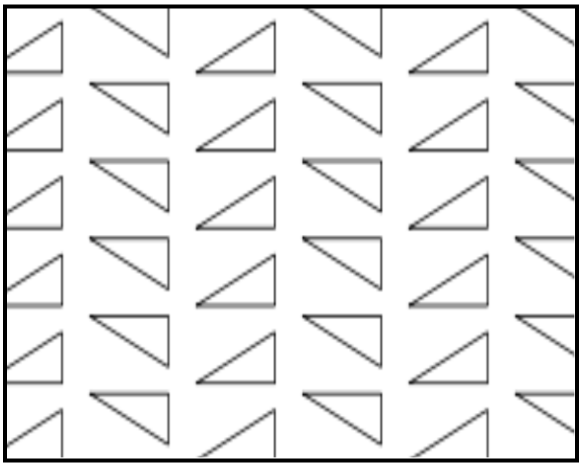


Figure 3.10. pg group

Figure 3.11. Usage of pg group
(Source: Wikipedia, 2010)

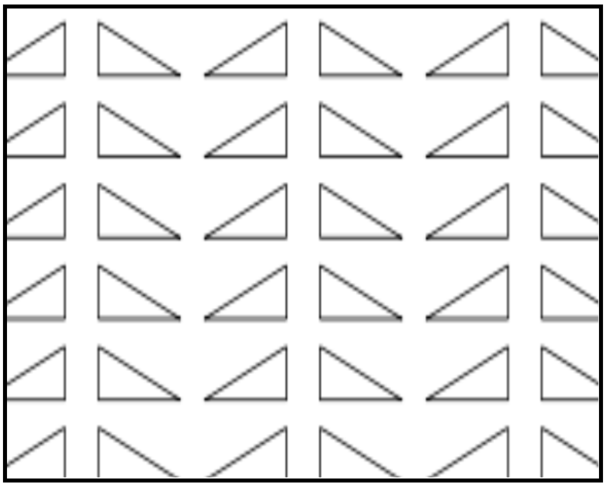


Figure 3.12. pm group

Figure 3.13. Usage of pm group
(Source: Wikipedia, 2010)

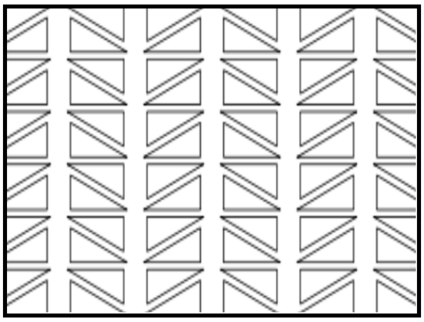


Figure 3.14. pmg group

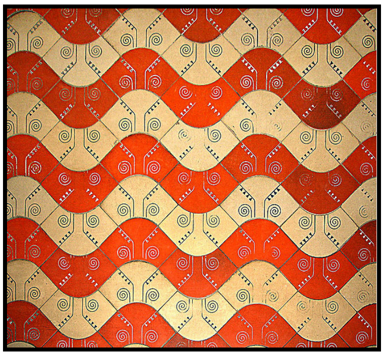


Figure 3.15. Usage of pmg group
(Source: Wikipedia, 2010)

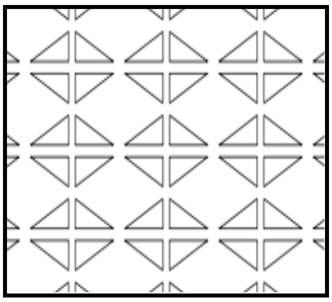


Figure 3.16. pmn group

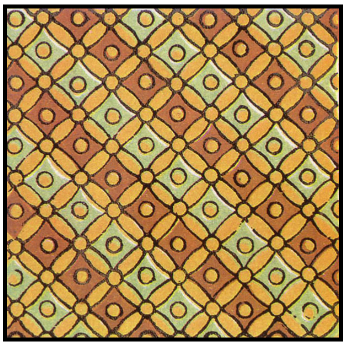


Figure 3.17. Usage of pmm group
(Source: Wikipedia, 2010)

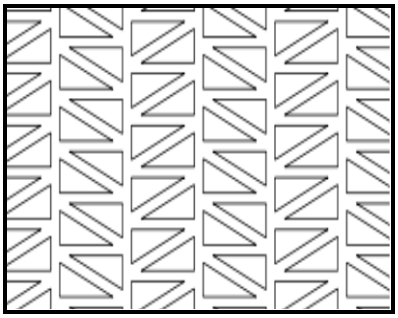


Figure 3.18. pgg group

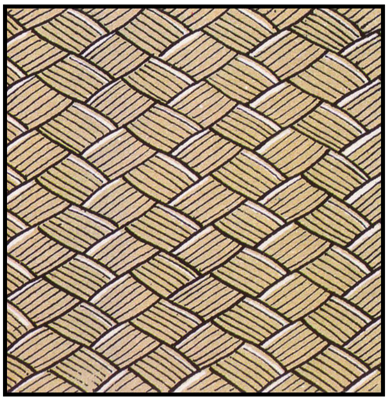


Figure 3.19. Usage of pgg group
(Source: Wikipedia, 2010)

3.2.3. Square Lattice

Group of p4, p4m and p4g have square lattice (Figure 3.20) framework when they are repeated.

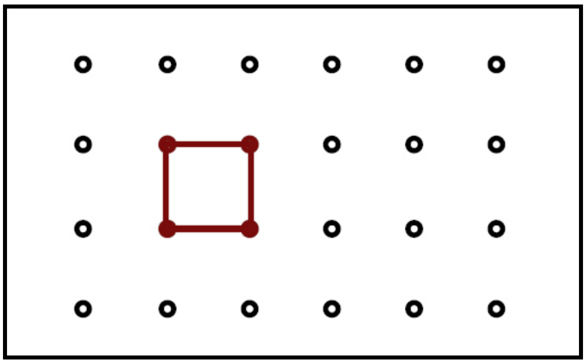


Figure 3.20. Rectangular Lattice

p4 (Figure 3.21) group has a 90° rotation that is a rotation of order 4. It also has rotations of order 2. The centers of the order-2 rotations are midway between the centers of the order-4 rotations. There are no reflections. The second group of p4g (Figure 3.22) is similar like the previous group, p4. This group contains reflections and rotations of orders 2 and 4. There are two perpendicular reflections passing through each order 2 rotation. However, the order 4 rotation centers do not lie on any reflection axis. There are four directions of glide reflection. Moreover, p4m (Figure 3.23) has both order 2 and order 4 rotations. This group has four axes of reflection. The axes of reflections are inclined to each other by 45° so that four axes of reflections pass through each order 4 rotation centers. Every rotation center lies on some reflection axes. There are also two glide reflections passing through each order 2 rotations, with axes at 45° to the reflection axes. p4m is very common and is easy to recognize because of its square lattice.

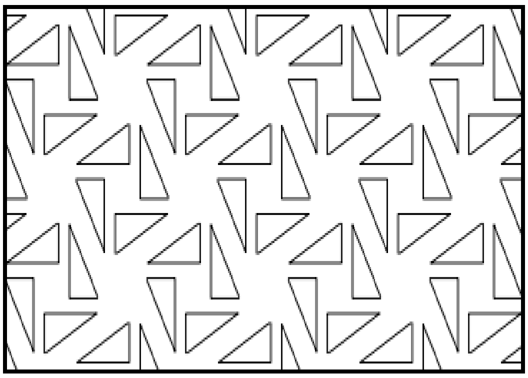


Figure 3.21. p4 group

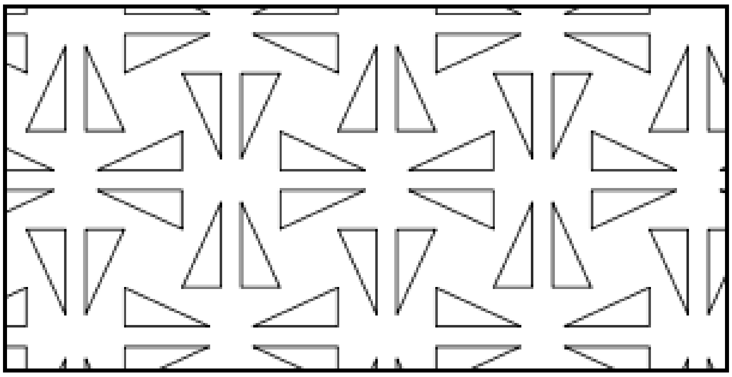


Figure 3.22. p4g group

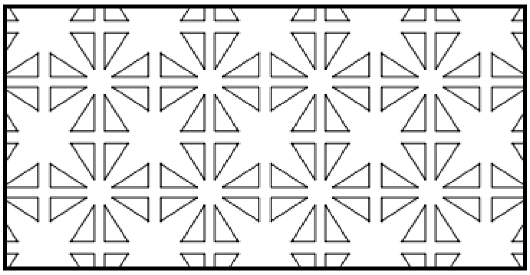


Figure 3.23. p4m group

3.2.4. Centered Rectangular Lattice

The centered rectangular type of lattice (Figure 3.24) form generated two type of wallpaper patterns that is called cm and cmm.

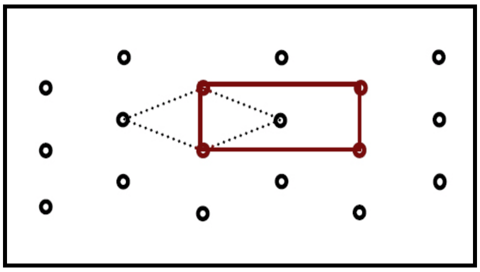


Figure 3.24. Centered Rectangular Lattice

cm group (Figure 3.25, 3.26) contains reflections and glide reflections with parallel axes. There are no rotations in this group. The translations may be inclined at any angle to each other, but the axes of the reflections bisect the angle formed by the translations, so the lattice is rhombic. Also, cmm (Figure 3.27, 3.28) has perpendicular reflection axes, as group pmm, but it also has rotations of order 2. The centers of the rotations do not lie on the reflection axes.

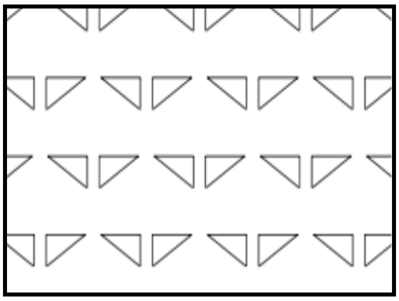


Figure 3.25. cm group

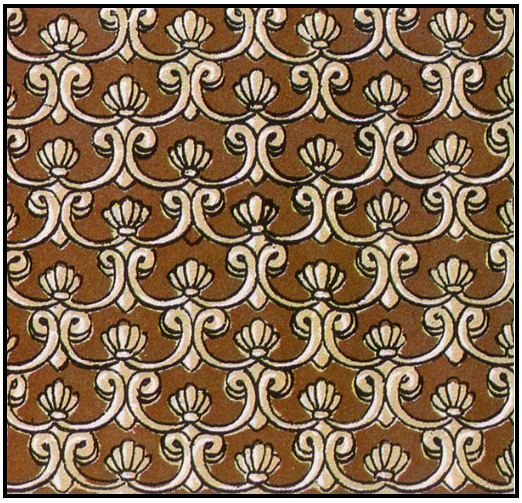


Figure 3.26. Usage of cm group
(Source: Wikipedia, 2010)

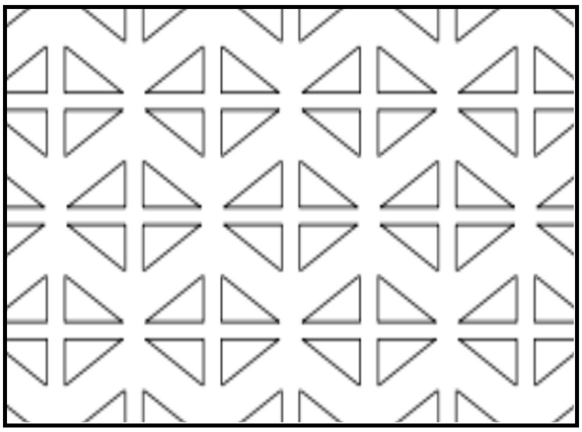


Figure 3.27. cmm group

Figure 3.28. Usage of cmm group
(Source: Wikipedia, 2010)

3.2.5. Rhombic (Hexagonal) Lattice

Group of p3, p31m, p3m1, p6 and p6m have rhombic (hexagonal) lattice (Figure 3.29).

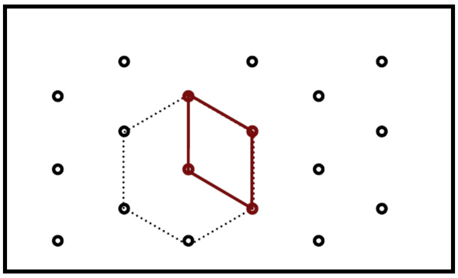


Figure 3.29. Rhombic (Hexagonal) Lattice

p3 (Figure 3.30) is the one of the simplest group that contains a 120°-rotation, that is, a rotation of order 3. It has no reflections or glide reflections. p31m (Figure 3.31) contains reflections (whose axes are inclined at 60° to one another) and rotations of order 3. Some of the centers of rotation lie on the reflection axes, and some do not. There are some glide-reflections. In addition to two kind of lattice, p3m1 (Figure 3.32) is similar to the last as it contains reflections and order-3 rotations. The axes of the reflections are again inclined at 60° to one another, but for this group all of the centers of rotation do lie on the reflection axes. There are some glide-reflections. Moreover, p6 (Figure 3.33) contains 60° rotations, that is, rotations of order 6. It also contains rotations of orders 2 and 3, but no reflections or glide-reflections and finally, p6m is the most complex group that has rotations of order 2, 3, and 6 as well as reflections. The axes of reflection meet at all the centers of rotation. At the centers of the order 6 rotations, six reflection axes meet and are inclined at 30° to one another. There are some glide-reflections.

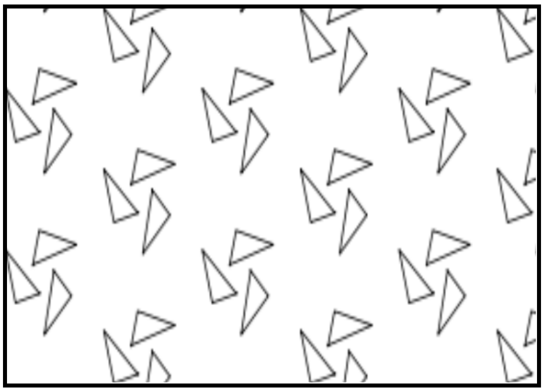


Figure 3.30. p3 group

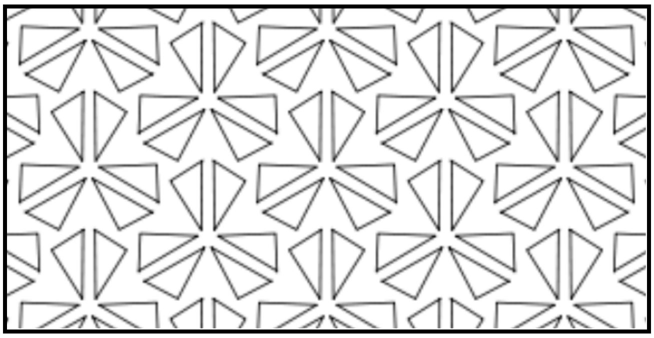


Figure 3.31. p31m group

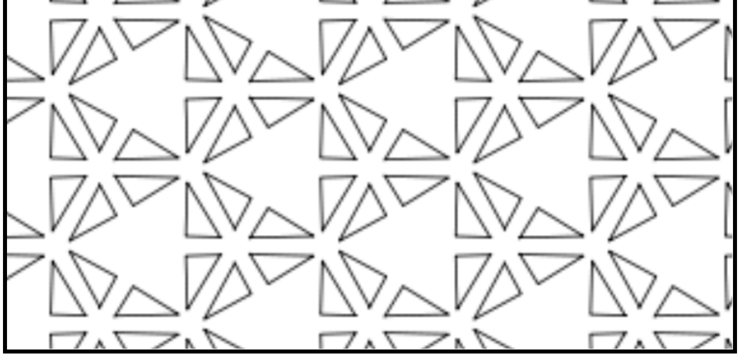


Figure 3.32. p3m1 group

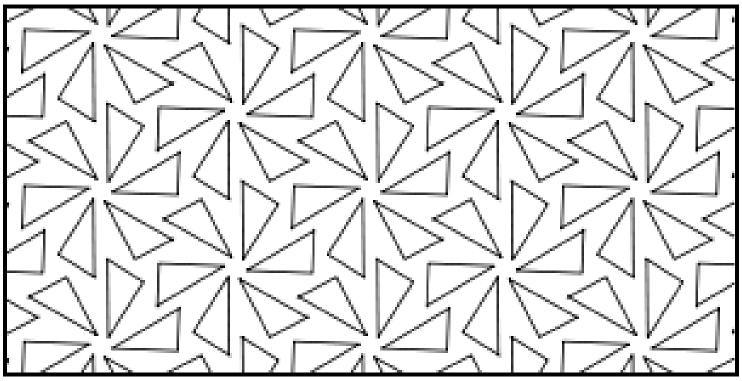


Figure 3.33. p6 group

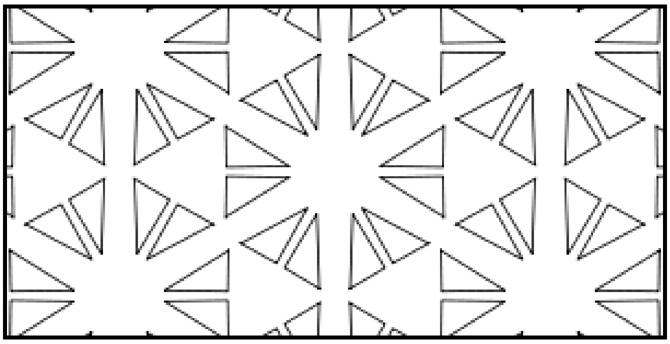


Figure 3.34. p6 group

Symmetry has been a major role for the ornamental art in Islamic architecture. Many researchers have interested in the relationship between symmetry groups and Alhambra tessellation. According to Grünbaum “groups of symmetry had no relevance to artists and artisans who decorated the Alhambra. They certainly could have produced equally attractive ornamentation in any of the symmetry groups that anybody wished them to do so (Figure 3.35). Naturally, nobody did, since nobody knew about symmetry groups for the next five centuries. Thus it is only our infatuation with the idea that any attractive ornamentation must be explained in group-theoretic terms that leads us to try to find them there” (Grünbaum, 2006). Nevertheless, the question of which of the seventeen wallpaper groups are represented in the Alhambra has been discussed and researched from many scientist. In her 1944 thesis, Edith Müller found 11, and not 17 as has often been claimed. Two more were described in Branko Grünbaum, Zdenka Grünbaum and G.C. Shephard.

Figure 3.35. Symetry groups of Alhambra tesselations
(Source: KTH, 2009)

To summarize, it can be seen that symmetry is an important method to cover a plane with tessellation technique. By using planar isometry the plane can be covered by the polygons.

3.3. Type of Tessellation

Many classification of tessellation have been established in the literature. This thesis will focus on two forms. One of them is based on iteration way and the second one is based on their shape.

3.3.1. Way of Iteration

The first one of the classifications is considered with respect to the tessellation patterns iteration way. It other words, to generate a framework in a way of tessellations main unit pattern repetitions. This process is benefited from the symmetry that is explained in the previous part of this chapter.

3.3.1.1. Periodic and Non-periodic Tessellation

Patterns that do not repeat in a linear direction are called non-periodic tilings (Figure 3.36). On the contrary to this “a tiling is periodic if there is finite section of the tiling that can be translated in two nonparallel directions to recreate the entire tiling" (Kinsey and Moore, 2002) (Figure 3.37).

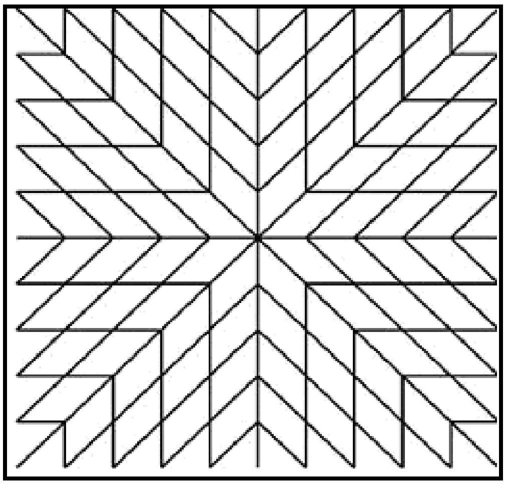


Figure 3.36. Non-Periodic tiling

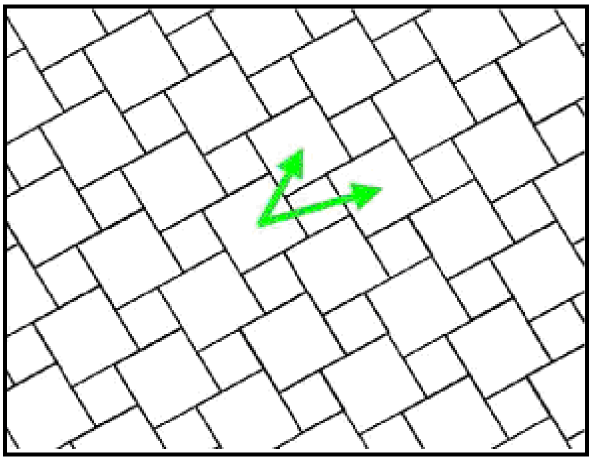


Figure 3.37. Periodic tiling

Konyalı deals with the periodic and non-periodic tessellations by considering to Dutch artist M.C.Echer on her study. She explained periodic tessellation by using one of the most famous paintings from Echer that is called the fish and the birds (Figure 3.38). She points out that the symmetry group of the painting has two independent translations, moreover, it has no rotation or reflection (Konyalıoğlu, 2009) .

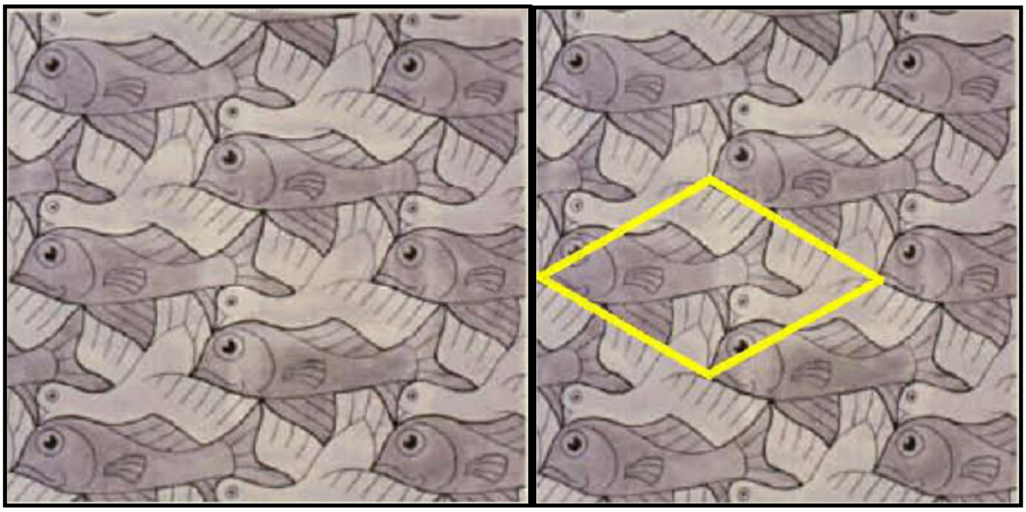


Figure 3.38. Fish and Birds
(Source: Konyalıoğlu, 2009)

Decagon is the single tile, which can tile periodically and non-periodically on the plane. Figure 3.39 displays the periodic tessellation that is generated by decagon tile. If the decagons are rotated in multiples of 36 degrees the tiling became non-periodic, Figure 3.40 shows the non-periodic decagon tessellation.

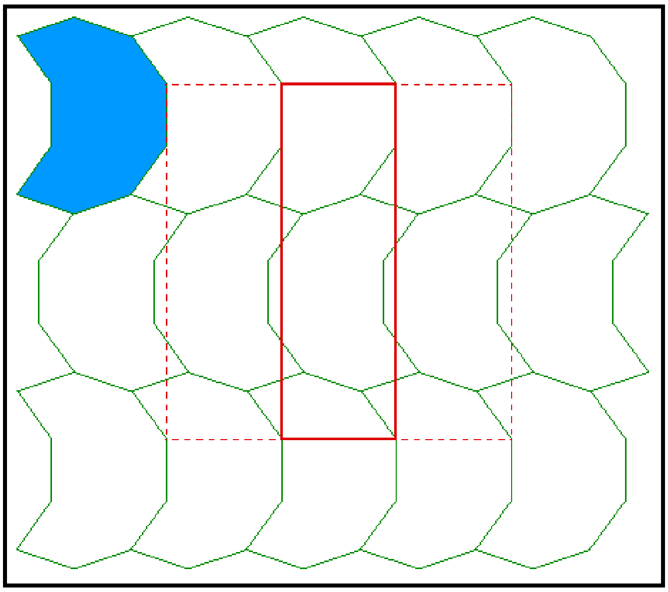


Figure 3.39. Decagon periodic Tessellation
(Source: UWA, 2010)

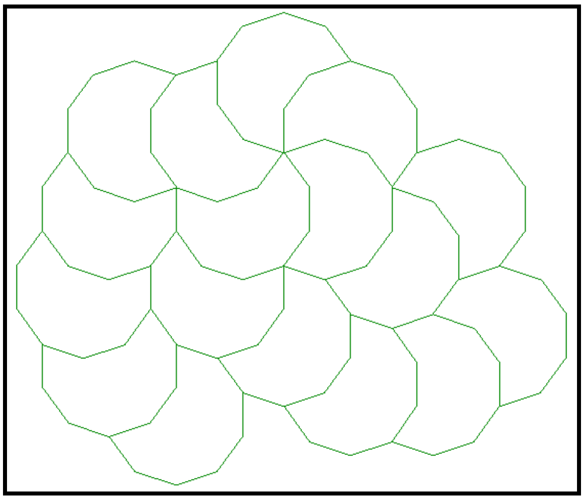


Figure 3.40. Decagon non-periodic Tessellation
(Source: UWA, 2010)

Many tiles can cover the plane by periodic tessellation, also as it can be seen above there is only one tile that cover the plane in both periodic and non-periodic way. Yet, there should be a question that how about purely non-periodic tessellation, it means that figure cover the plane but never form a periodic tessellation.

Kinsey and Moore (2002), in their book deal with the answer of the question that whether or not there is a set of tiles that can tile the plane only in a non-periodic way on. They said that “We adopt the language of Grünbaum and Shephard and call such a set of tiles aperiodic. Upon hearing the question, most people suspect that there is not an aperiodic set of tiles. Most mathematicians agreed until 1964, when Robert Berger produced such a set. His original set contained over 20,000 different tiles. Another mathematician, Raphael Robinson, found an aperiodic set containing only five tiles” (Figure 3.41). But the most well-known set of aperiodic tiles are the Penrose tiles.

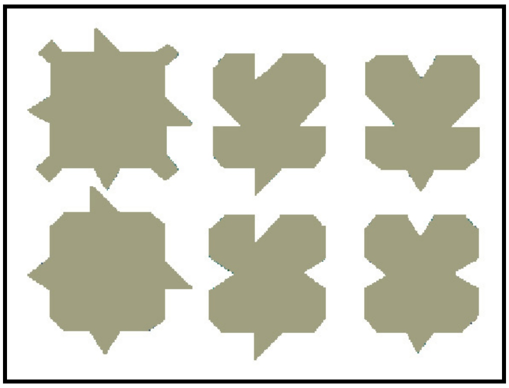


Figure 3.41. Robinson aperiodic tiling

3.3.1.2. Aperiodic Tesselleation

As it can be seen on the previous part of this chapter, rectangles, triangles and regular hexagons generally use to cover a plane with repetitive or periodic patterns. These shapes have two-fold, three-fold, four-fold or six-fold symmetry; however, regular pentagons with their fivefold symmetry do not tile a plane without any gaps or overlaps.

"In the 1970's, new tilings were discovered that not only were non-periodic but could not be rearranged to be periodic". Roger Penrose developed a set of prototiles related to a regular pentagon that can tile the plane .One set of two prototile becomes known as "kite" and "dart". Eric A. Lord points out that the kite and dart are generated by dissecting decagon (Lord, 1991) (Figure 3.42).

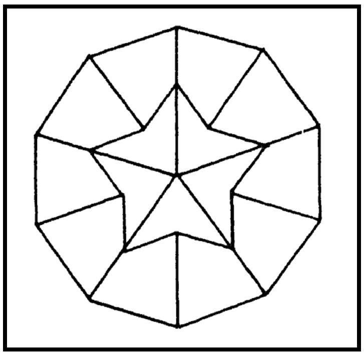


Figure 3.42. Kite and dart from decagon
(Source: Lord, 1991)

The interior angles of the kite are 72°, 72°, 72° and 144°. Also, interior angle of dart are 36°, 72°, 36° and 216° (Figure 3.43).

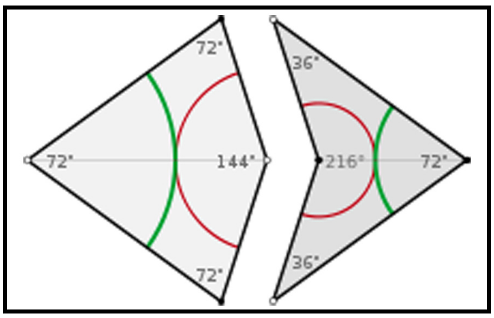


Figure 3.43. Interior angles of dart and kite
(Source: Wikipedia, 2010)

There are two ways to match kite and dart on the plane. One of them is match the vertices (Figure 3.43) another is to use a pattern of circular arcs to constrain the placement of the tiles (Figure 3.44). By using those rules there are only seven ways tiles can be put around a vertex.

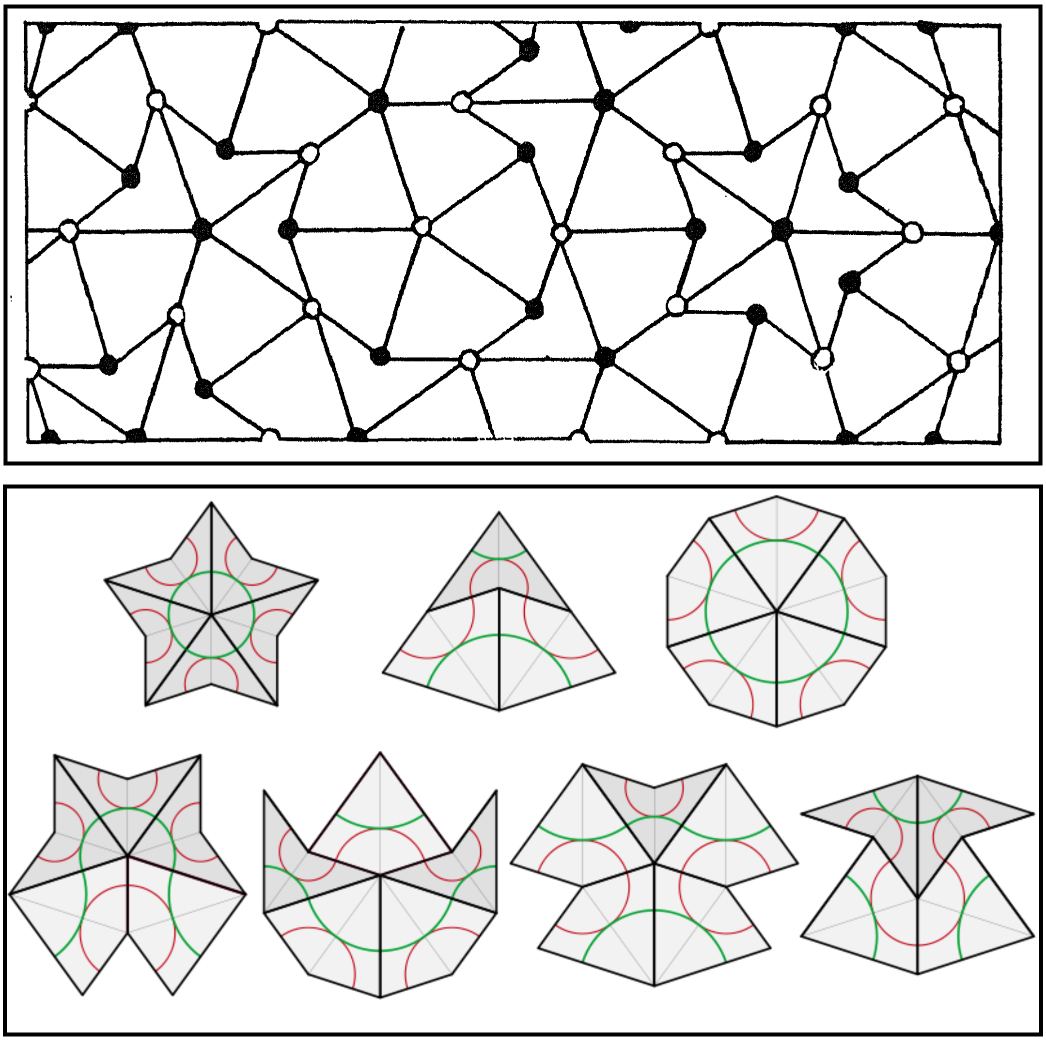


Figure 3.44. Matching Kite and Dart
(Source: Lord, 1991, Wikipedia, 2010)

Another example of this type of a-periodic tiling was discovered by Roger Penrose and consists of two rhombuses (Figure 3.45, 3.46) (Tennant, 2008), a fat one with internal angle of 72° and 108° and a skinny one with internal angles of 36 ° and 144° . Penrose, introduce two tiles along with a set of rules for how the tiles must be put together.

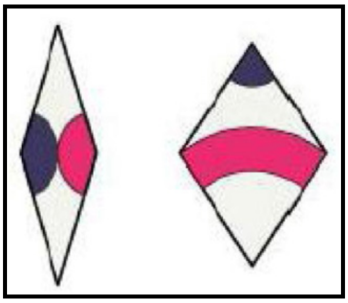


Figure 3.45. Penrose rhombs
(Source: Tennant, 2008)

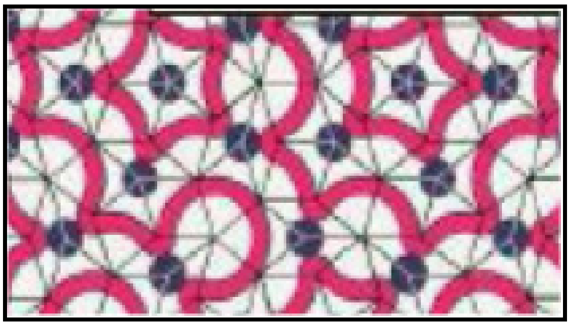


Figure 3.46. Penrose aperiodic tilings
(Source: Tennant, 2008)

Penrose tiling (Figure 3.47) may be constructed by using both reflection symmetry and fivefold rotational symmetry. Because of its non-periodic properties penrose tiling lacks any translational symmetry. Two penrose rhombuses will tessellate infinitely without repeating exactly the same pattern so the tessellation may be likened to an irrational number. Moreover, the ratio of fat rhombus and skinny rhombus approaches $(1+\sqrt{5})/2$, the golden ratio used through out the history by many architects, artist to reach pleasing proportions.

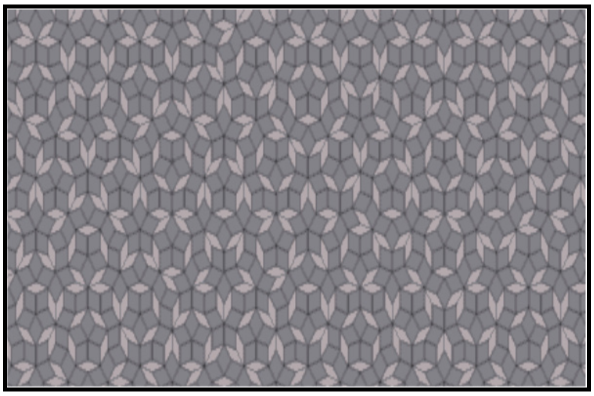


Figure 3.47. Penrose Tesselation
(Source: Daviddarling, 2010)

Foreign Office Architects use Penrose tessellation and point out that “The practicalities of construction meant that such a pattern was too complicated to achieve but the Penrose tiling provided the inspiration for the final design used. Similar patterns can be found in medieval Islamic architecture in the form of Girih, geometric star and polygon strapwork. Such patterns were usually periodic with a cell repeated in the same orientation within a lattice” (Bizley, 2010). This is very important construction method for Islamic architecture and facade ornamentation.

3.3.2. Polygon Shape

When analyzing a tessellation pattern, this thesis interested in the closed figures, or tiles and aims to analyze relationship between polygon and tessellation patterns to reach kinetic planar surfaces method because of the fact that planar surface should be designed by using some polygonal platforms. So, this part of the chapter tries to answer following questions;

- What kind of shapes will tessellate the plane without overlapping or leaving shapes?
- Why will certain shapes tessellate and others not?
- How can tessellation combine regular polygon to cover a plane?
- How many different type of method to generate tessellation design?

Tessellation is usually represented by the number of sides of the polygons around any cross point in the clockwise or anti-clockwise order. For instance, (3^6) represents a tiling in which each of the points are surrounded by six triangles, 3 is the number of the sides of a triangle and superscript 6 is the number of triangles around the referred point (Figure 3.48).

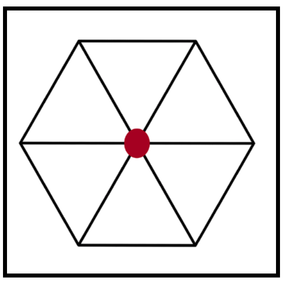


Figure 3.48. 3^6 Tiling

Tessellation by regular polygons was the first kind of tessellation to be the subject of the mathematical research that was studied by Kepler in 1619. Polygon (Figure 3.49) is the general classification of the closed plane figure with the n sides. Polygons are named according to the number of sides and the interior angle (Table 3.3).

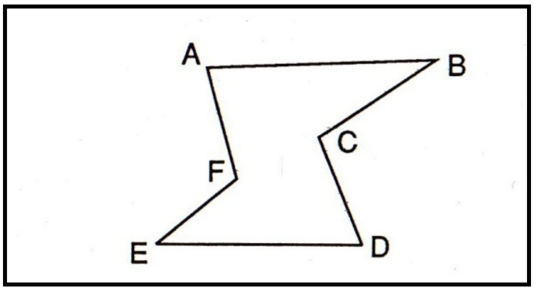


Figure 3.49. Polygon (hexagon)

AB, BC, CD, DE, EF, AF are the side of the polygon and every side of polygon meet at their end point which is called vertices (A, B, C, D, E, F).

Table 3.3. Names of the polygons

SIDES AND ANGLE	NAME
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon
13	13-gon
n	n-gon

Moreover, each tessellation is a closed topological disk whose boundary is a single simple closed curve. By this it is meant to be a curve whose ends join up to form “loop” and which has no crossings or branches. For most of the tessellations considered here, such an intersection may be empty or may consist of a set of isolated points and arcs. In these cases the points will be called vertices of the tessellation and arcs will be called edges. Polygons have vertices and edges, so it would be confusion to some terminology. Because of this reason, Grünbaum and Shephard (1986) have explained the differences between corners and vertices, sides and edges in the case of a polygonal tessellation on their book that the points A, B, C, D, E, F, G are corners but A, C, D, E, G are vertices of the tessellation, AB, BC, CE, EF, FG and GA are sides while AC, CD, DE, EG and GA are edge (Figure 3.50).

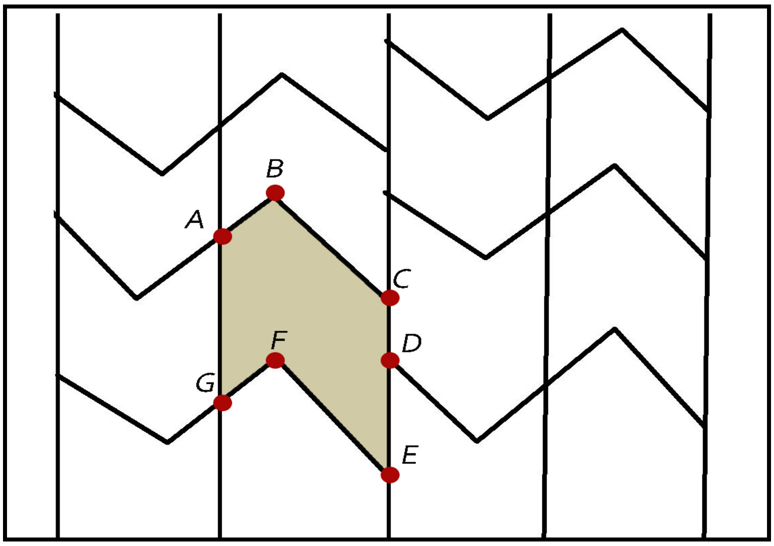


Figure 3.50. The illustration of tessellation element

3.3.2.1. Which Shapes Can Tessellate and the Reasons

However, there are many kinds of polygons that can be generated but how many of them can cover the plane without any gap or overlapping. Let’s begin with triangle.

Triangle is the simplest polygonal shape and also, it was important to understand which kind of polygon can tessellate the plane .Any kind of triangle can tessellate the plane (Figure 3.51) this property is the basis for almost every type of polygonal tessellation (Seymour and Britton, 1989).

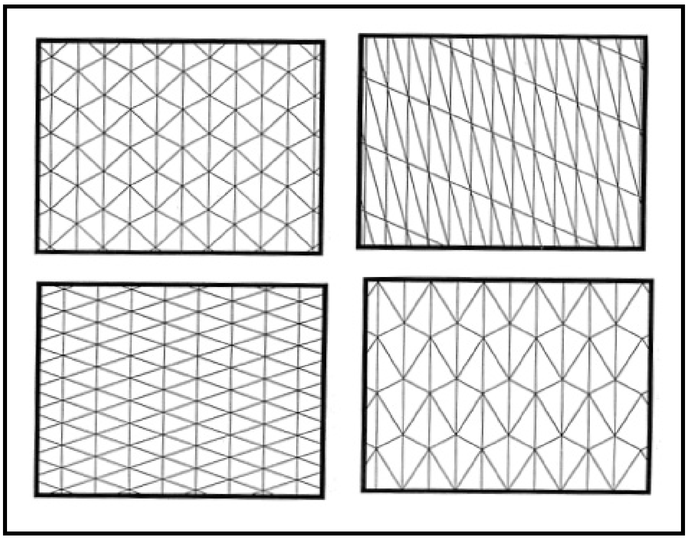
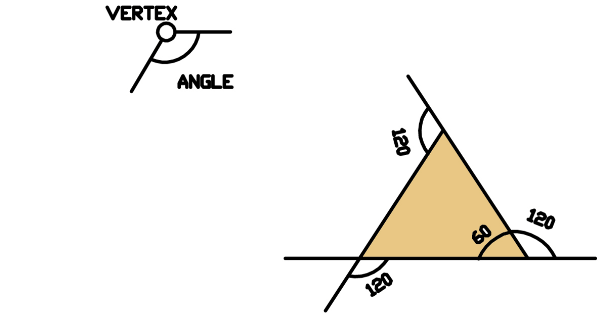


Figure 3.51. Tessellation with different types of triangles
(Source: Seymour and Britton, 1989)

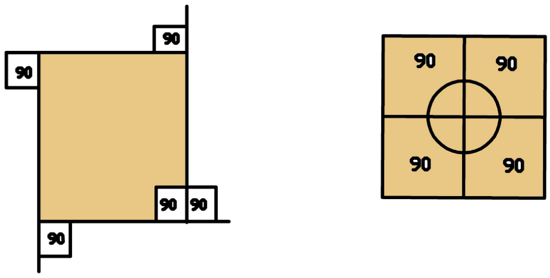
According to Seymour and Britton only triangle and quadrilateral polygon can tessellate the plane by themselves. Their common property is their interior angle. The interior angle of the triangle is 180°. Seymour and Britton describe this property with a picture (Figure 3.52).

Figure 3.52. Demonstration that the sum of the angles of a triangle equals 180°
(Source: Seymour and Britton, 1989)

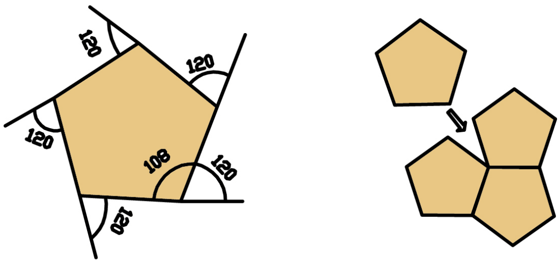
Moreover, interior angles of the quadrilateral polygon are 360°. This is the major important point to find which regular polygon can tessellate the plane. In this case, it should be compare with some polygons to understand its difference from the others. For instance, pentagon cannot tessellate the plane with themselves. The generalization for the general polygon is that if the interior angle of any polygon can be divided into 360° exactly, this polygon can tessellate the plane by themselves. Jaspreet Khaira displays five polygons and their proporties to tessellate a plane in a figure (Figure 3.53).



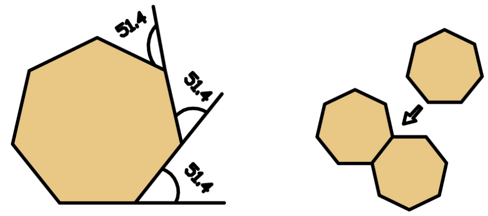
Triangle-The three exterior angles must add to make 360 °. Therefore each one must be (360 /3) 120 °. The interior angles are (180-120) 60 °. 60 is a factor of 360 and so the equilateral will



Square- The four exterior angles must add to make 360 °. Therefore each one must be (360/4) 90 °.90is a factor of 360 and so a square will tessellate.



Pentagon- The five exterior angles must add to make 360 °. Therefore each one must be (360/5) 72 °. The interior angles are (180-72) 108 °.60 is a factor of 360 and so the equilateral will tessellate.



Hexagon- The six exterior angles must add to make 360 °. Therefore each one must be (360/60) 60 °. The interior angles are (180-60) 120 °.120 is a factor of 360 and so a hexagon will tessellate.

Triangle-The three exterior angles must add to make 360 °. Therefore each one must be (360 /3) 120 °. The interior angles are (180-120) 60 °. 60 is a factor of 360 and so the equilateral will tessellate.

Figure 3.53. Proporties of the Regular Polygon
(Source: Khaira, 2009)

Regular polygons combine with each other with edge-to-edge. This means that every side of every tile is an edge of the tessellation. From the point of view, Grünbau and Shephard inquired the possibilities of edge-to-edge tilings on the plane. They say that; the interior angle at each corner of a regular n-gon {n} is (n-2) /n radians (or 180 (n-2)/n degrees) so that if an n1- gon {n1}, an n2- gon {n2},…., an nr –gon {nr}, meet at a vertex of tiling then $\frac{n1-2}{n1}+\dots\dots\dots+\frac{nr-2}{nr}=2$

At the end of their examination of relationship between interior angles and to tessellate plane by regular polygon; they claim that there are 21 types of vertices that arise from different orders in which the n-gons meeting at a vertex can be arranged (Figure 3.54). Moreover, there are 17 species which of a vertex in a tiling is determined by the number of n-gons that meet at the vertex (Figure 3.55) (Grünbaum and Shephard, 1986).

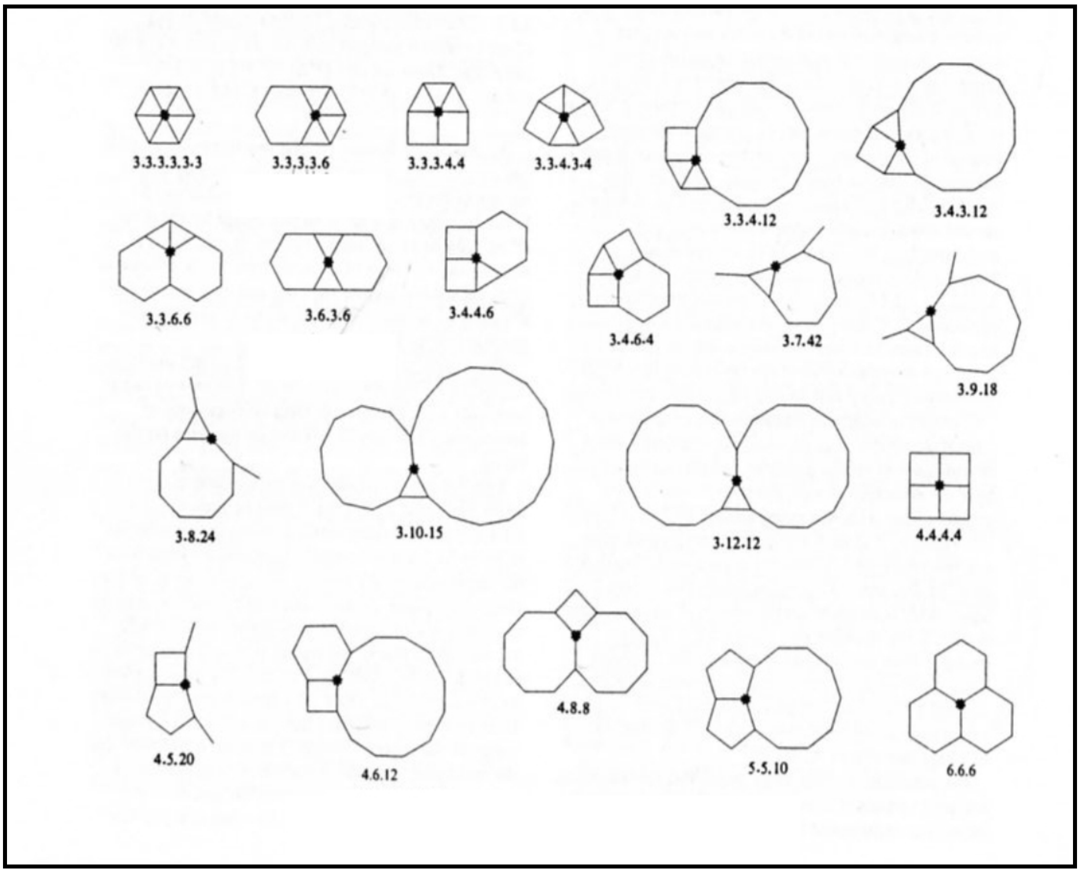


Figure 3.54. 21 types of vertices that are possible with regular polygonal tiles
(Source: Grünbaum and Shephard, 1986)

Species number	$n = 3$	4	5	6	7	8	9	10	12	15	18	20	24	42	
	Number of n-gons meeting at a vertex														
1	6														3.3.3.3.3.3
2	4			1											3.3.3.3.6
3	3	2													3.3.3.4.4 3.3.4.3.4
4	2	1							1						3.3.4.12 3.4.3.12
5	2			2											3.3.6.6 3.6.3.6
6	1	2		1											3.4.4.6 3.4.6.4
7	1				1									1	3.7.42
8	1					1							1		3.8.24
9	1						1				1				3.9.18
10	1							1		1					3.10.15
11	1								2						3.12.12
12		4													4.4.4.4
13		1	1									1			4.5.20
14		1		1					1						4.6.2
15		1				2									4.8.8
16			2					1							5.5.10
17				3											6.6.6

Figure 3.55. The Possible Species and Types of Vertices for Edge-to-edge tiling by regular polygon (Source: Grünbaum and Shephard, 1986)

3.3.2.1.1. Regular Tessellations

Regular tessellation includes a fixed number of specific n-gon at every vertex. According to Kinsey and Moore (2002) “Regular tessellation consists of repeated copies of a single regular polygon, meeting edge to edge so that every vertex has the same configuration.” There are three regular tilings of the plane (Figure 3.56).

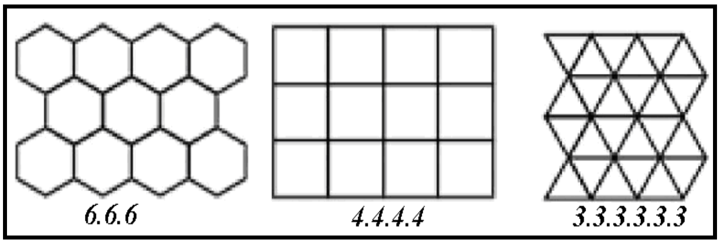


Figure 3.56. Regular tessellations

3.3.2.1.2. Semiregular Tessellations

Two or more convex regular polygons are placed such that the same polygons in the same order surround each polygon vertex is called semi regular tessellations, or sometimes Archimedean tessellations. In the plane, there are eight such tessellations (Figure 3.57).

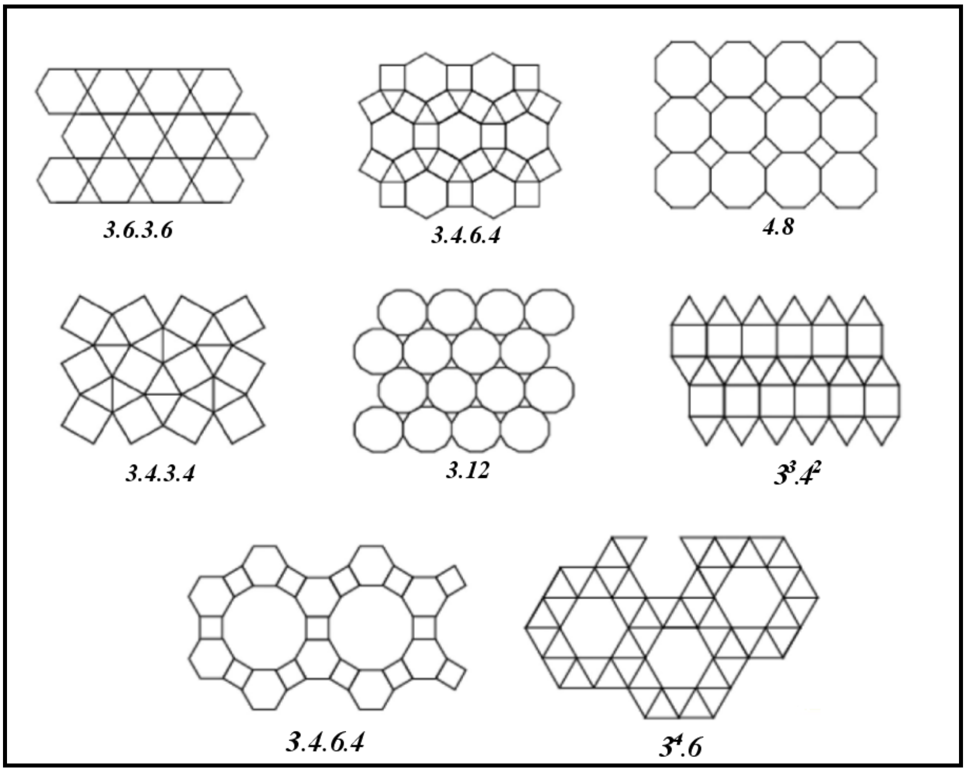


Figure 3.57. Semiregular tessellations
(Source: Wolfram, 2009)

3.3.2.1.3. Demiregular Tessellations

There is some kind of discussion for the meaning of demi-regular tessellation (Figure 3.58). Some of the authors define them as orderly composition of regular and semi-regular tessellation; however, the others define demi-regular tessellation as a tessellation having more than one transitivity class of vertices. Grünbaum and Shephard explain the demiregular tessellation as a k-uniform tessellation.

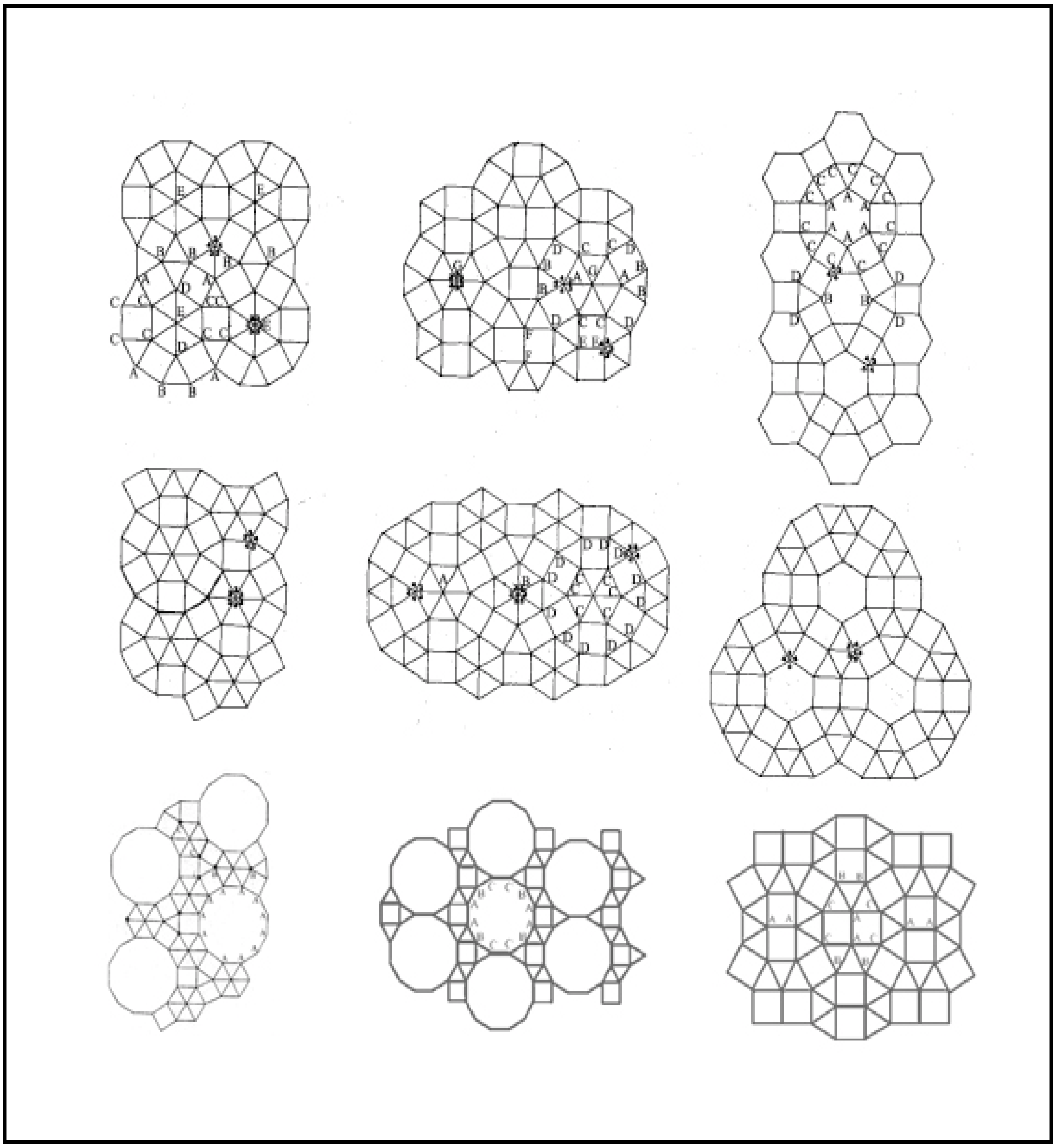


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Figure 3.58. Demiregular tessellations

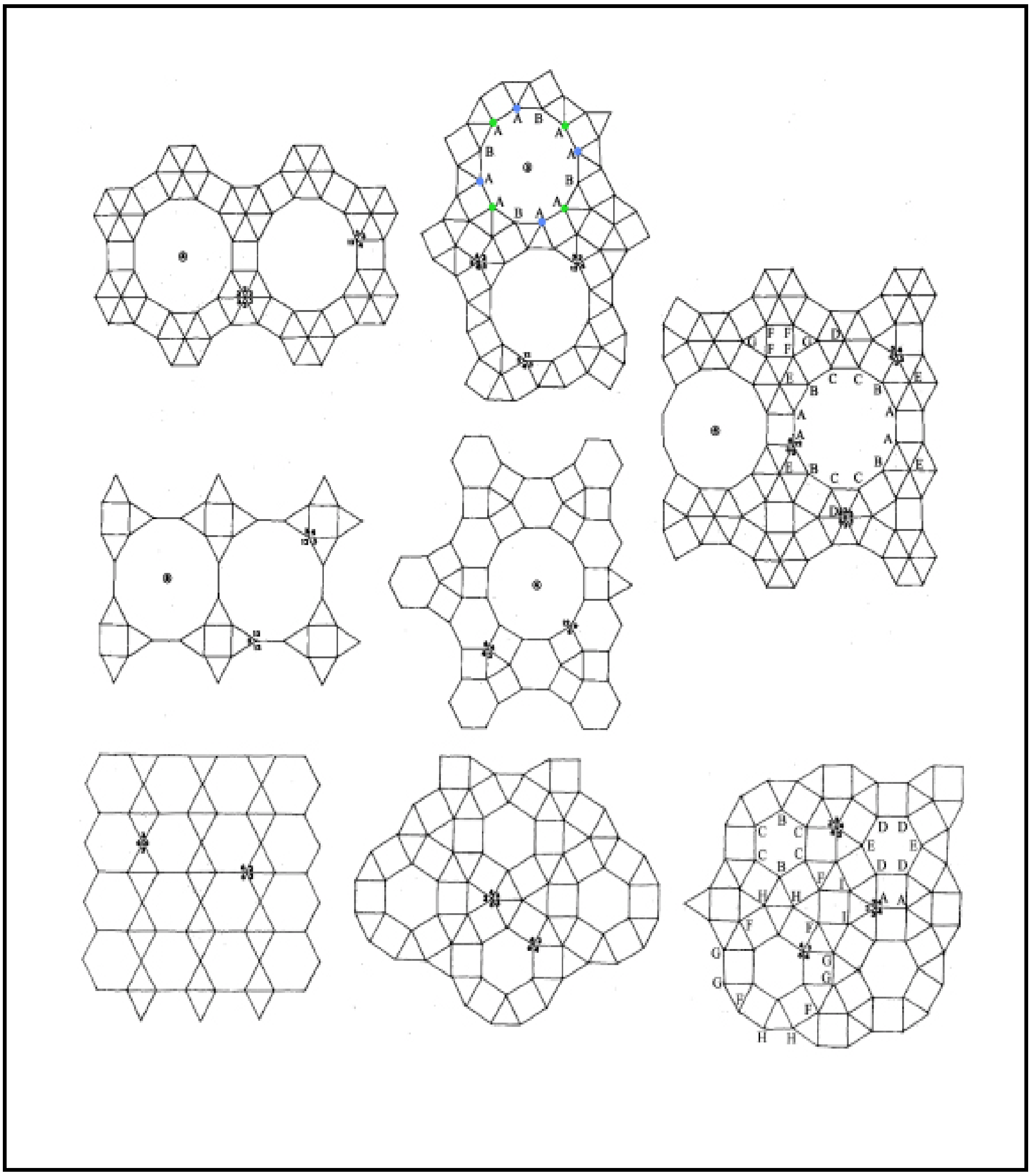


Figure 3.58. (cont.)

3.4. Dual of Tessellation

Every regular polygon has a center point called a cetroid. This centroid is equidistant from the sides and also equdistant from the vertices.

The dual of tessellation (Figure 3.59, 3.60) is formed by joining the center of each polygon as a vertex and joining the centers of all neighboring polygons. The number of sides remains the same. Moroever, Grünbaum and Shephard describe the dual tessellation on their book that, “2 tilings are said to be dually situated to each other if they lie in the same plane, every vertex of one is an interior point of a tile of the other, every tile of one contains precisely one vertex of the other, and crosses, just one edge of the other. Moreover, they point out that, tessellation and its dual are homomorphic to one of the two dually situated tessellations. Thus, any motion or change of scale affects each other.

The triangular and hexagonal tessellations are duals of each other, while the square tessellation is its own dual.

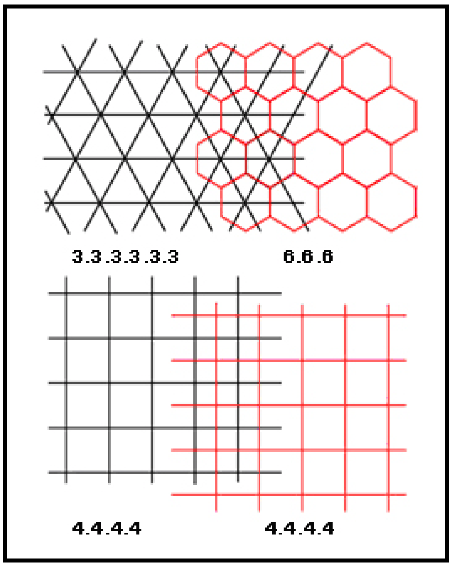


Figure 3.59. Duals of regular tessellations.

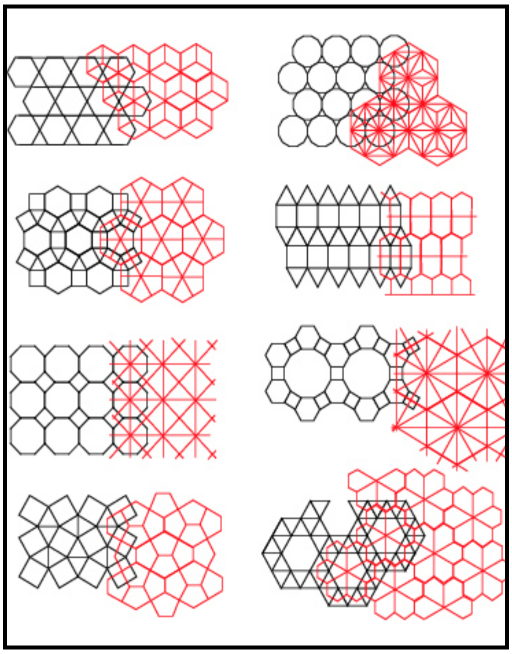


Figure 3.60. Duals of semiregular tessellations.

CHAPTER 4

PLANAR MECHANISMS

This research aims to provide the knowledge about the possibility of the kinetic tessellation on planar surfaces. Thus, this chapter is concentrated on the engineering discipline to understand the behavior of the kinetic structure. The discipline of the mechanism science can be divided into two parts, statics and dynamics. Statics deal with the analysis of stationary systems where time is not a factor, whereas dynamics deal with systems that move throughout the time. Also dynamics can be split into two categories as kinetics and kinematics. Kinematics can be described as the study of motion without regarding the forces while kinetics studies the forces on systems in motion (Norton, 2004).

4.1. Common Definitions in Mechanism Science

This section will present some brief information about the basic definitions in mechanism science.

4.1.1. Types of Motions on Planar Surfaces

In plane or two dimensional Euclidean spaces there are three types of motions, as pure rotation, pure translation and complex motion. In the case of pure rotation the body possesses one point (center of rotation) that has no motion with respect to the stationary frame of reference. All other points on the body describe arcs about that center. In pure translation all points on the body describe parallel (curvilinear or rectilinear) paths. A reference line drawn on the body changes its linear position but does not change its angular orientation. Finally in complex motion there exists a simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and angular orientation (Norton, 2004).

4.1.2. Kinematic Elements of Mechanisms

The individual bodies that make up a mechanism are called the members or link. From the mechanical point of view, a link is assumed to be completely rigid. Links in a mechanism or mechanical manipulator are connected by pairs. The connection between two links is called a joint or a kinematic pair. The joints can be classified in four categories (Norton, 2004),

- the type of contact between the elements, line, point or surface.
- the number of degrees of freedom allowed at the joint.
- the physical closure of the joint: either force or form closed.
- the number of links joined (order of the joint).

Kinematic pairs are divided into two types as higher and lower kinematic pairs with respect to their type of contact. If two elements contact each other with a substantial surface area they are called lower kinematic pairs. The forms of lower-pair elements are geometrically identical, one of them is hallow and the other is solid. On the other hand, if the pair elements are in contact at a point or along a line, they are called higher kinematic pairs (TSAI 1999). Before proceeding further in the lower and higher kinematic pairs, degrees of freedom concept of the joints should be clearly explained. The degrees of freedom allow independent motions between two elements that are joined. For instance, revolute pair allows one rotational independent motion and the slider joint allows one translational independent motion between the joined links (Figure 4.1). As a result they are called as one degrees of freedom joints or pairs.

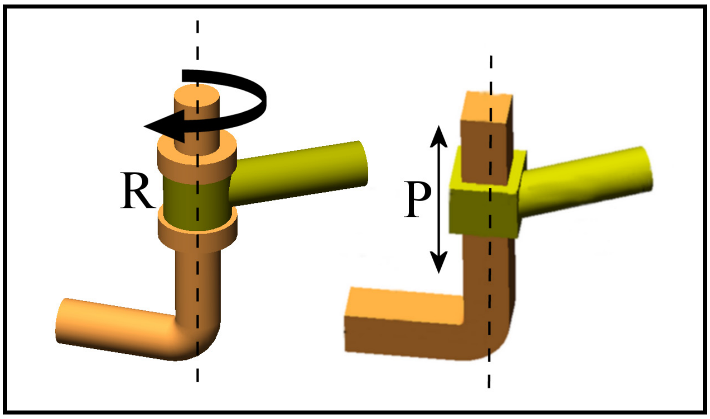


Figure 4.1. Revolute joint and slider joint motions

Similarly if the joint is said to be two degrees of freedom, than it has two independent relative motions in combinations of PP, RR or PR.

4.1.2.1. Lower Kinematic Pairs

As it is discussed, if the two elements of any joint have an area of contact between each other, they are called as lower kinematic pairs. Although various kinds can be introduced, most common ones are revolute, prismatic, helical, cylindrical and spherical joints.

4.1.2.1.1. Revolute Joint

Revolute (R) joints (Figure 4.2) permit two connected elements to rotate with respect to each other around a rotation axis. Revolute joint can also be called as a pin joint, a turning pair or a hinge pair. As mentioned earlier this type of joint has one degree of freedom and imposes five constraints to its elements.

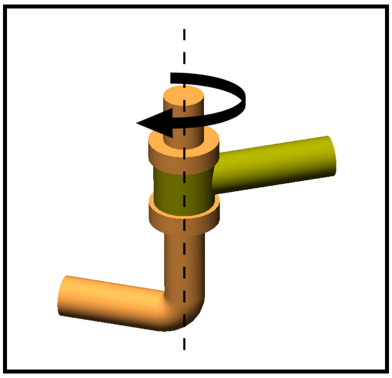


Figure 4.2. Revolute joint

4.1.2.1.2. Prismatic Joint

Similar to the revolute joints, prismatic (P) joints (Figure 4.3) have one degree of freedom and permit two connected elements to translate with respect to each other along a translation axis. This type of joint also imposes five constraints to its elements.

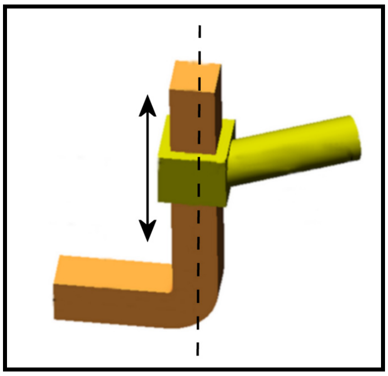


Figure 4.3. Prismatic joint

4.1.2.1.3. Helical Joint

Helical (H) joints (Figure 4.4) allows two connected elements to rotate around, and translate along an axis. Although they are related with two different types of motions, helical joints have one degree of freedom due to the fact that translational and rotational motions are dependent to each other by the pitch (p) of the screw. Helical joints can also be referred as screw joints.

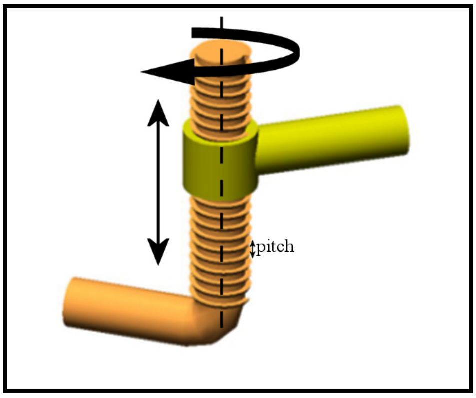


Figure 4.4. Helical joint

4.1.2.1.4. Cylindrical Joint

Cylindrical (C) joints (Figure 4.5) allows two connected elements to rotate around, and translate along an axis similar with the helical joints. On the other hand both of these motions are independent and the joints have two degrees of freedom where they impose four constraints.

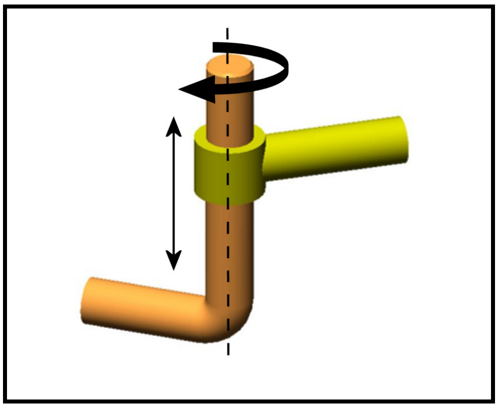


Figure 4.5. Cylindrical joint

4.1.2.1.5. Spherical Joint

Spherical (S) joints (Figure 4.6) allow two connected elements to rotate freely with respect to three axes where the center of the axes and the spherical joint coincide. Spherical joint has three degrees of freedom and imposes three constraints so no translations between the paired elements are permitted.

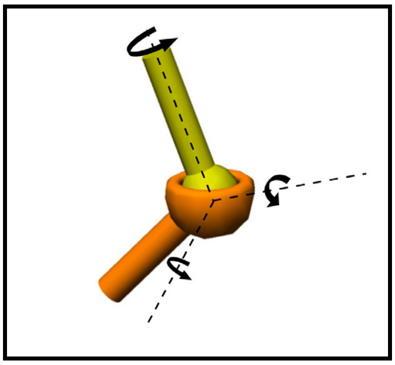


Figure 4.6. Spherical joint

4.1.2.2. Higher Kinematic Pairs

If the two elements of any joint has a point or a line of contact between each other, they are called as higher kinematic pairs. Sphere in cylinder, cylinder on plane and sphere on plane are the common examples of the higher kinematic pairs.

4.1.2.2.1. Sphere in Slot Joint

Sphere in slot (S_c) joints (Figure 4.7) allow four independent motions between their connected elements as three rotational motions around the sphere axes and one translation along the cylinder axis. The joint has four degrees of freedom, imposes two constraints to its elements and has a line type of contact.

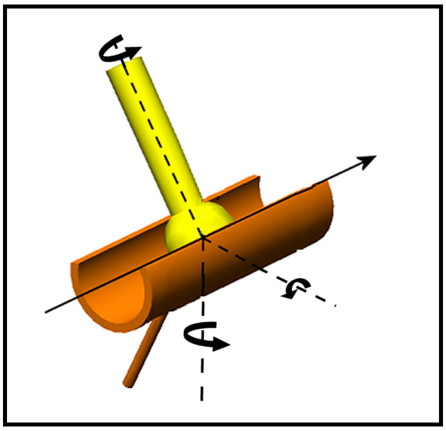


Figure 4.7. Sphere in slot joint

4.1.2.2.2. Cylinder on Plane Joint

Similar to the sphere in slot joint, cylinder on plane (C_p) joints (Figure 4.7) have four degrees of freedom. They allow four independent motions between their connected elements as two rotational motions around the cylinder and plane perpendicular axes and two translation motions along the plane axes. The joint imposes two constraints to its elements and has a line type of contact.

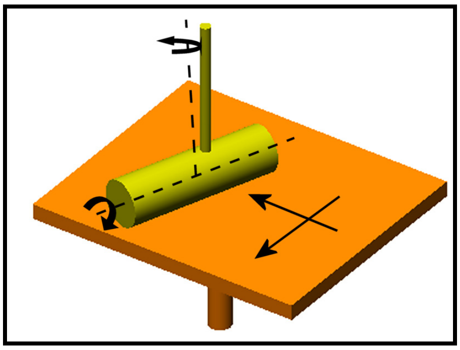


Figure 4.8. Cylinder on plane joint

4.1.2.2.3. Spherical on Plane Joint

Unlike the previous two higher kinematic pairs spherical on plane (S_p) joints (Figure 4.9) have five degrees of freedom and point type of contacts. They allow five independent motions between their connected elements as three rotational motions around the sphere axes and two translation motions along the plane axes. The joint imposes only one constraint to its elements.

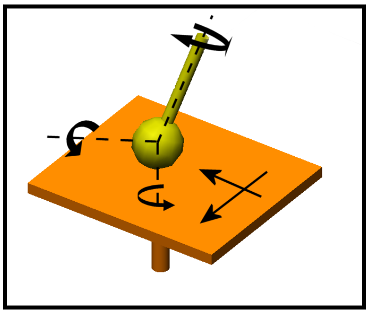


Figure 4.9. Sphere on plane joint

In order to summarize all the introduced lower and higher kinematic pairs, a table can be given for the ease of use (Table 4.1).

Table 4.1. Common lower and higher kinematic joints

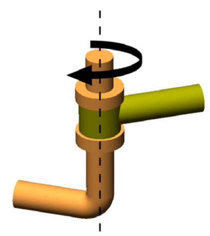
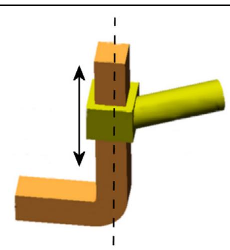
<table>
<tr><th>Name</th><th>Relative Motions</th><th>Degrees of Freedom</th><th>Geometry of Contact</th><th>Representation</th></tr>
<tr><td>Revolute</td><td>R</td><td rowspan="2">1</td><td rowspan="2">Area</td><td></td></tr>
<tr><td>Prismatic</td><td>P</td><td></td></tr>
</table>

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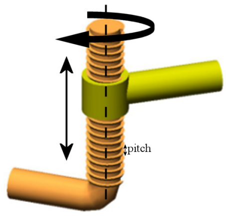
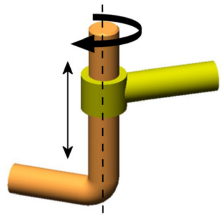
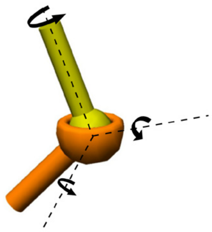
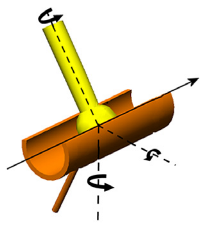
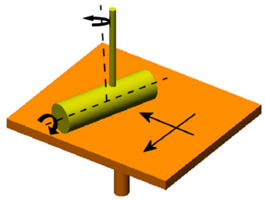
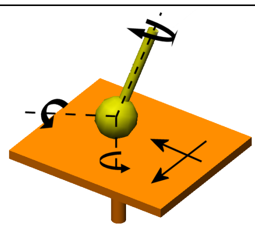
Table 4.1. (cont.)

Name	Relative Motions	Degrees of Freedom	Geometry of Contact	Representation
Helical	RP	1	Area	
Cylindrical	RP	2		
Spherical	RRR	3		
Sphere in Cylinder	RRRP	4	Line	
Cylinder on Plane	RRPP			
Sphere on Plane	RRRPP	5	Point	

4.1.3. Kinematic Chains, Mechanisms and Machines

A kinematic chain is an assembly of links that are connected by joints. Each link in a valid kinematic chain should have at least two distinct joints. If a link has k joints, it is called a k-nary link. Similarly, if a joint connects k links then it is called a k-nary joint (Schmidt, 2006). As an example, links containing only two pair element connections are called binary link, three pair element connections are called ternary link and so on (Figure 4.10). In addition to this, every link in the chain should be connected to the other links. If the chain forms one or more closed loops, it is called closed kinematic chain, otherwise the chain is referred as open (Uicker et al., 2003)

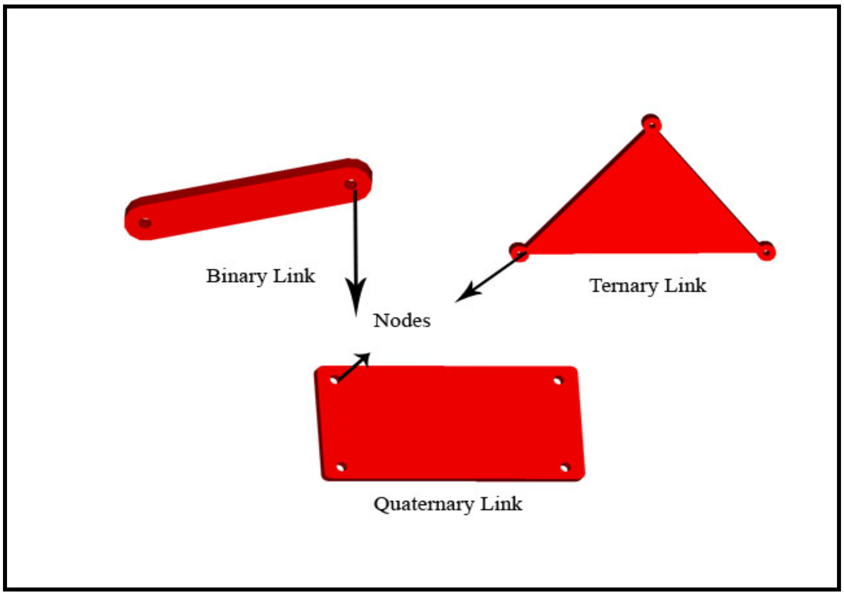


Figure 4.10. Links of different order

A kinematic chain generally refers to a movable chain whose links are connected with pairs and if one of the links of kinematic chain is fixed to the ground, it is called mechanism (Figure 4.11). Fixed link sometimes called base. Tsai (1999) claims in his book that, in a mechanism, one or more links might be assigned as the input links. If input link moves with respect to the base (fixed) link all the other links will move according to the kinematical constraints. In addition to this, Yan (1998) has defined the differences between rigid chain and structure in his book that, a rigid chain shows to an immovable chain that is connected with revolute pairs and simple joints, but if one of the links in a rigid chain is fixed, it is called structure (Figure 4.12).

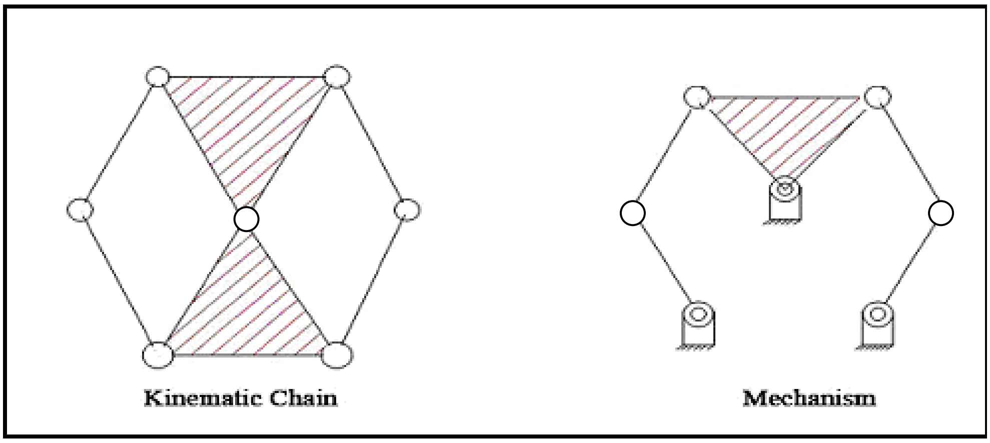


Figure 4.11. Kinematic chain and mechanism

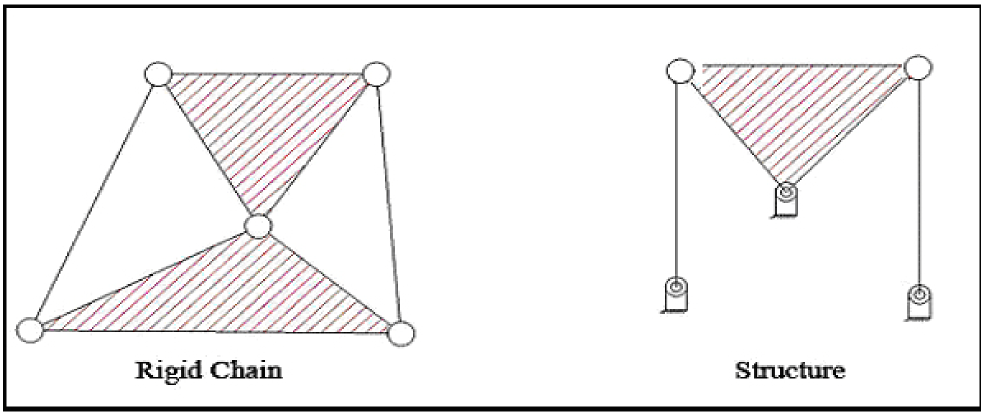


Figure 4.12. Rigid chain and structure

Robert Norton (2004) emphasized in his book that machine means a combination of resistant bodies arranged to compel the mechanical forces of nature to do work accompanied by determined motions. Sometimes the terms mechanism and machine are confused. Tsai explained the differences between mechanism and machine. He said that "an assembly of parts is called mechanism it is used only for the transmission of motion, and it is called machine if it is used to transform external energy into useful work."

4.2. Planar Mechanisms

Mechanisms can be categorized with respect to the relative motion of the rigid bodies, such as, planar spherical and spatial mechanism. In a planar mechanism, all of the relative motions of the rigid body stay in one plane or in parallel plane. Uicker et al.

(2003) added for the definition of planar mechanism that, "this characteristic makes it possible to represent the locus of any chosen point of a planar mechanism in its true size and shape on a single drawing or figure". Planar mechanisms use only revolute and prismatic pairs. On planar mechanism all revolute axes should be normal to the plane of motion and all the prismatic pair axes should be parallel to the plane. The plane four-bar linkage, the plane cam and follower and slider crank are familiar examples of planar mechanisms.

4.3. Design of Mechanisms

In general definition design means a plan or drawings before producing something to see its function or workings. Designing something is vital but also very complicated issue. The process of designing something generally depends on the designer and the discipline of the designer.

This thesis is an interdisciplinary study between architectural, mathematical and mechanical area. As it is mentioned before, the main aim of the thesis is to develop a methodology for kinetic planar mechanism in the light of mathematical knowledge. So, it is very important to analyze the design with both architectural aspect and mechanical aspect to see the place of mathematic on the design process for both disciplines. From this point of view, this part of the chapter will be concentrated on the ways of mechanical engineers in designing mechanisms and their design.

Robert L. Norton (2004) defines the engineering design process on his book as; "…….the processes of applying the various technique and scientific principles for the purpose of defining a device, a process or a system in sufficient detail to permit its realization…….Design may be simple or enormously complex, easy or difficult, mathematical or nonmathematical; it may involve a trivial problem ore one great importance".

Yan (1998) mentions about the engineering design and mechanism and machine design process and show this process on a simple flow chart (Figure 4.13).

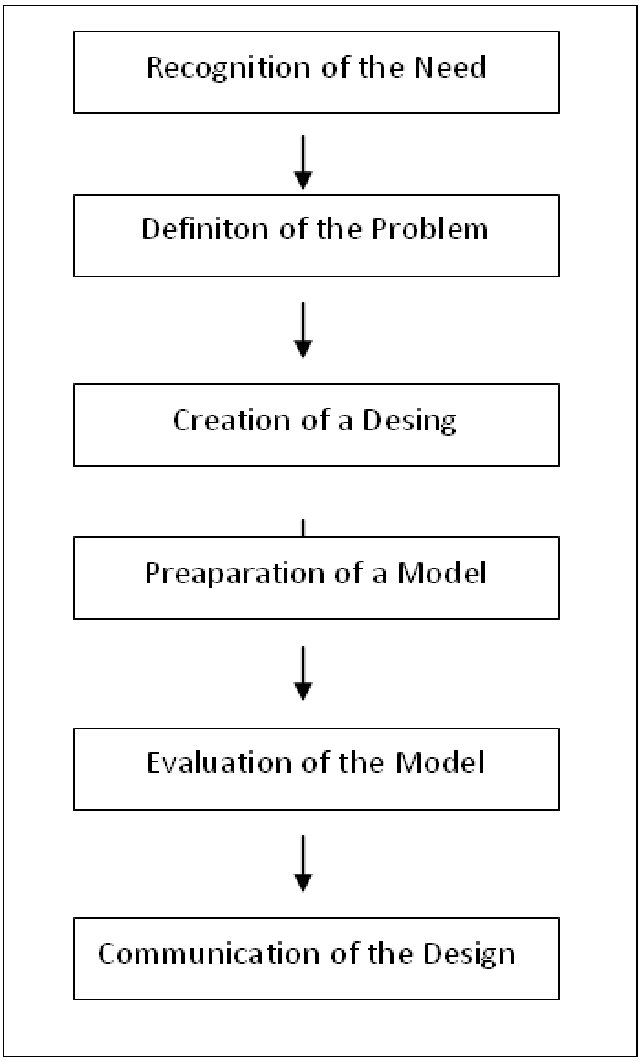


Figure 4.13. Engineering Design Process
(Source: Yan, 1998)

Robert Norton (2004) said that many engineers care about the analysis that means to decompose, to take apart, to realize into its constituent parts during the design processes. However, before any system can be analyzed it must exist, so the first step should be synthesis, which means combining all the parts together. In addition to Norton thoughts, Kent and Noss (2002) point out the processes of engineering design as "…..Even the simplest joint between a column and a beam in a building is so complex you could spend six months analyzing it…..But in building you approximate hugely because you have to get it done in a day, and there is nothing wrong with that, part of the art of structural design is learning how to approximate".

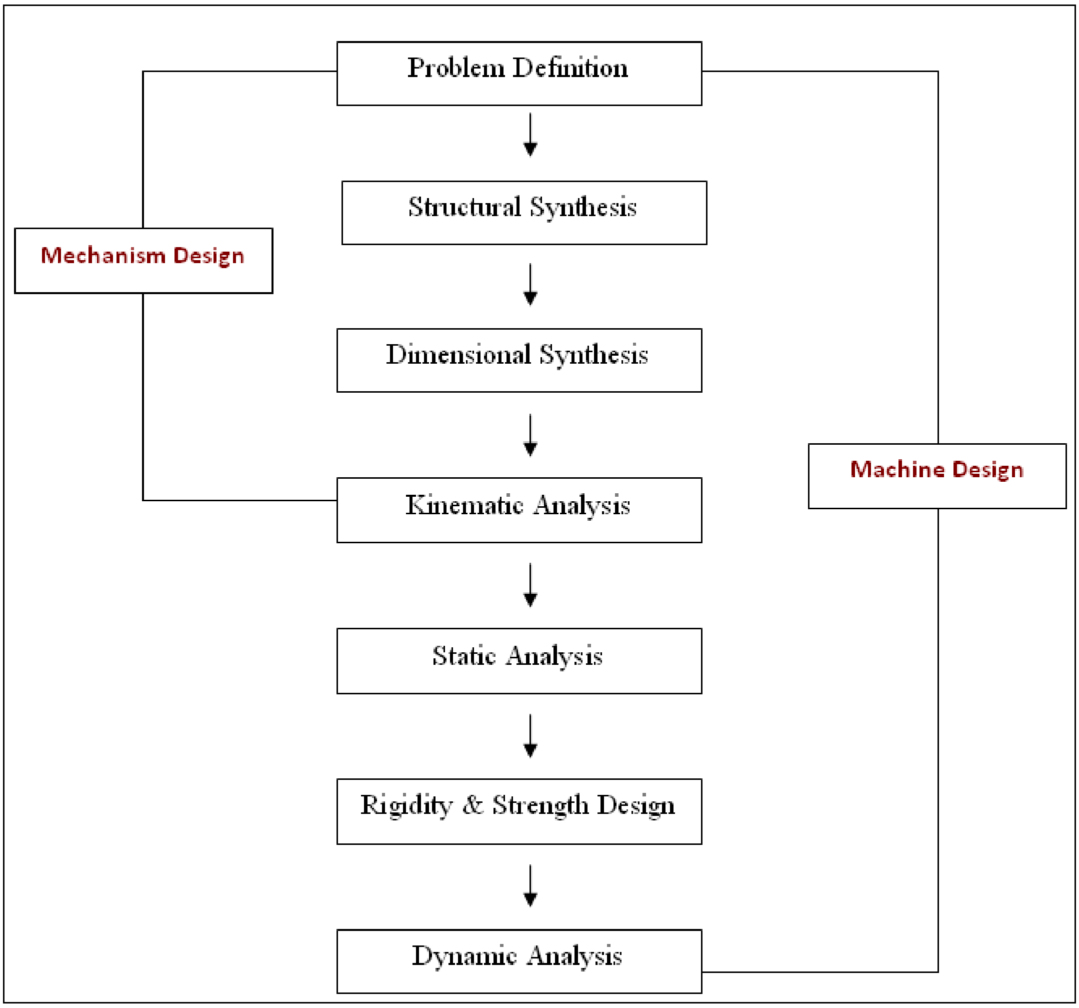


Figure 4.14. Mechanism and machine design process
(Source: Yan, 1998)

4.3.1. Synthesis of Mechanisms

Synthesis of mechanism (Figure 4.15) is the main and earliest stage of mechanism design processes, which can be divided into three stages: type synthesis, number synthesis and dimensional synthesis. Type synthesis and number synthesis can also be called structural synthesis of mechanism. According to Alizade and Bayram, structural synthesis of mechanism is the most important steps for designing mechanical system, using structural synthesis; it is possible to define correct manipulator or robot structure for the mechanism (Alizade and Bayram, 2004). This thesis will focus on type synthesis and number synthesis processes on mechanism design.

Synthesis of mechanisms tries to answer following questions during the whole processes. How many links and joints are required for a desired degree of freedom, what are the link types and how many of each are needed for this link set, how many different link sets satisfy the desired degree of freedom, and how many unique topologies are available from which to choose are those typical questions.

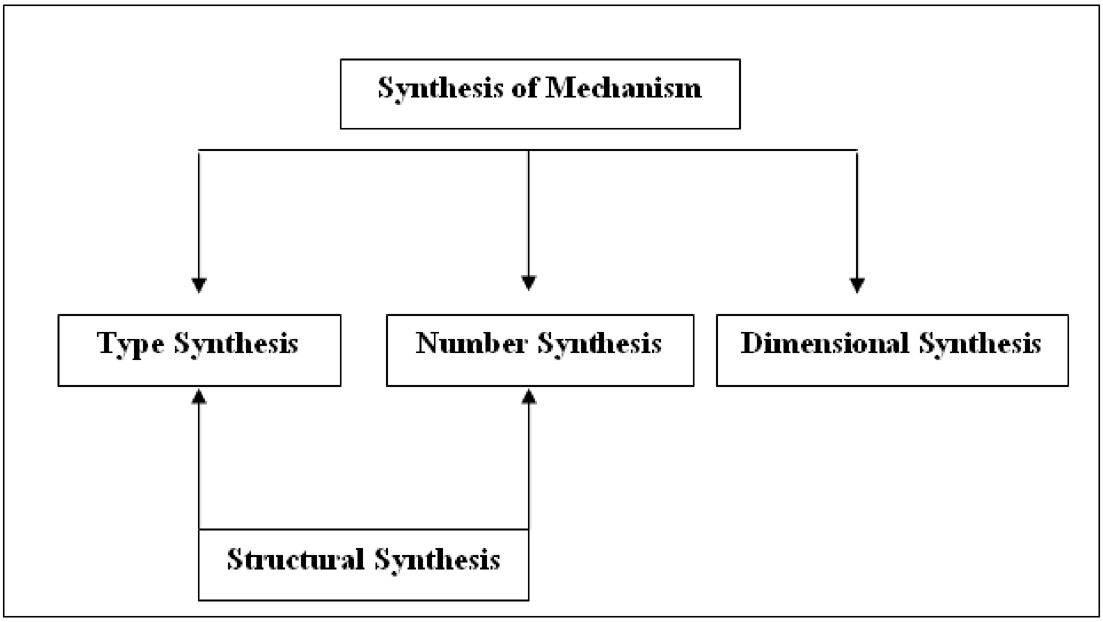


Figure 4.15. Synthesis of mechanism

4.3.1.1. Type Synthesis

Type synthesis of mechanism is a very important part in mechanism and machine theory. The main aim of the type synthesis is to determine all mechanism topologies that satisfy the structural requirements. Type synthesis focuses on selecting and combining various mechanism types such as gear, cam, linkage, belts and pulleys etc. (Pucheta and Cordona, 2008)

Type synthesis mainly consider with three principles .The first principles is to chose simple mechanism and short kinematic chain. This principles provides to reduce weight and decrease cost and accumulated errors, moreover, it improves reliability and increase precision and efficiency. On the other hand the second principle of the type synthesis is to reach smaller size of mechanism and the third one is to reach good dynamic characteristics.

4.3.1.2. Number Synthesis

Number Synthesis is the second step of the synthesis of mechanism and it concentrates on the number of links, and joints to obtain certain mobility (Shigley, 2003).

4.3.1.2.1. Degrees of Freedom of Mechanisms

Degrees of Freedom (DoF) is one of the first fundamental concerns of both design and analysis of mechanisms. Degrees of freedom can also be referred as mobility (M). Mobility of mechanism can be determined directly by counting of the number of links and the number of types of joints that it includes.

On the previous section of this chapter the definition of kinematic chain is explained. Norton, explained the mobility of mechanism by considering the kinematic chains, or mechanisms that might be either open or closed. He claims that a closed mechanism will have no open attachment points or nodes and may have one or more degrees of freedom.

Alizade et al. (2006) explained the historical origin of mobility of mechanism on their research. The earliest study of the calculation of independent loops was done by Euler. Then, the first structural formulas were created by Chebyshev, Sylvester, Gruebler, Somov and Gokhman. The concept of the structural formulas and simple structural groups were developed in the first half of the 20 th century, the productive results to find general methods for determination of the mobility of any mechanism had been obtained by many researchers.

As this study is restricted with planar mechanism, this chapter tries to explain degrees of freedom (mobility) in planar mechanism by using two different mobility formulations.

One of the simplest mobility formulations was created by Gruebler. Before explaining the Gruebler equation, it should be consider that, each link of a planar mechanism has three degrees of freedom when moving relative to the fixed link before they are connected. Robert Norton (2004) also added that, a system of l unconnected links in the same plane will have $3l$ DoF, where the two unconnected links have a total of six DoF. When these links are connected by full joints, they will lose two DoF, leaving four DoF. This reasoning leads to Gruebler's equation for planar mechanisms that are composed of lower kinematic pairs as

$$M = 3(l-1) - 2j \tag{4.1}$$

where M is the mobility, *l* is the total number of links including ground link, j is the total number of one degree of freedom pairs.

As a result of further developments of robotic science, new parameters in the structural formulas describing the real physical essences should be created to be more suitable for the use in practice (Alizade., et al., 2006). Alizade and Freudenstein formulations are one of the important ones for this direction. According to this formulatio

$$\begin{gathered} M = \sum f_i - \sum_{k=1}^{L} \lambda_k + q - j_p \\ L = N - C - B \end{gathered} \tag{4.2}$$

where f_i is the total degrees of freedom of the i^{th} kinematic pair, λ_k is the space or subspace number of the k^{th} independent loop, L is the total number of independent loops, N is the total number of elements on platforms, C is the total number of connections between the platforms, B is the total number of platforms, q is the number of excessive links and j_p is the total number of passive joints. As this study deals with the planar cases (λ=3), the equation will become;

$$M = P_1 - 3L + q - j_p \tag{4.3}$$

where, P_i is the total number of one degree of freedom joints.

CHAPTER 5

DESIGN OF KINETIC PLANAR SURFACE

The main purpose of this study is to develop a methodology to design kinetic planar surfaces with mathematical tessellation techniques in the light of architectural, mechanical and mathematical interdisciplinary approach. This chapter consists of two parts. First part of the chapter focus on regular planar surfaces while second part of the chapter deal with the irregular planar surfaces. Both of the methodologies try to find an answer of the question that which forms or size of platform and linkages should be chosen and how should they join to reach kinetic planar surfaces.

5.1. Method for Kinetic Regular Tessellation

This thesis hands out the tessellation with regarding to their shape that is regular tessellation and semi regular tessellation with respect to the platform shape. This study restricted with regular tessellation. As mentioned on chapter 3 there are only three regular tessellations; square, triangle and hexagonal tessellation. The planar surface will be covered with regular platform to be reached kinetic regular tessellation. In this case, the most important point is, the planar mechanism reaches the regular tessellation on the first point of the motion and on the last point of the motion, which means out of the motion. This aim come to mind to the question that "How can we reach regular kinetic tessellation with using regular platform? This study tries to answer the question with developing some method.

As mentioned above, the main question is how can we reach regular kinetic tessellation with using regular platform? To answer the question this method with based on duality of the tessellation that is described in chapter 3. The method is inspired from the duality of tessellation. As mentioned before any geometric forms dual shape can be reached by using duality. In another words the geometric form transforms to another unique form, and that new form can also be transformed back in to the first form (hexagonal can transform triangle while triangle can transform hexagonal thanks to duality), similar to the motion of a kinetic structure.

Within the scope of this study, the main question requiring realizations of this method are which type of platform can be used, which type of link and joint can be added to the platform, how can platform and link can be combine to reach kinetic mechanism. Therefore, this methods examines the kinetic tessellation into following order consist of 3 steps successively building upon each other.

The purposes of the first step of the methodology to determined type of the platform which are compose of regular kinetic tessellation. The second step of the methodology aims the determined the type of the link by the help of duality of the tessellation. Finally, the last step of the methodology will be the determination of the combination order of the platforms and the links.

5.1.1. Kinetic Square Tessellation

Square tessellation is one of the regular tessellations that can be defined as {4, 4} (Figure 5.1).

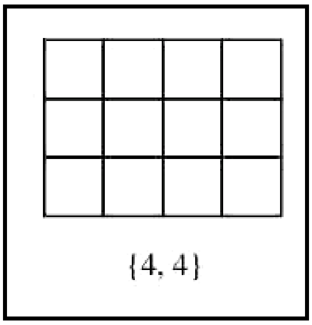


Figure 5.1. Square tessellation

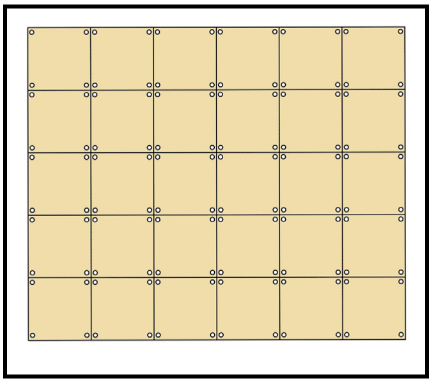


Figure 5.2. Closed form of square tessellation

The first step of the methodology is to find the form of the first link by considering to regular square tessellation (Figure 5.3).

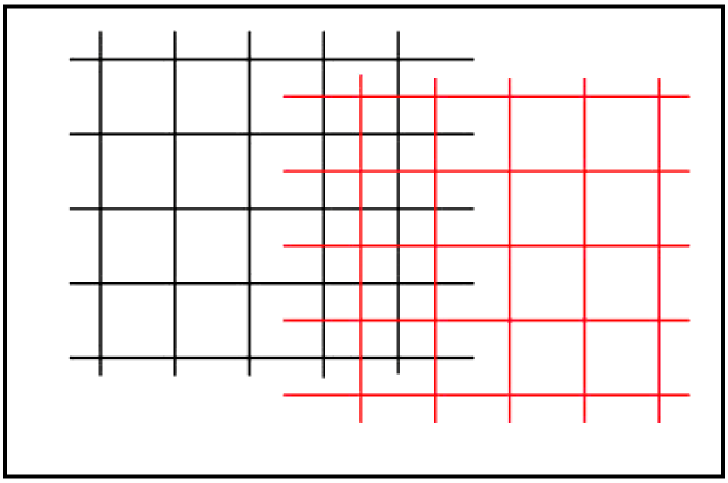


Figure 5.3. Dual tessellation of the square tessellation

Black lines show the real part of the square tessellation while the red lines show the dual of the square tessellation. Those two tessellations give the desired information of the form of the platform and the link.

Form of the link is determined by the square tessellation. In this step, determination of the vertices of the tessellation is an important point. The length of the links will be determined by pointing one vertex to the others around (Figure5.4).

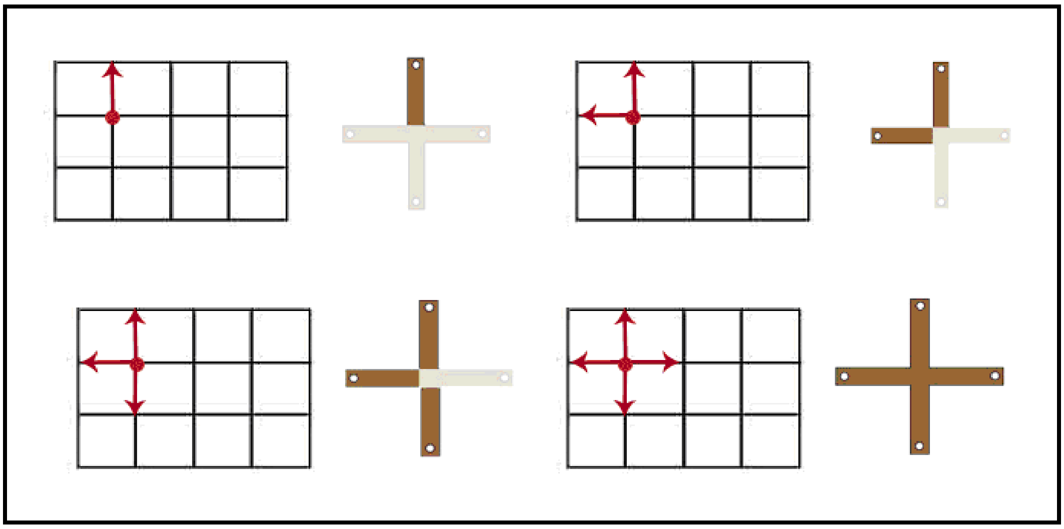


Figure 5.4. Process of obtaining the form of the first link

In the second step dual of the tessellation provides the form of the platform of the mechanism (Figure 5.5). The dual of the square tessellation is its own dual that is formed by taking the center of each polygon as a vertex and joining the centers of adjacent polygons. In this step, the line is drawn successively by starting from the vertices point until the same vertices point is being reached (Figure 5.5).

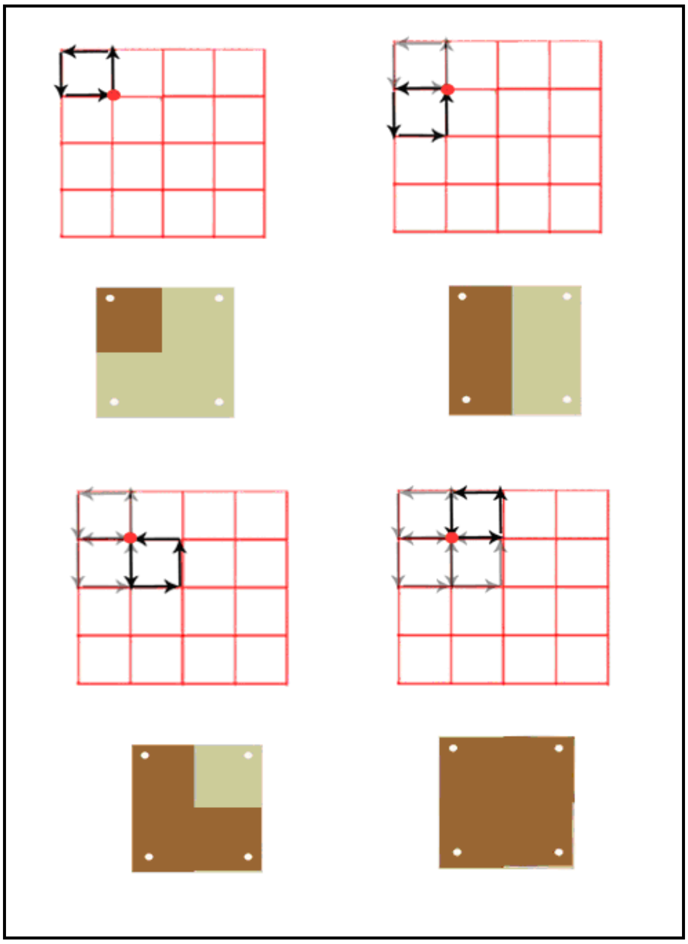


Figure 5.5. Process of obtaining the form of the platform

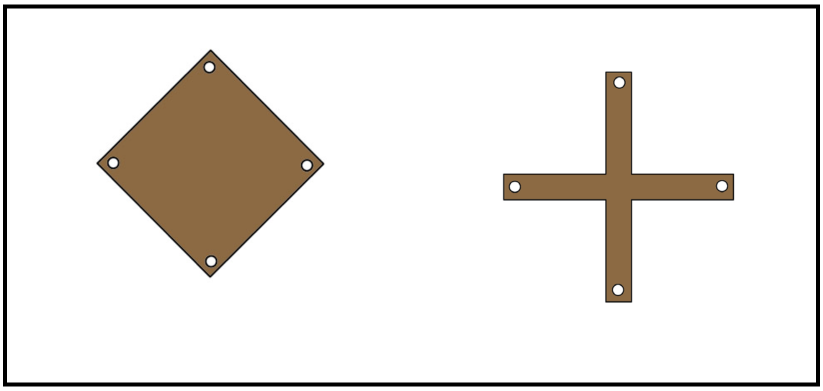


Figure 5.6. Form of square tessellation platform and link

The final step is to determine the placement of combination of link and platform. To achieve this aim we have to look at the intersection of the square tessellation and it's dual. Firstly, platform of the mechanism will be placed to the point where the square tessellation and its dual intersect, and then the links of the mechanism will be added to that platform (Figure 5.7-5.9).

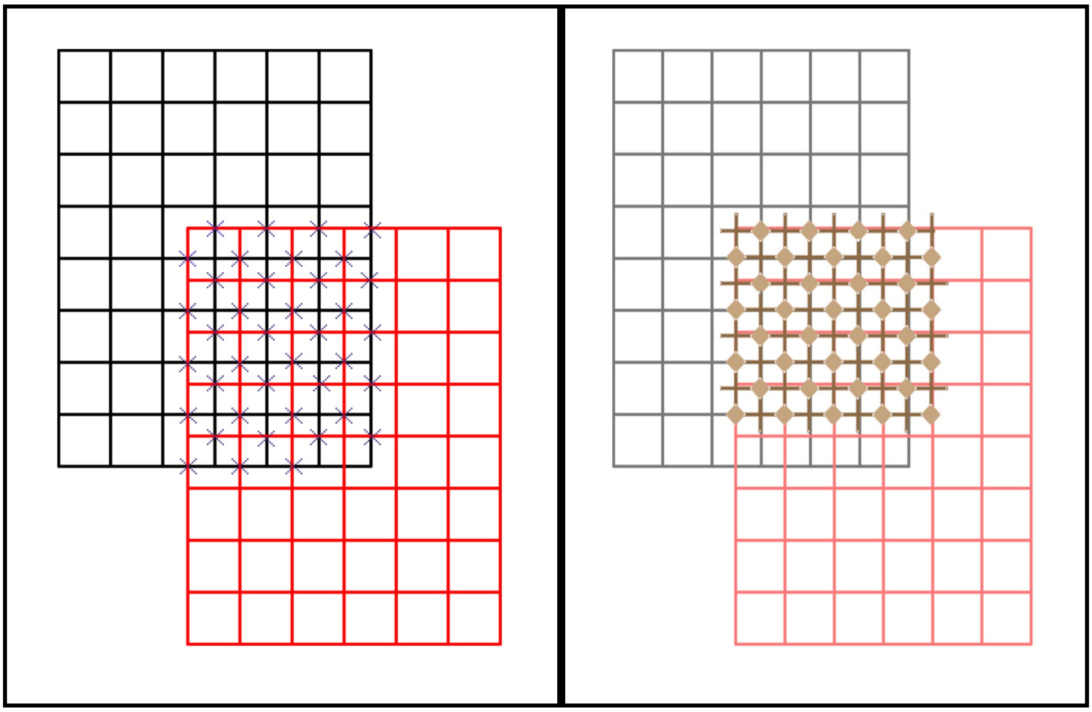


Figure 5.7. Placement of the kinetic square tessellation

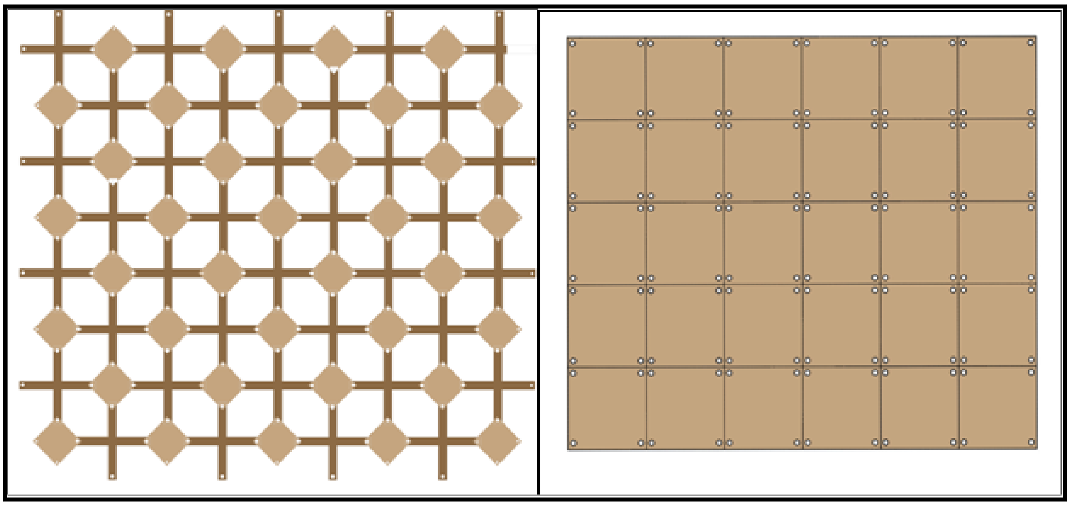


Figure 5.8. Open and closed forms of the kinetic regular tessellation

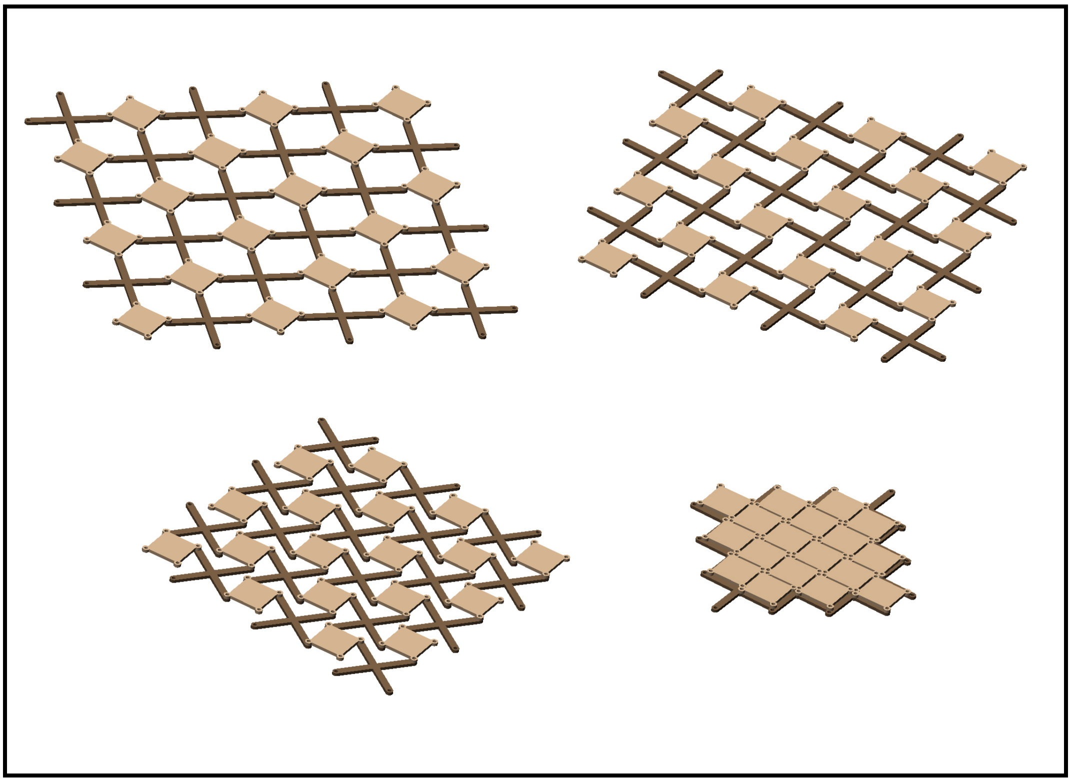


Figure 5.9. Kinetic square tessellation

5.1.2. Kinetic Hexagonal Tessellation

Hexagonal tessellation is named {6, 3} which is described at Figure 5.10.

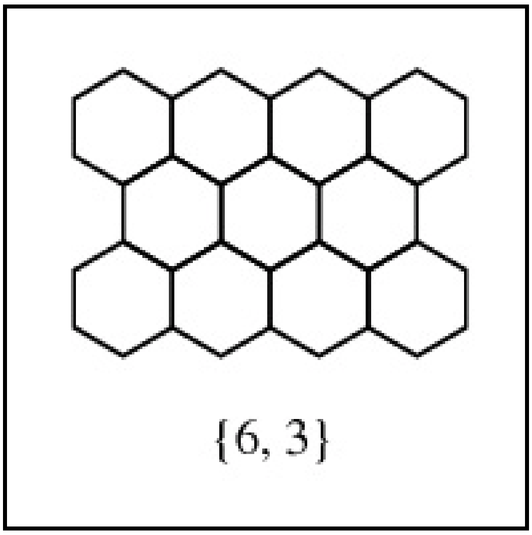


Figure 5.10. Hexagonal tessellation

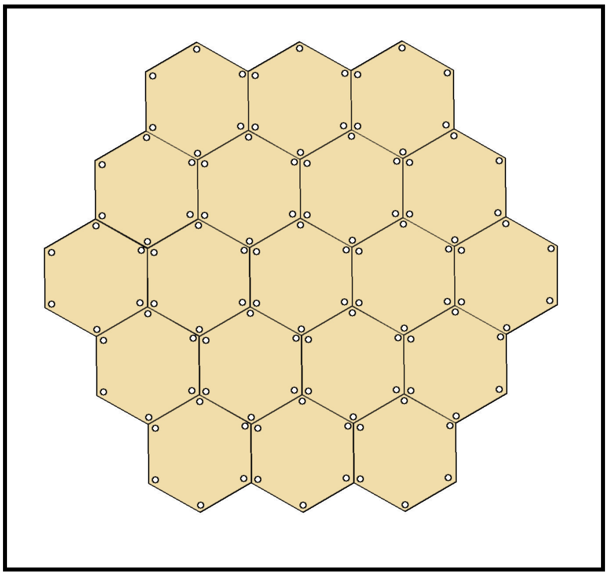


Figure 5.11 Closed form of the kinetic Hexagonal Tessellation

As it is mentioned above, the first step of the methodology is to find the form of the first link (Figure 5.12) by considering to regular hexagonal tessellation.

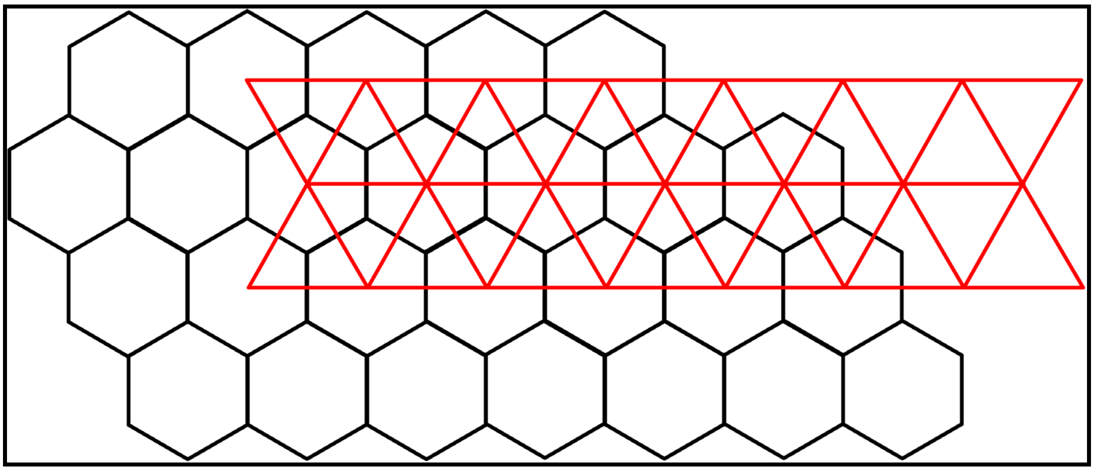


Figure 5.12 Dual tessellation of the hexagonal tessellation

As it can be seen from the Figure 5.12 black line shows the real part of the hexagonal tessellation while the red line shows the dual of the hexagonal tessellation. Form of the link is determined by hexagonal tessellation. The length of the links will be determined by pointing one vertex to the others around as the first step of kinetic square tessellation method.

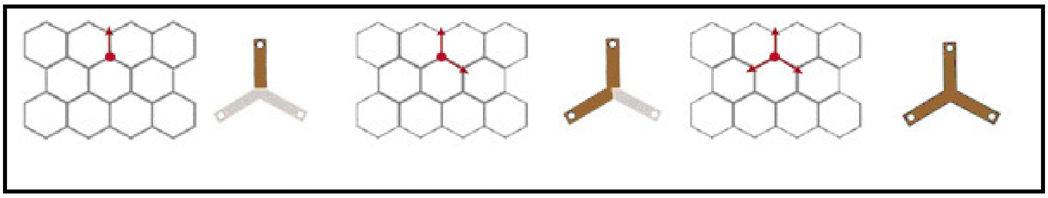


Figure 5.13 Process of obtaining the form of the first link

In the second step dual of the tessellation provides the form of the platform (Figure 5.15) of the mechanism. The dual of the hexagonal tessellation is triangular tessellation that is formed by taking the center of each polygon as a vertex and joining the centers of adjacent polygons. In this step, the line is drawn successively by starting from the vertex point until the same vertex point is being reached as it is mentioned on the second step of the kinetic square tessellation .

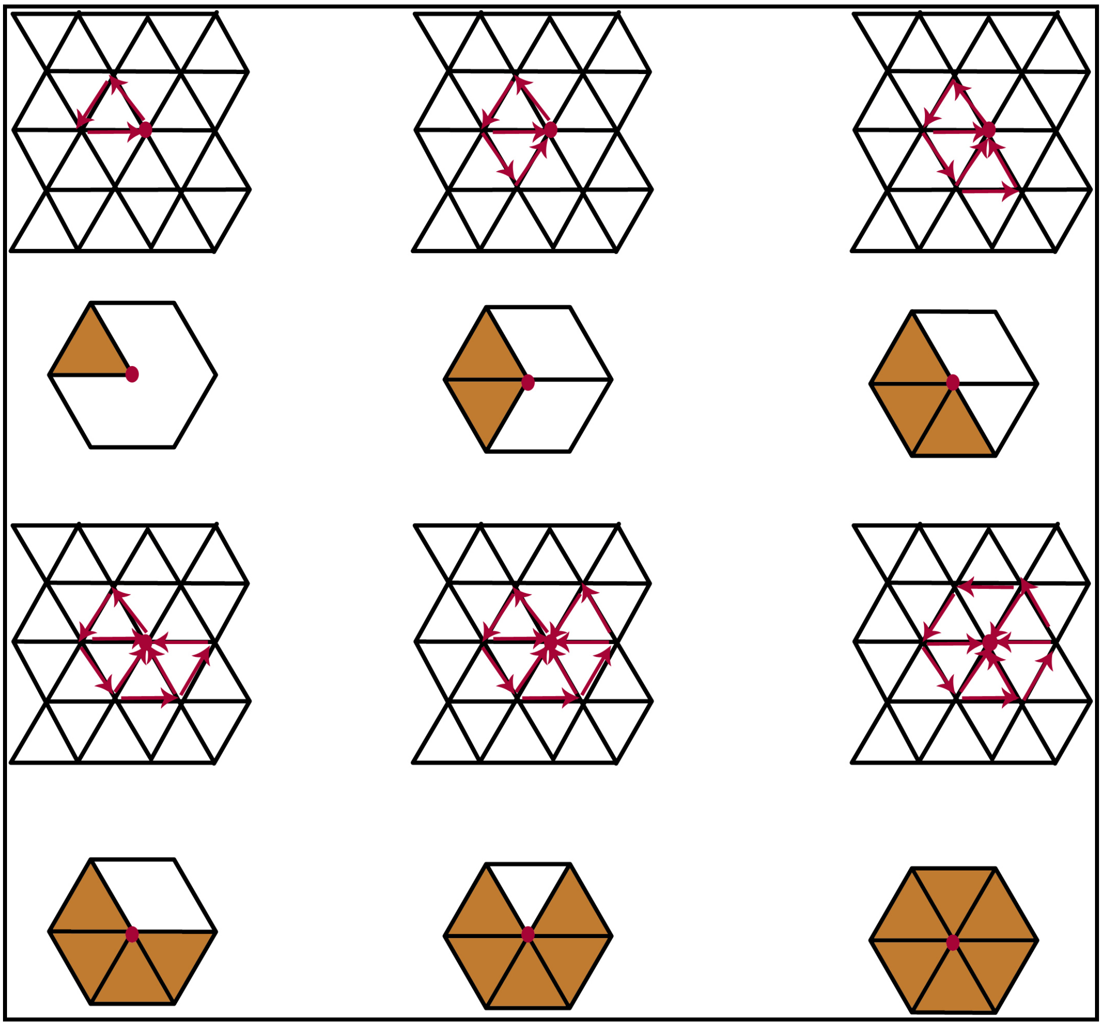


Figure 5.14. Process of obtaining the form of the platform

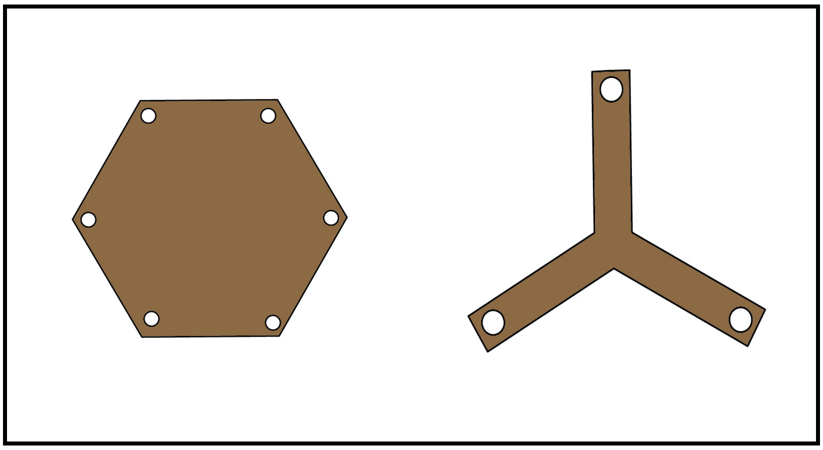


Figure 5.15. Form of hexagonal tessellation platform and link

The third step is determined the placement of the combination of link and platform. As we mentioned above, intersection of the hexagonal tessellation and it's dual that is triangular tessellation. Firstly, platform of the mechanism will be placed to the point where the hexagonal tessellation and its dual intersect, and then the links of the mechanism will be added to that platform (Figure 5.16).

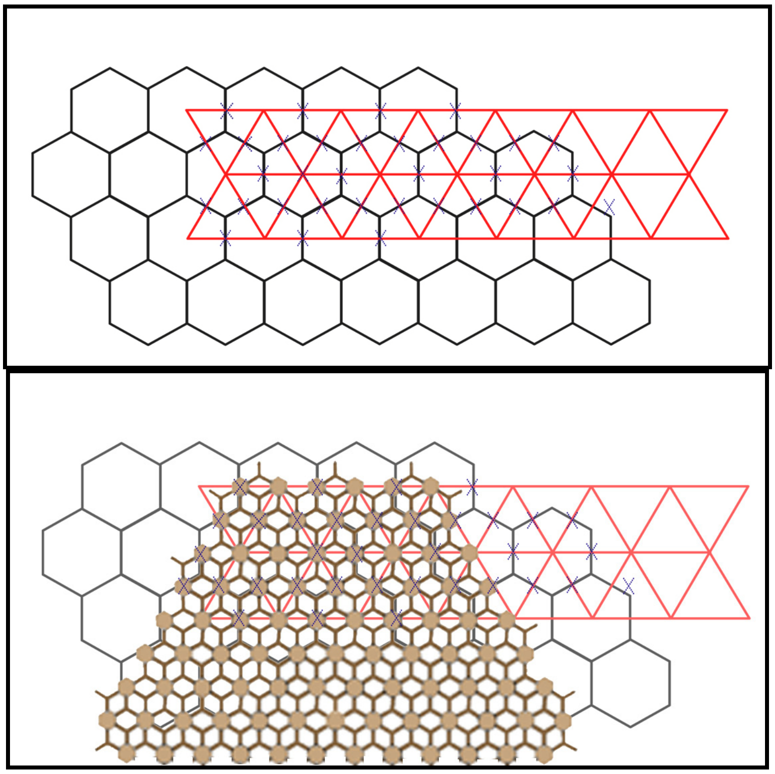


Figure 5.16. Placement of the kinetic hexagonal tessellation

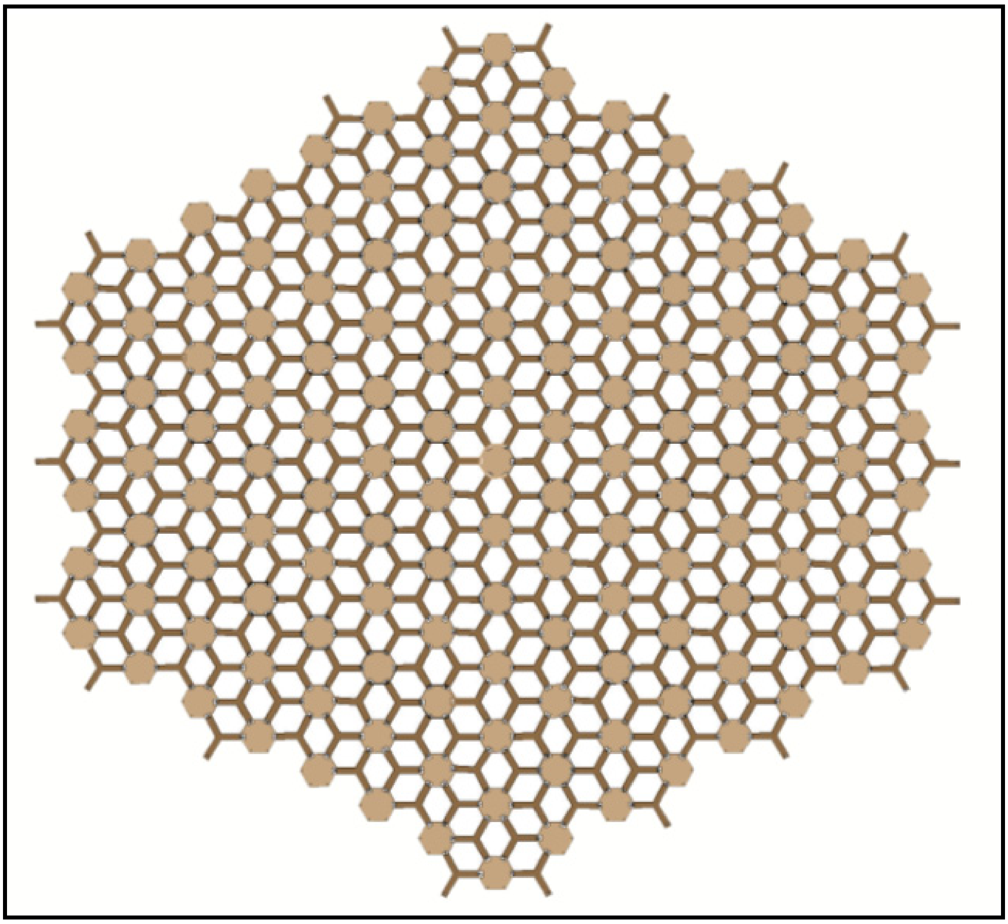


Figure 5.17. Open form of the kinetic hexagonal tessellation

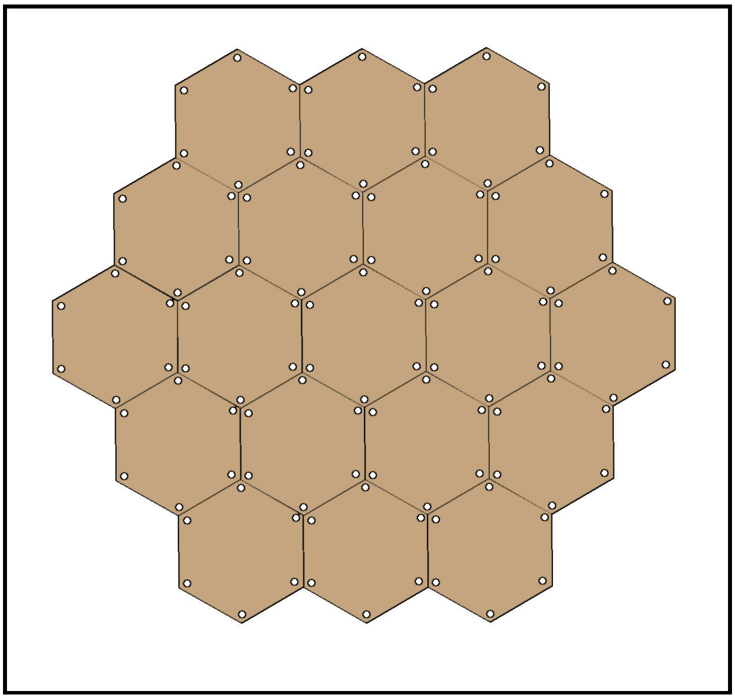


Figure 5.18 Closed form of the kinetic square tessellation

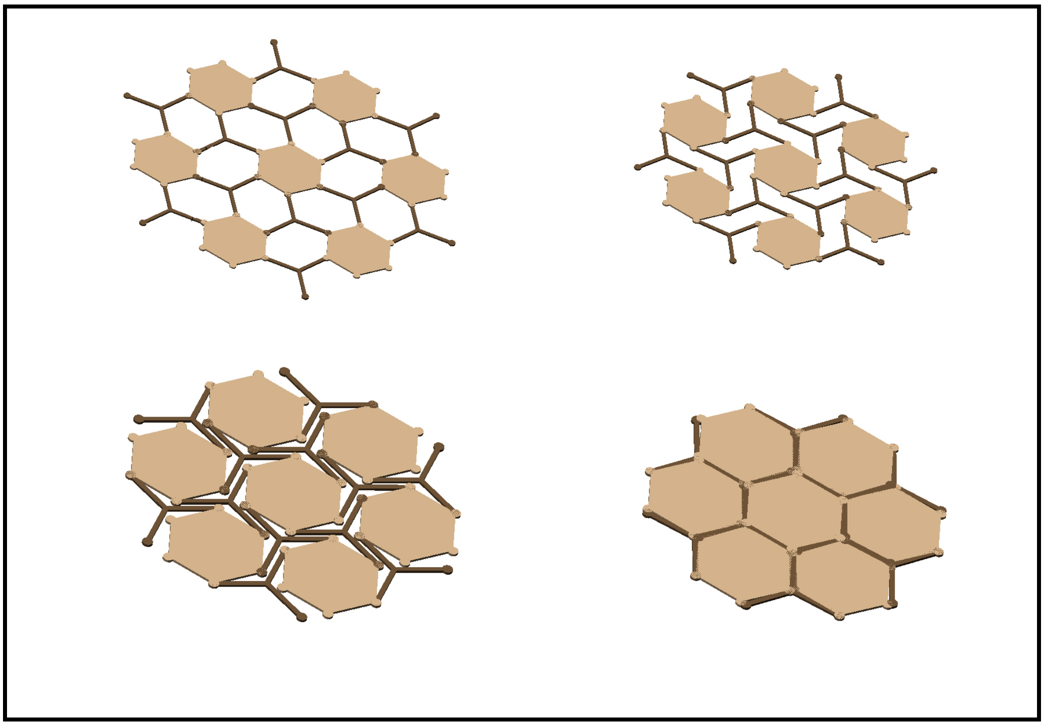


Figure 5.19. Kinetic hexagonal tessellation

This methodology can be applied to triangular tessellation too. Due to triangle and hexagonal tessellation dual of each other, the process of triangular tessellation is near the hexagonal one. In this case, form of the first link is consisting of 6 legs and platform is triangle (Figure 5.20).

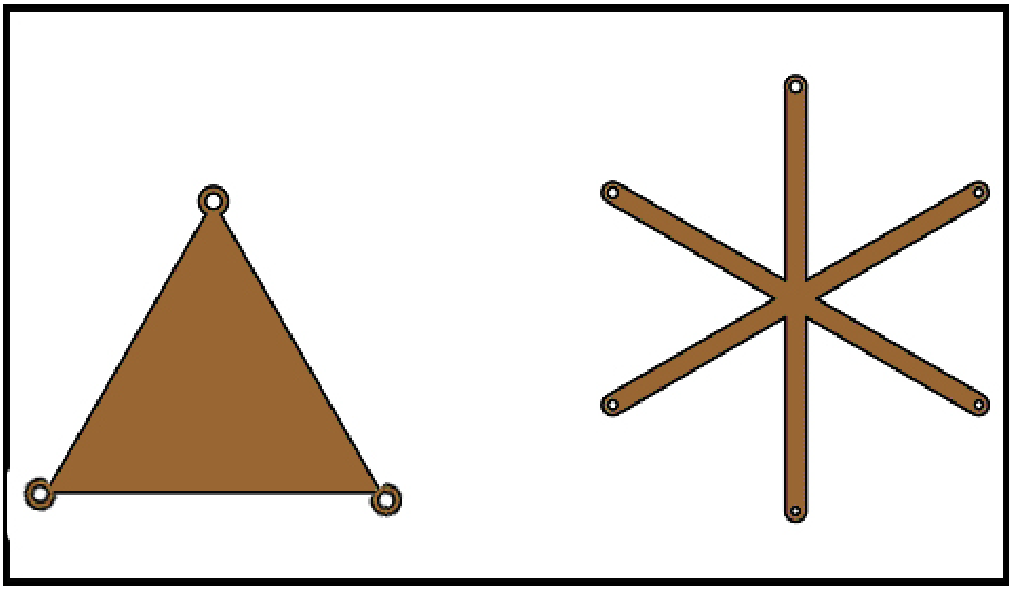


Figure 5.20. Platform and link of triangle tessellation

5.1.3. Mobility of the Kinetic Regular Tessellation

After explaining method of the regular kinetic tessellation, now let's construct the mobility calculation algorithm of the kinetic regular tessellation by using two different mobility formulations (Equations 1-2).

According to Freudenstein – Alizade, mobility criterion formulation, where f_i is the degrees of freedom of i[th] kinematic pair, λ_k is the space or subspace number of the k[th] independent loops, L is the total number of independent loops, q is the number of excessive links and j_p is the number of passive joints.

$$\sum f_i - \sum_{k=1}^{L} \lambda_k + q - j_p = M \tag{5.1}$$

According to Grübler mobility criterion formulation, where l is the total number of links including ground link and P_i is the total number of joints with i[th] degrees of freedom.

$$\lambda(l-1) - \sum_{i=1}^{\lambda-1} (\lambda - i)P_i + q - j_p \tag{5.2}$$

As the study deals with the planar cases (λ =3), the equations will become;

$$P_1 - 3L + q - j_p = M \tag{5.3}$$

$$3(l-1) - 2P_1 + q - j_p = M \tag{5.4}$$

In order to start the process the smallest mobile element of the regular tessellation should be considered (Figure 5.21). As shown in table 5.1 if the equation variables are inserted to the formulations, the mobility of the smallest system will be calculated as unity (M=1).

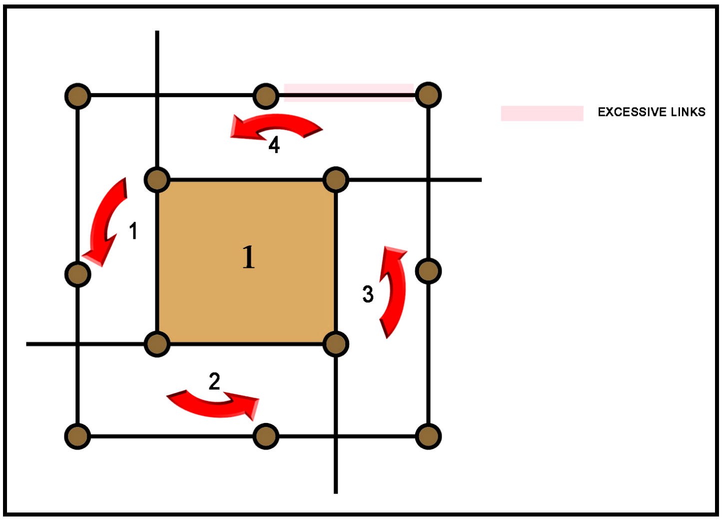


Figure 5.21. The smallest mobile element of regular tessellation

Table 5.1. Equation variables of Figure 5.21

L	l	j_p	q	P_1
4	9	0	1	12

After calculating the mobility of the first module, let's iterate the system by sharing loops (Figure 5.22). As seen in figure, two modules are sharing one loop and the equation variables are shown in table 5.2.

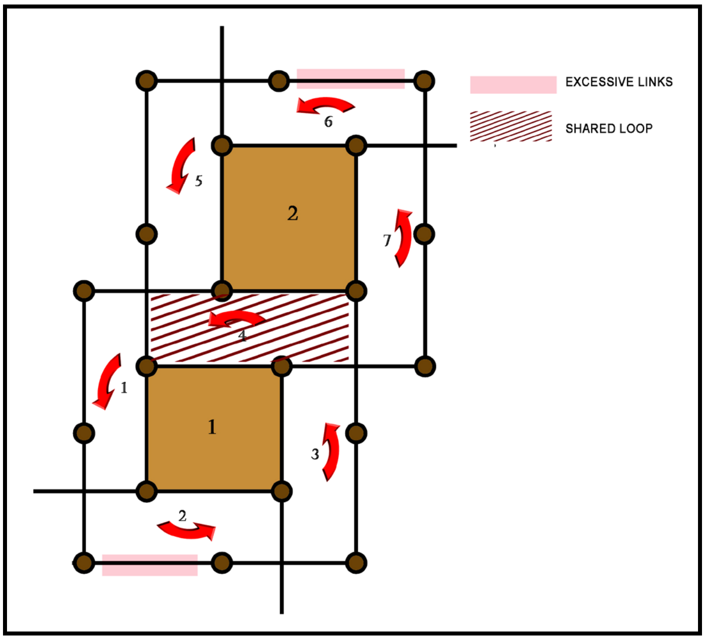


Figure 5.22. The first iterated mobile regular tessellation modul

Table 5.2. Equation variables of Figure 5.22

L	l	j_p	q	P_1
7	14	0	1+1	20

Note that each addition of module that shares only one loop will add one more excessive link to the system. If the mobility of the resulting system is to be calculated, it will be revealed again as unity (M=1). Now let's iterate the structure with one more module.

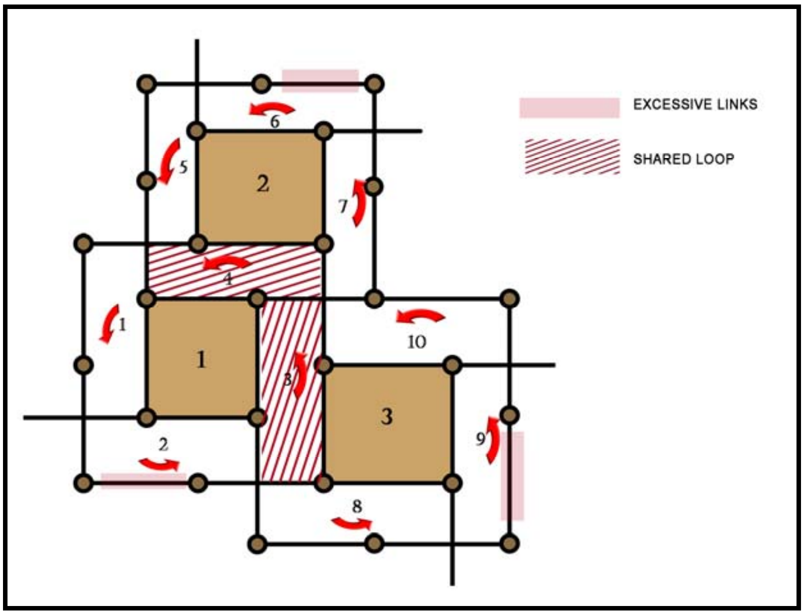


Figure 5.23. The second iterated mobile regular tessellation module

Table 5.3. Equation variables of Figure 5.23

L	l	j_p	q	P_1
10	19	0	2+1	28

Similar to the second iteration the third module (Figure 5.23) will add an additional excessive link to the system. Using the parameters in table the mobility of the resulting system will also revealed to be one (M=1).

After two identical iterations, let’s carry out the most important one. This time the iterated module will share two loops with the previous system (Figure 5.24).

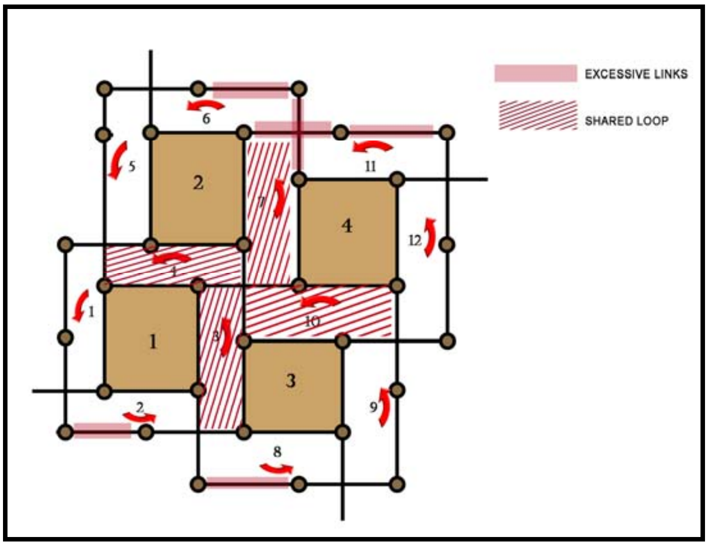


Figure 5.24. The fourth iterated mobile regular tessellation module

Table 5.4. Equation variables of Figure 5.24

L	l	j_p	q	P_1
12	20	0	4+1	32

Using the new variables of the system (Table 4), the mobility of the system will again be calculated as unity.

It should be again noted that, the most important point in the last iteration is the fact that sharing two loops with one additional module will add two more excessive links to the system instead of one. If the iteration procedure is combined, a new theorem can be introduced to the literature related with the mobility calculations of the regular tessellations.

Theorem: *Number of redundant or excessive links is equal to the one plus the number of loops that will be shared during the whole iteration process.*

5.2. Method for Kinetic Irregular Surfaces with Regular Tessellation

5.2.1. Irregular Surfaces

Mathematical tessellation technique is used for covering planar surfaces. The method of covering a planar surface with regular tessellation which is explained in the last part of the chapter is used to tessellate regular geometric form. For example square, hexagon, triangle etc (Figure 5.25).

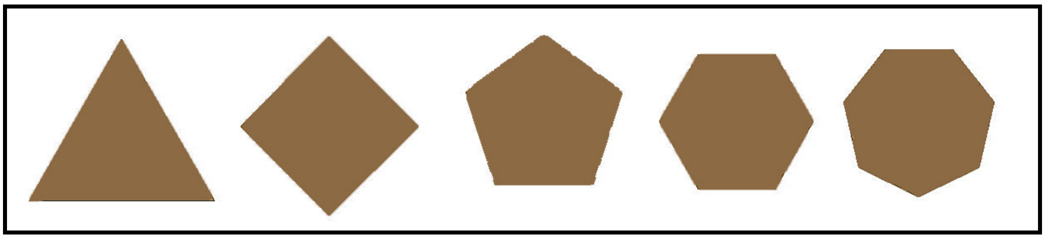


Figure 5.25. Regular geometric surfaces

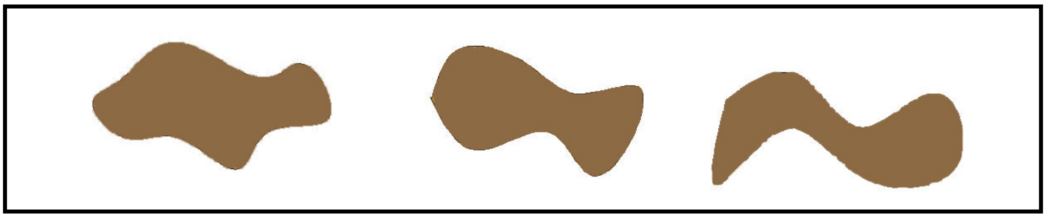
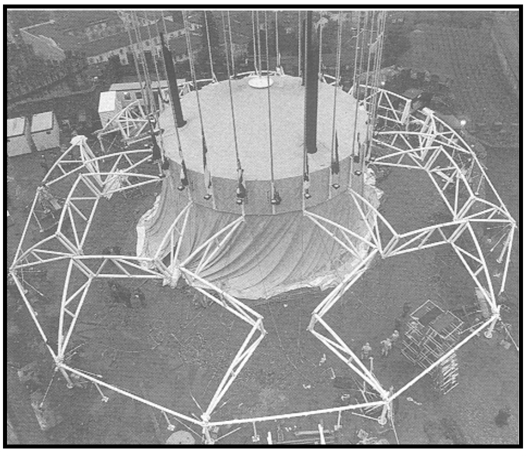


Figure 5.26. Irregular geometric surfaces

Though, regular geometric form is not sufficient for more complex surface design. Especially, after Modernism, architects tried to find different forms in their design, so, contemporary architecture (both static and kinetic), have used to irregular form to their building .This study have focus on kinetic architecture.

Kinetic aspect of the architecture suggests mainly a new concept of form with kinetic parts that is capable of being transformable, deployable or foldable. Mostly, kinetic buildings reach its changeable form with kinetic structure; however, kinetic structure is not enough to cover the all area of the building. To solve the problem, researchers generally use membrane or any kind of flexible, flat material to cover among the kinetic structures (Figure 5.26-5.32). In order to prevent this common behaviour kinetic surfaces are needed. In the light of this, irregular kinetic surfaces gain importance for this thesis. Figure 5.27 and Figure 5.29 display some kinetic structures, Figure 5.28 and Figure 5.30 show the kinetic surface where its form can be changed according to the kinetic structural movement.



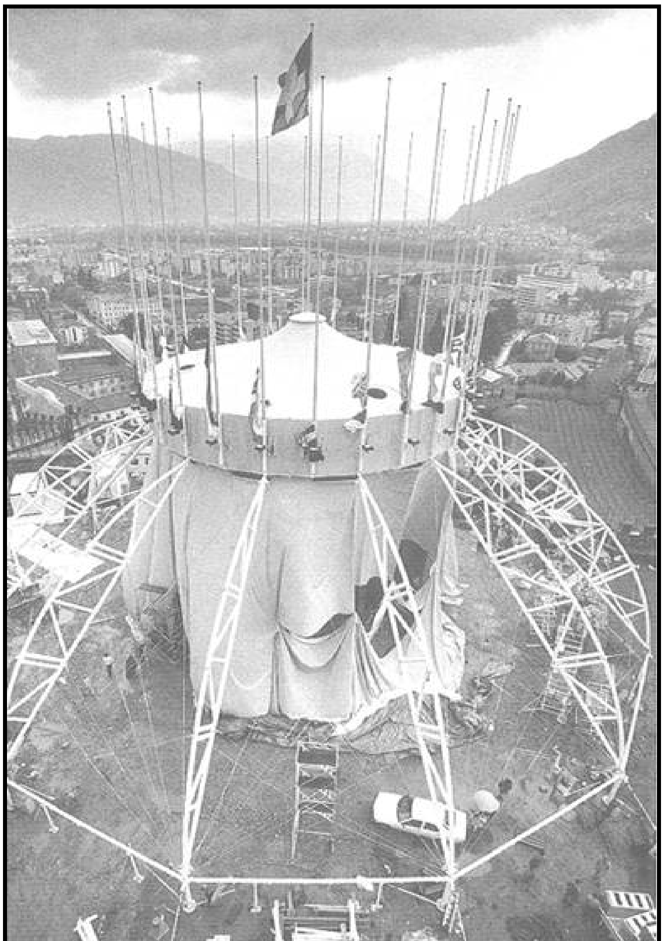
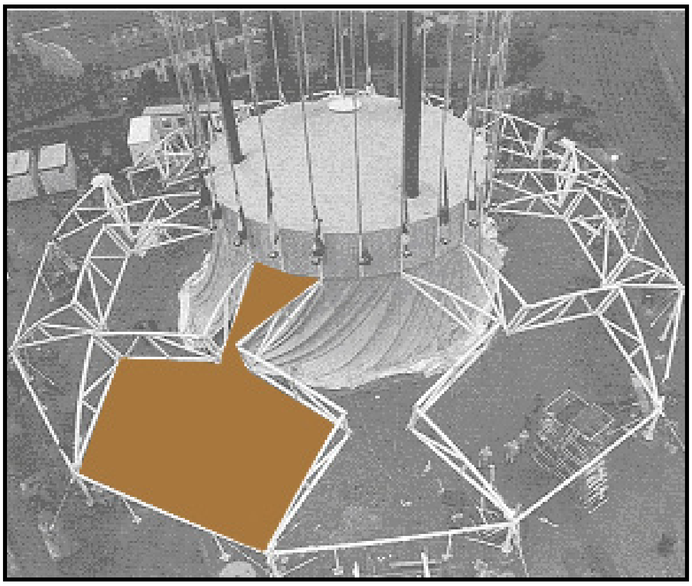


Figure 5.27. Kinetic structures
(Source: Korkmaz, 2004)



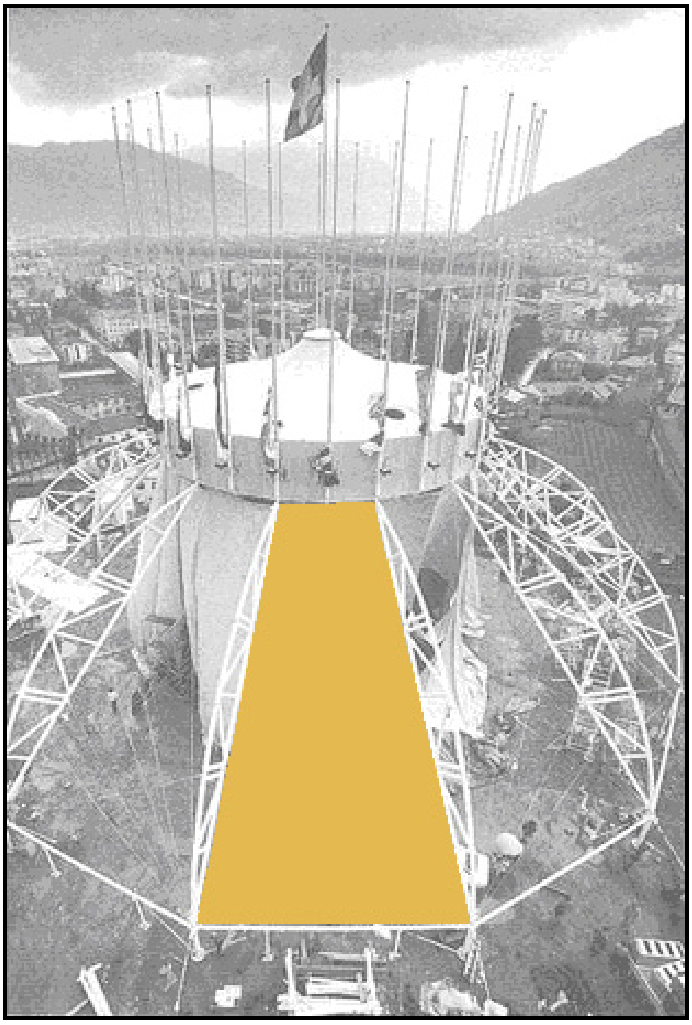


Figure 5.28. Kinetic structures and kinetic surface between them
(Source: Korkmaz, 2004)

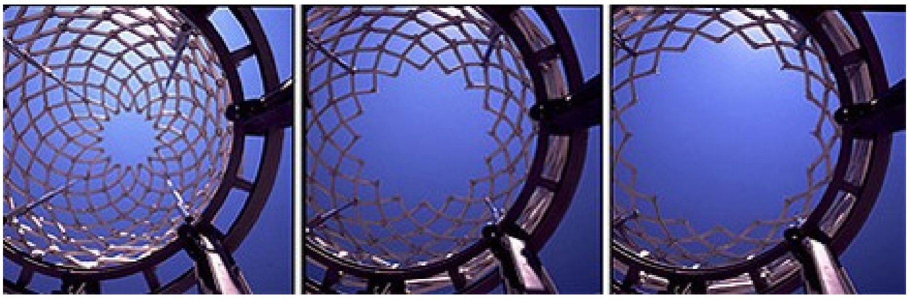


Figure 5.29. Kinetic Structures
(Source: Hoberman, 2010)

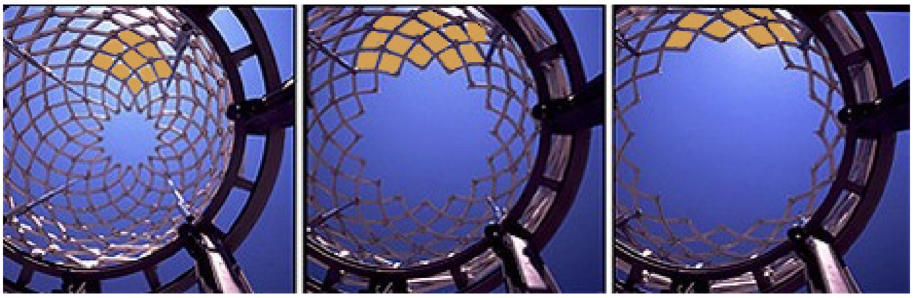


Figure 5.30 Kinetic Structures and kinetic surface between them
(Source: Hoberman, 2010)

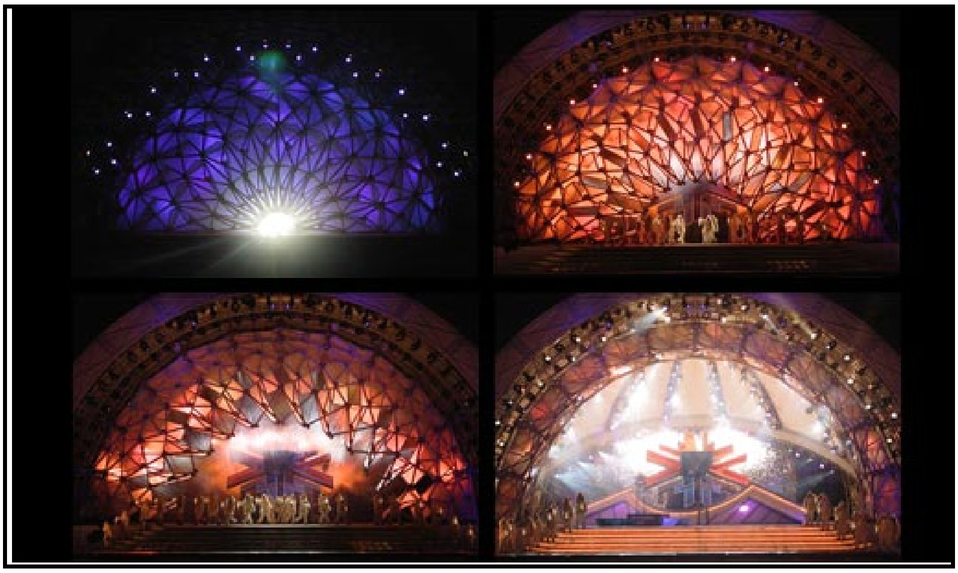


Figure 5.31 Hoberman arch
(Source: Hoberman, 2010)

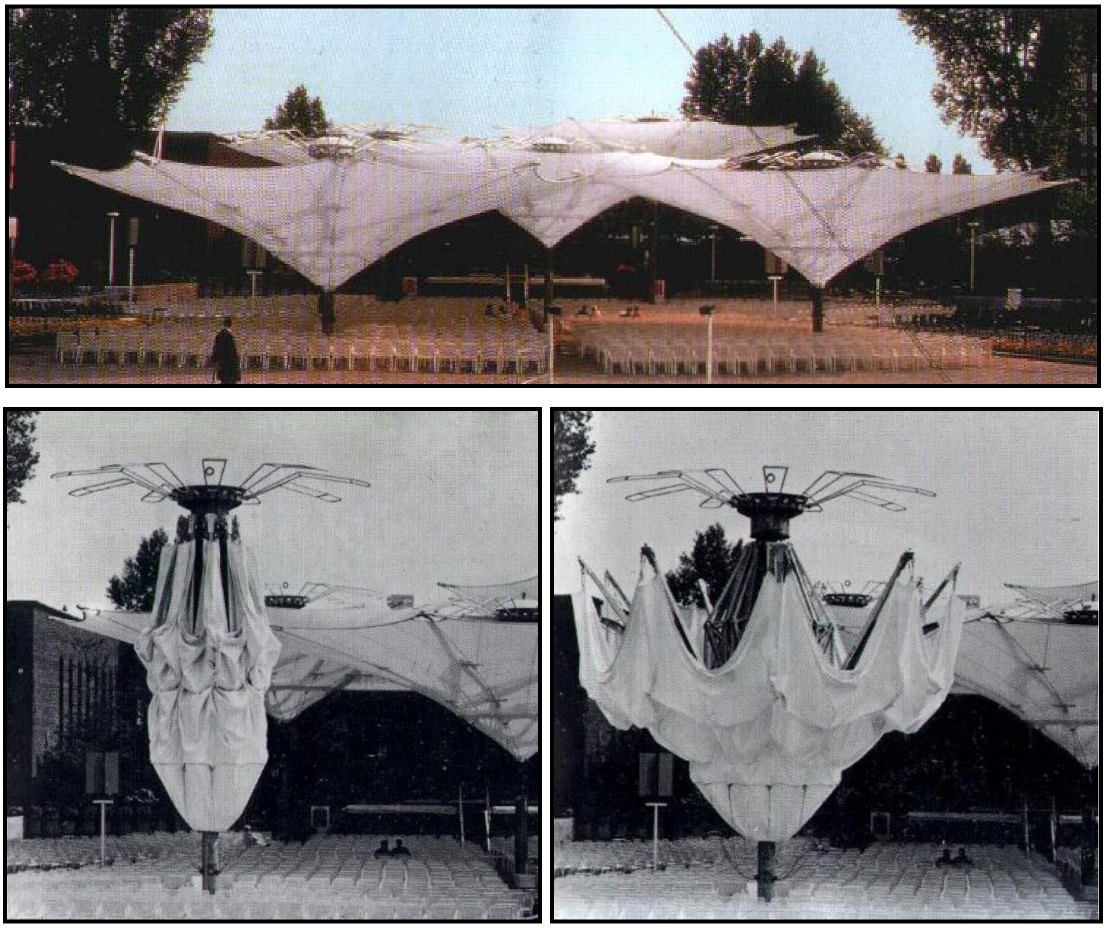


Figure 5.32 Umbrellas at the Cologne Garden exhibition, 1971
(Source: Korkmaz, 2004)

5.2.2. Fractal Geometry and Irregular Tessellation

The method of the kinetic irregular tessellation consists of two steps. The first step includes the main part of the method, and it was explained before in 5.1. In this step type of platform can be determined thanks to duality of tessellation. After determining the platform of tessellation, the next step of the method can be passed according to form of irregular kinetic planar surface.

The main idea of this method comes from geometry of nature. Natural life obtains itself with its small part. Human body, geometric of a leaf etc, consists with his geometry and order. Natural life includes fractal geometry or golden ratio to derive them. Figure 5.33 displays some fractal geometry example from the natural life.

Avnir, Biham, Lidar and Malcai said that "*fractals are beautiful mathematical constructs characterized by a never-ending cascade of similar structural details that are revealed upon magnification on all scales" (*Avnir, Biham, Lidar and Malcai, 1998).

This is the main and simple definition of fractal geometry. In addition to this, fractal geometry can be explained as self-similar sets of fractional dimension. It means that self-similar patterns composed of smaller-scale copies of themselves (Brown and Witschey, 2003).

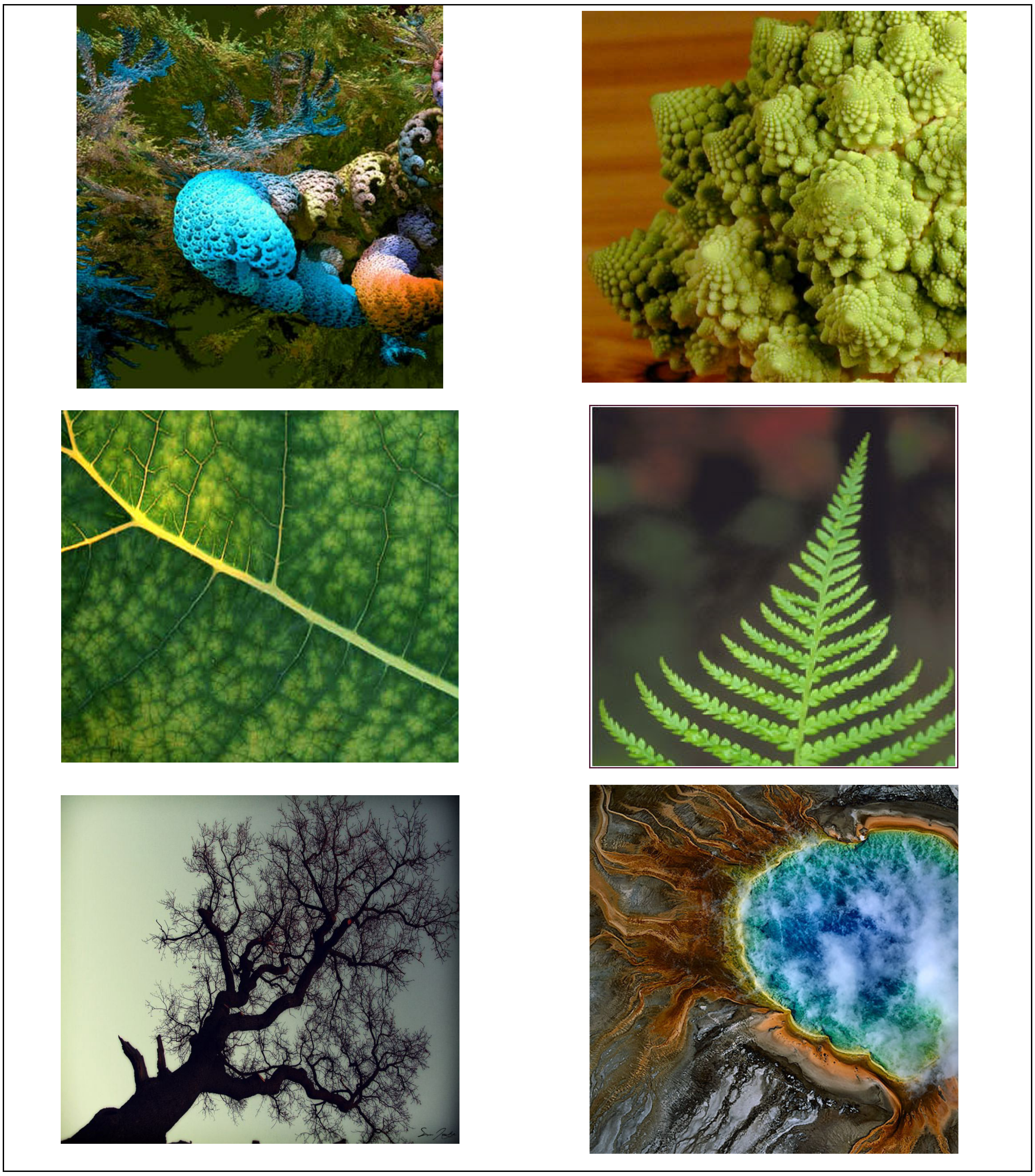


Figure 5.33. Examples of fractal geometry in natural life
(Source: Miqel, 2010)

David Gibson explained this iteration with basic figure (Gibson, 2002). Figure 5.34 and Figure 5.35 display basic iteration of fractal pattern. First pattern in Figure 5.34 is a star pattern, then as a second step it iterated 10 times with the same form but different size. Moreover, the third figure is a pattern that is iterated 10 times to the second form, this processes continue according to pattern form. On the Figure 5.35 the unique pattern is a hexagonal, and it is iterated six times on the second pattern, visa versa.

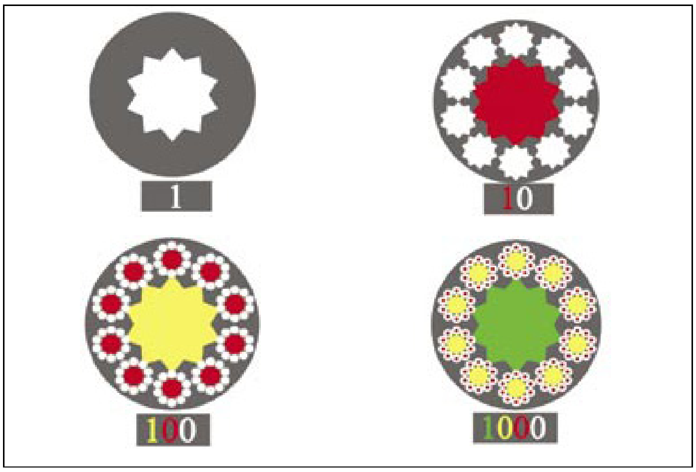


Figure 5.34 Fractal Iteration
(Gibson, 2002)

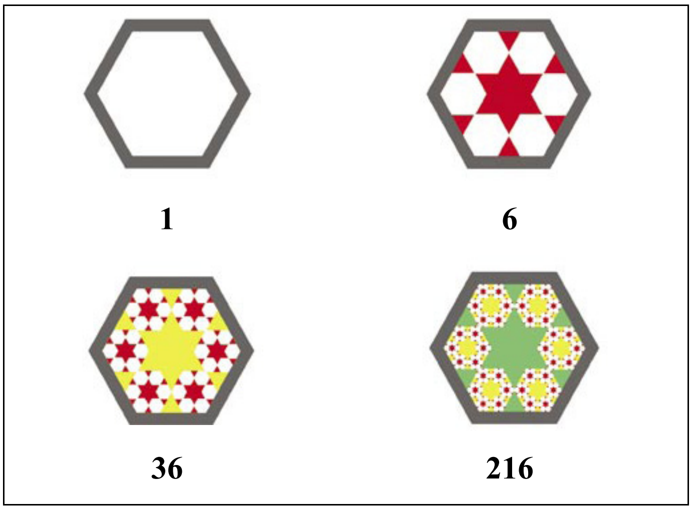


Figure 5.35 Fractal Iteration
(Gibson, 2002)

The method of kinetic irregular tessellation will be basic in this iteration concept. The main unit of the pattern is regular planar mechanism that is obtained with method 1. The irregular tessellation method consists of 3 steps successively building upon each other.

1-Determining the unique regular tessellation form by using regular kinetic tessellation method (Figure 5.36)

2-Determining to proportion of the unique regular tessellation (Figure 5.37)

3-To join new platforms to each other with the same order (Figure 5.38)

In the first step of this method, firstly main part of the planar tessellation should be determined according to method 1. For instance, the irregular planar will be cover with regular hexagonal platform. The form and size of the platform and placement of them is formed by method 1. This will be main part of the mechanism.

In the second step, next part of the pair should be produced with 1/50 scale. This new part will be second part of mechanism. After this decision, the main part of mechanism and second part of mechanism joint each other with rate of two and connecting with revolute pair.

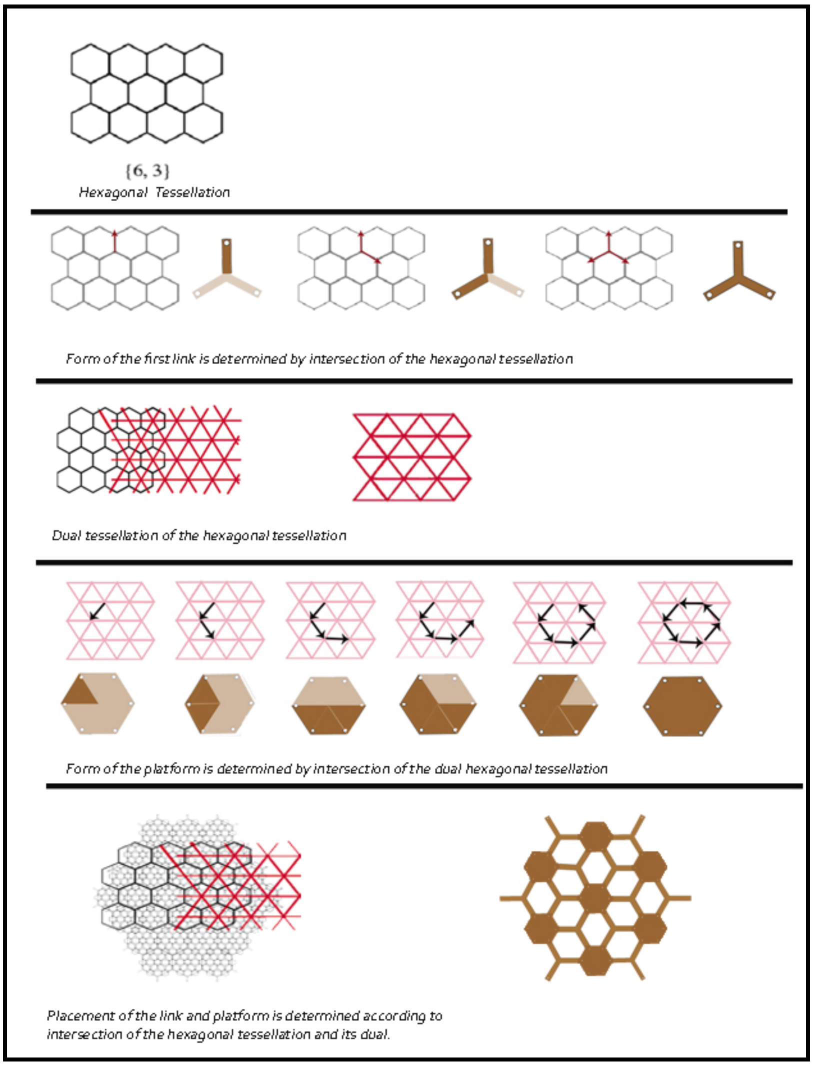


Figure 5.36 First step of irregular kinetic tessellation method.

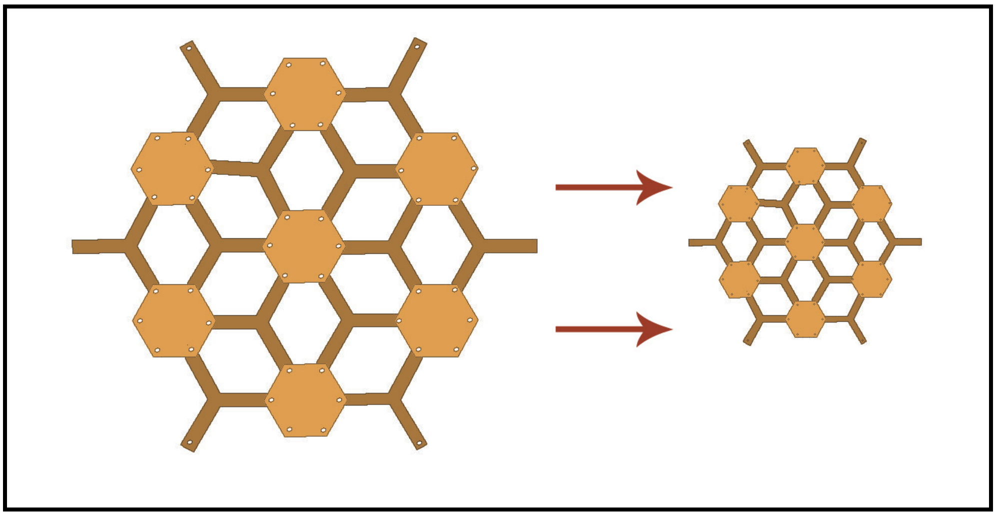


Figure 5.37. Second step of irregular kinetic tessellation method, main part of the system and the second part of the system

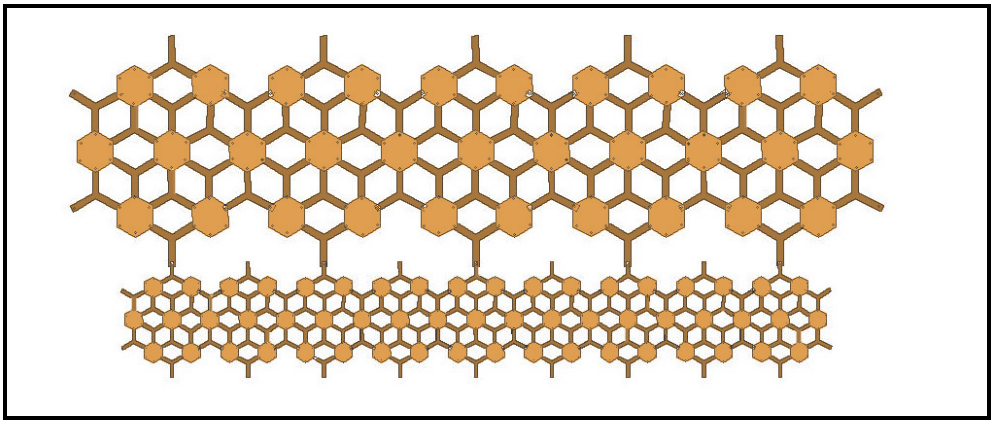


Figure 5.38. Third step of irregular kinetic tessellation method.

This systematics can be continued according to the form of the planer surface. If that surface formed with more cylindrical or agonic shape, the second part of platform should be smaller step by step.

Importance of this methodology

1-This method provides easiness to design a kinetic regular planar surface when choosing platform and link. It makes easier fast for huge surface especially on the huge building facade.

2-Thanks to this method, any kind of irregular geometric form could be kinetic with regular kinetic tessellation.

3-Moroever, whatever this method is derived with many part of regular platform and links, every step of the movement, mobility of the mechanism is 1. This feature makes this mechanism easier to apply even variable surface.

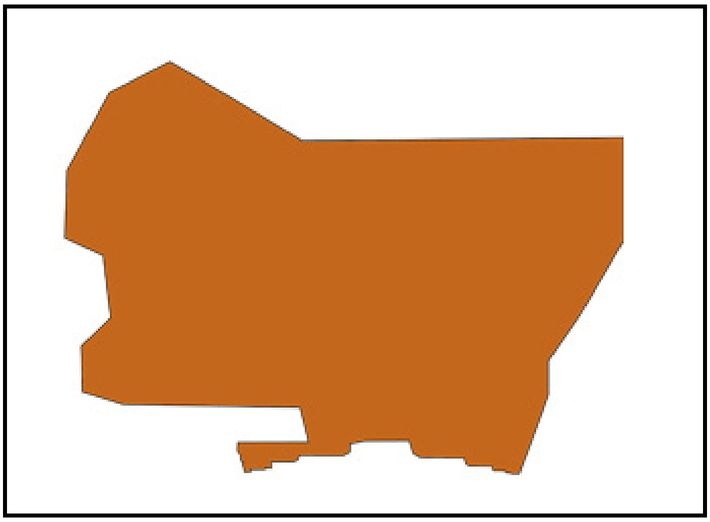


Figure 5.39. Irregular geometrical form

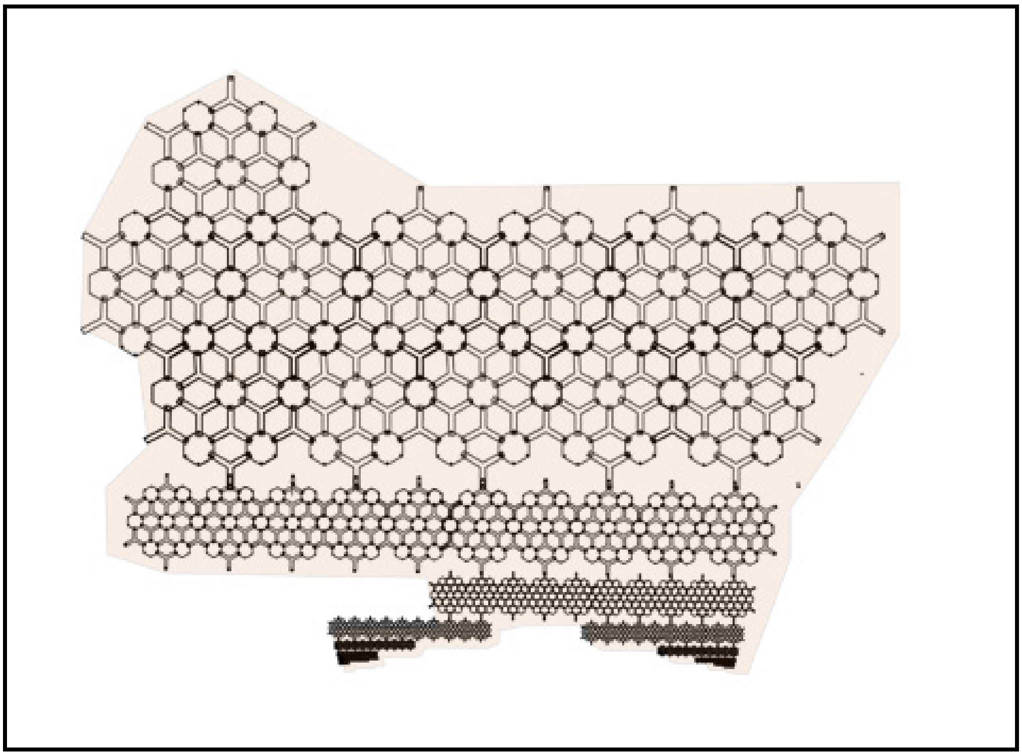


Figure 5.40. Kinetic irregular tessellation

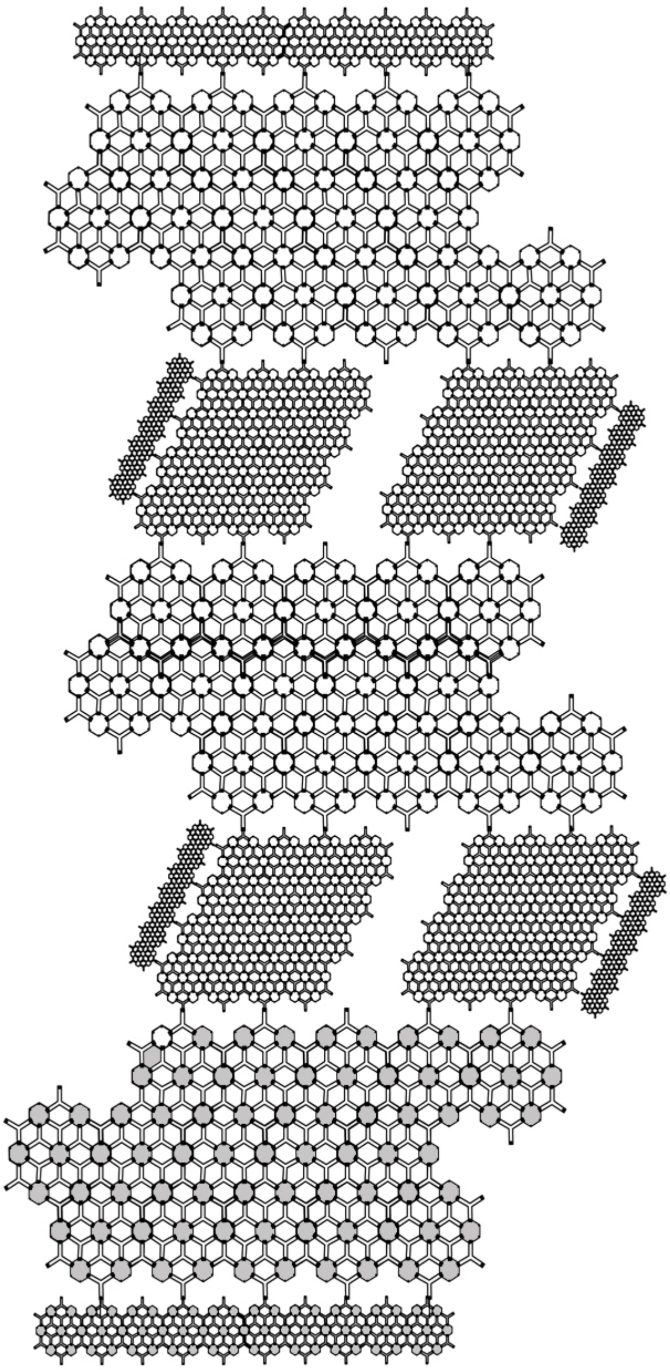


Figure 5.41. Kinetic irregular tessellation

CHAPTER 6

CONCLUSION

Processes of design are crucial for the designers. Many architects and artist have been benefited from mathematics and geometry during the processes of the design. Also they have developed some methods and techniques. Surface design is one of the important interests for both architects and artists through out the history. All periods of architecture have always considered surface design. This thesis proposes a method for the design processes of kinetic planar surfaces with architectural, mathematical and mechanical interdisciplinary approaches.

Throughout the history, architecture have been constituted a relationship between mathematic only by static architecture. On the contrary to this, due to rapid change in activities of modern society, needs of adaptation has emerged and by developing technical properties and construction development kinetic architecture has gained importance. As a result of literature review in the kinetic architecture area, it has been seen that, researcher and designer usually consider a particular type of mechanism. Therefore, they can easily control mechanism structure, number of joints and links at the same time. However, when researchers design variable building types, they have to combine and control many elements. Thus kinetic architecture needs a methodology aspect to develop its ability on variable building parts easily and rapidly. This thesis develops an aspect of the relationship between design processes and architecture by considering kinetic aspect upon developing a method for kinetic planar surfaces.

To accomplish the above mentioned objective of the study, firstly, this thesis considers the tessellation technique since it gives a theoretical background for the issue to cover planar surfaces without any gaps or overlaps. The usages of tessellation on the planar surfaces in architecture have been examined in three different periods; ancient period, Islamic architecture period and contemporary architecture period. By the help of this examination, it can be seen that tessellation technique has been used since the time humans begin to build, and it reached its best level with Islamic architecture period. One of the reasons to this is the fact that, figure of animal or human is prohibited on the facade of buildings in the Islam. It is feared that the depiction of the human form is idolatry and thereby a sin against God. So the designers tried to reflect their thoughts by the complex patterns. As a result they have developed geometry and used mathematics for their designs. Due to advanced construction materials and technology, the usage of tessellation again has gained importance on the facade of the buildings with the new concepts.

After this discussion, the main properties of tessellation technique are examined in chapter 3. This is an interdisciplinary study that aims to combine mathematical knowledge and mechanical knowledge to develop new aspects to the architecture. Thus, to learn the main properties of the tessellation is very important aim for the thesis. This chapter tries to analyze such question that, which shape can tessellate the planar surfaces, what kind of iteration types exist, and how polygons are combined by classifying the tessellation in mainly two parts. In this frame especially classification with respect to polygonal shape is crucial; the platform that is used in the kinetic planar surfaces is formed and connected by this point of view. It is understood that all polygonal shapes can not be combined to fit a planar surface without any gaps or overlaps. There exist three regular and eight semi- regular tessellations. This thesis deal with three regular tessellations (square, triangular and hexagonal) and develop a method by considering them.

After analyzing the main properties of tessellation technique, chapter 4 has studied planar mechanisms. This will be the theme of the motion on the planar surfaces and main properties of linkages and joints. To accomplish the main objective of the study, chapter 4 tries to answer the question of how can a mechanism be designed? Thus, the process of mechanism design is examined. It is understood that structural synthesis that is constituted with type synthesis and number synthesis is the fundamental part of the mechanism design process. In these steps type of the mechanisms, form and size of the platforms and links are determined. This is the one of the crucial points for the study because the process of the structural synthesis is also the main problem for designing variable building parts. However, as a result of literature survey on mechanism design, current processes usually based on the ease of use and the easiness of the solutions, there are not many methods or techniques for these processes by considering design and esthetic conditions.

In the above mentioned structure of thesis, a method is developed with architectural, mechanical and mathematical interdisciplinary approach. This method consists of two parts. First part of the thesis deals with regular surfaces covered by regular platforms. The form and size of the platforms and links and also their placements are determined by regular tessellation and their duals. All platforms and links are combined with revolute pairs. Visual Nastran 4 D is used to analyze the performance and motion of regular kinetic tessellation and mobility of these mechanisms are calculated by Alizade formulation. As a result of mobility calculation, it has been seen that the mobility of the smallest mobile element of the tessellation is 1. Also, when these mobile elements are iterated, the mobility of the mechanism is still conserved unity (M=1). In this frame the most important point is the fact that, when mobile elements are iterated the calculation of mobility is changing but the total mobility of the mechanism will be one. In that step it is understood that when a mobile element of tessellation is iterated, some excessive links will exist in the mechanism. After this a theorem is introduced that number of redundant or excessive links is equal to the one plus the number of loops that will be shared during the whole iteration process. The mobility of the system is always one and this feature makes this mechanism easier to be applied variable surfaces.

The second part of the method is based on the irregular surfaces. Covering irregular surfaces is very important feature for the designer, because it simplifies the design processes. The main objective of this method is to reach more irregular form that means agonic shape. The method of kinetic irregular surfaces with regular platforms is inspired from fractal geometry. Fractal geometry is a new and very complex subject related with many disciplines. This thesis is not examine fractal geometry deeply, but inspired from its iteration system. This method has very important features for design of kinetic surfaces for both architecture and mechanical engineering field.

In above mentioned properties of the methodology, following conclusions are found to be important.

One of the main concepts of the kinetic architecture is to fulfill the life conditions that are changing rapidly. Because of this, kinetic building parts should be constructed easily and cheaply for variable buildings. This methodology can supply this features. By using regular kinetic tessellation method, form, size and placement of the platforms and links can be determined easily and it makes to design more complex patterns with simple constructions fastly.

Many kinetic architects and researchers have deal with the kinetic structure. At the end of the designs the surfaces among kinetic structures usually are covered by textile or flexible material. By using kinetic irregular tessellation method these surfaces can be covered with kinetic building parts.

Kinetic architecture is a controversial interdisciplinary approach between architecture and mechanism science. However, rapid change in activities and environmental conditions effected human sources and traditional aspect of architecture cannot be sufficient for future, there are a few examples both in application of structures as building parts and also in research areas. Due to this, kinetic architecture deserves more serious consideration.

Finally, it should be added that mathematics is a kind of language to tell, to understand, to discuss, to analyze and even to ***THINK***. From the thousands of years ago scientists and designers use it as a tool and language, so, the relationship between mathematical knowledge, mechanism science and building parts deserves more serious consideration in kinetic architecture.

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APPENDIX A

GLOSSARY

Adaptable: Able to adjust to new conditions.

Chain: Several links connected by pairing elements constitute a chain.

Change: To make or become different.

Ceiling: The upper interior surface of a room or other similar compartment

Degree of Freedom: Number of independent inputs required to determine the positions

Dual Tessellation: The dual of a regular tessellation is formed by taking the center of each polygon as a vertex and joining the centers of adjacent polygons.

Façade: The principal front of a building, that faces on to a street or open space.

Kinetic: adj. of or produced by movement.

Kinetics: Sub branch of solid mechanics, which deals with the action of forces on bodies.

Lattice: A partially ordered set in which every subset containing exactly two elements has a greatest lower bound or intersection and a least upper bound or union.

Link: Each component part of a mechanism is called a link

Linkage: A linkage consists of links (or bars) generally considered rigid, which are connected by joints.

Mechanism: A mechanical device that has the purpose of transferring motion and/or force from a source to an output.

Motion: n. the action or process of moving or changing place or position.

Ornament: A thing used or serving to make something look more attractive but usually having no practical purpose, especially a small object such as a figurine

Plane: A two-dimensional area in geometry.

Planar: of or pertaining to a geometric plane.

Polygon: A plane figure with many sides

Tessellation: To cover a plane without any gaps or overlaps.

Tilling: Synonym meaning of tessellations.

Regular: (of a polygon) having all sides and angles equal.

Self-similarity: Two or more objects having the same characteristics. In fractals, the shapes of lines at different iterations look like smaller versions of the earlier shapes.

Static: adj. (physics) (of force) acting by weight without producing movement.

Transform: ~sth/sb (from sth) (into sth): to change the appearance or character of sth/sb completely.