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The Circuit Implementation of a Wavelet Function Approximator

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Abstract. This paper describes the analog synthesis of a wavelet function approximator using sigmoidal mother wavelet. Any finite energy multivariate function can be approximated by this analog circuit using the multiresolution approximation property of the wavelet decomposition. The approximator circuit includes bipolar junction transistors, operational amplifiers and linear passive circuit elements.

Key Words: wavelets, analog synthesis, system identification

I. Introduction

With the advent of artificial neural networks, the analog computation has received much attention over the last ten years. A methodology for implementing neural network architecture using VLSI circuits has been developed in [1]. It has been shown that any continuous multivariate function can be approximated arbitrarily closely by artificial neural networks [2,3]. The approximate identity neural networks for analog synthesis of nonlinear static and dynamical systems have been proposed in [4] and BJT and MOS devices are used for realization. Recently, in order to approximate arbitrary nonlinear functions, wavelet network inspired by both feedforward neural networks and wavelet decompositions using a learning algorithm of backpropagation type has been proposed in [5]. The identification of static and dynamical systems using wavelet network have attracted many researchers since the wavelet analysis has been successfully applied for analyzing signals both in space and frequency domain with different resolution levels [6–8].

In this study, the analog circuit implementation of a wavelet function approximator using sigmoidal type mother wavelet proposed in [9] has been realized and the mother wavelet has been implemented by bipolar junction transistors using the implementation of sigmoids proposed in [4]. The wavelet coefficients and parameters have been calculated using stepwise-selection by orthogonalization algorithm [6].

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The wavelet decomposition and the approximation to a given function by wavelets will be given briefly in the next section of this paper. Then, the circuit implementation of sigmoidal mother wavelet and the function approximator has been presented in Section III. In Section IV, some examples for function approximation have been given using the proposed circuit.

II. Wavelets and Function Approximation

Any function $f(x) \in L^2(R^n)$ can be approximated by linear combination of the members of wavelet family. The wavelets are obtained by the translations and dilations of a wavelet function $\Psi : R^n \to R$ which is called mother wavelet which satisfying the admissibility condition [10].

$$f(x) = \sum_i W(a_i, b_i; \Psi)\Psi_i(a, b) \tag{1}$$

where $\Psi_i(a, b) = a^{d/2}\Psi(a(x - b))$ for $i \in Z$ is a member of the wavelet family and $a \in R, b \in R^d$. In order to ensure the approximation is valid, the wavelets should constitute a frame [11]. Selecting the frame approach gains more freedom on the choice of the mother wavelet function. The frames are the generalizations of orthonormal bases for Hilbert spaces.

In [9], a sigmoidal mother wavelet has been proposed satisfying admissibility condition which has been found by combining three sigmoids as

$$\Psi(x) = s(x+k) - 2s(x) + s(x-k) \tag{2}$$

where $s(x) = \tanh(qx)$, (q : constant) and $0 < k < \infty$. The dilated and translated wavelets form an affine frame for dilation stepsize $a_0 = 2$ and the translation stepsize $0 \leq b_0 \leq 3.5$ where the wavelets can be obtained as

$$\Psi(a_i, b_i) = a_0^{nd/2}\ \Psi\left(a_0^n(x - mb_0)\right) \tag{3}$$

with $m, n \in Z$ and d is the dimension.

In this study, the sigmoidal mother wavelet parameters have been chosen as $k = 2$, $q = 1$, and dilation and translation stepsizes have been selected as $a_0 = 2$ and $b_0 = 0.01$ where $\Psi_i : R \to R$. The examples of approximation have been given in Section IV of this paper.

One of the ways of approximating the functions in higher dimensions is to use the tensor product of the one dimensional wavelet functions. For the construction of the tensor product wavelet frame, the multidimensional wavelet $\Psi(x)$ is defined as the tensor product of the 1-dimensional wavelet functions as

$$\Psi(x) = \Psi(x_1) \cdot \Psi(x_2) \cdots \Psi(x_n) \tag{4}$$

where x_i is the i^{th} element of the input vector x [11]. Then, the Fourier transform of the wavelet function is defined as

$$\hat{\Psi}(\omega) = \hat{\Psi}(\omega_1) \cdot \hat{\Psi}(\omega_2) \cdots \hat{\Psi}(\omega_n) \tag{5}$$

Each $\Psi_i(x_i)$, $i = 1, \ldots, n$ should satisfy the admissibility condition individually.

In this study, the tensor product of sigmoidal mother wavelets has been used for the approximation of the 2-dimensional functions. One example for the approximation of 2-dimensional function has been given in Section IV. The number of dimensions can be increased in the same manner for the approximation of higher dimensional function.

For a given input-output pair set $\{x_j, y_j \mid y_j = f(x_j), j \in Z\}$ the problem is to minimize the mean square error

$$MSE \triangleq \frac{1}{2} E\{(f(x) - f_w(x))^2\} \tag{6}$$

where E represents expected or mean value and the approximation function is given as

$$f_w(x) = \sum_i w_i \Psi(a_i, b_i) + cx + b_x \tag{7}$$

where w_i's are the coefficients of the chosen wavelets, cx is the linear term and the b_x is the bias term.

In this paper, the selection of the suitable wavelet basis and the initialization of the coefficients have been accomplished by the regression analysis techniques proposed by Zhang in [6]. The wavelets whose supports do not contain any data point are eliminated. To reduce the number of wavelets, the stepwise-selection by orthogonalization algorithm has been applied. The algorithm first selects the wavelet in that best fits the observed data, then repeatedly selects the wavelet in the remainder of that best fits the data while combining with the previously selected wavelets. Wavelet regression algorithm for the circuit implementation of the examples given in Section IV has been accomplished via the program written in MATLAB in [6].

III. Circuit Implementation

The sigmoidal mother wavelet function which has been proposed in [9] by Pati et al., can be implemented as the sum of three translated hyperbolic tangent functions as

$$\Psi(x) = \tanh(x-2) - 2\tanh(x) + \tanh(x+2) \tag{8}$$

Therefore, the mother wavelet can be obtained using hyperbolic tangent functions. The hyperbolic tangent function can be implemented by a dual-transistor pair. The collector current of a bipolar junction transistor can be formulated as

$$i_c = I_s e^{V_{be}/V_t} \tag{9}$$

where I_s is saturation current, V_{be} is base-emitter voltage, and V_t is the thermal voltage which can be determined as

$$V_t = kT/q \tag{10}$$

where k is Boltzman constant, q is the electron charge and T is the temperature.

With the analysis of the differential transistor pair given in [4], the difference of the collector currents can be calculated as

$$i_1 - i_2 = I_r \tanh\left(\frac{V_1 - V_2}{2V_t}\right) \tag{11}$$

which is shown in Fig. 1.

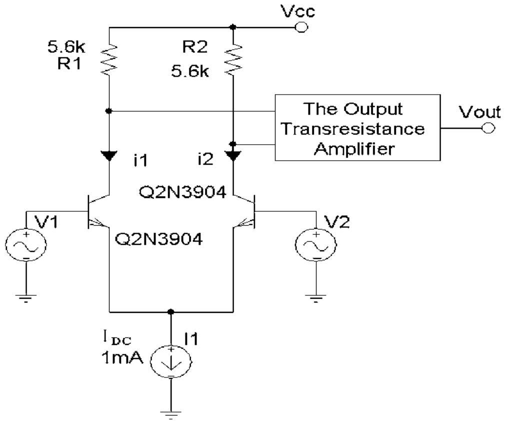

Fig. 1. The circuit implementing tangent hyperbolic function.

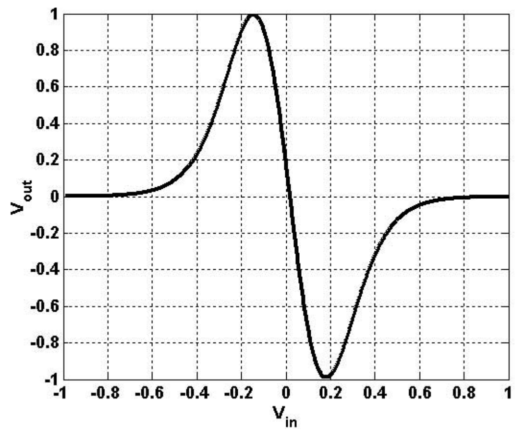

Fig. 3. The input-output voltage pair of the wavelet circuit.

By choosing the input $V_2 = 0$ and determining the difference, the output of tangent hyperbolic block (tanh) is determined as

$$V_{out} = A \tanh\left(\frac{V_1}{2V_t}\right) \tag{12}$$

where A is the circuit gain. Therefore, the sigmoidal wavelet function can be obtained using tanh blocks. The block diagram of the wavelet function is shown in Fig. 2 where the output voltage is

$$V_{out} = A_0 \Psi(V_{in}(t)) \tag{13}$$

The input-output voltage pair for wavelet block is shown in Fig. 3.

Any function $f(x) \in L^2(R)$ can be approximated by the dilations and the translations of the mother wavelet as shown in Fig. 4.

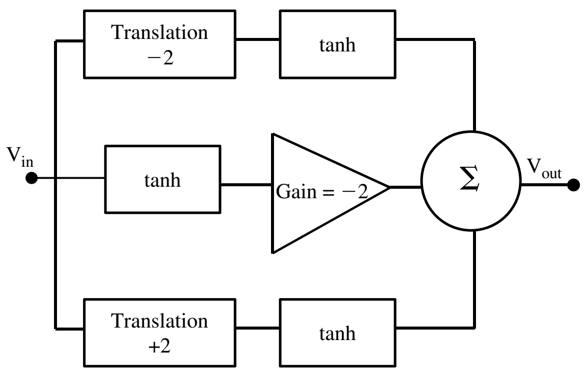

Fig. 2. The block diagram of wavelet circuit.

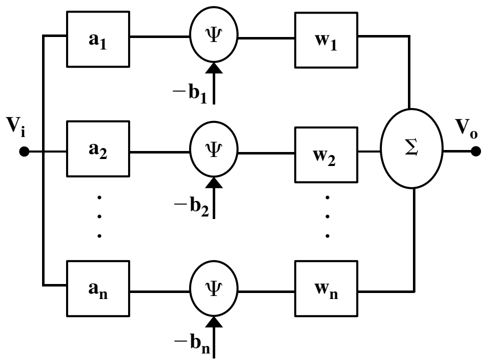

Fig. 4. The block diagram of approximation circuit.

The circuit implementations of the tensor product wavelets defined in equation (4) are obtained as the analog multiplication of the 1-dimensional wavelets. This operation has been implemented by the wavelet blocks and the analog multiplier macromodel AD633 of Analog Devices. The block diagram of one 2-dimensional (2D) wavelon is shown in Fig. 5 where the input voltage is $V_{in} = [v_{in1} v_{in2}]^T$.

IV. Applications

In order to determine the performance of the wavelet function approximator, some examples have been given. The realizations are done in two steps; in the first step the number of wavelons are chosen and the coefficients of function approximator including translation and dilation coefficients, a_i, b_i; weight

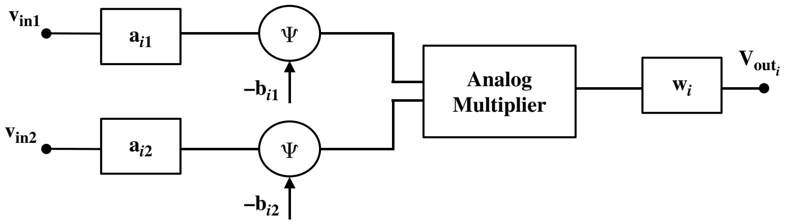

Fig. 5. The block diagram of 2D wavelon.

coefficients w_i; linear term c and bias term b_x, are calculated by "stepwise-selection by orthogonalization algorithm" [6]. In the second step, the circuit is set up by the selected wavelets chosen from the pre-constructed wavelet library for all possible dilation and translations and then simulated in PSpice.

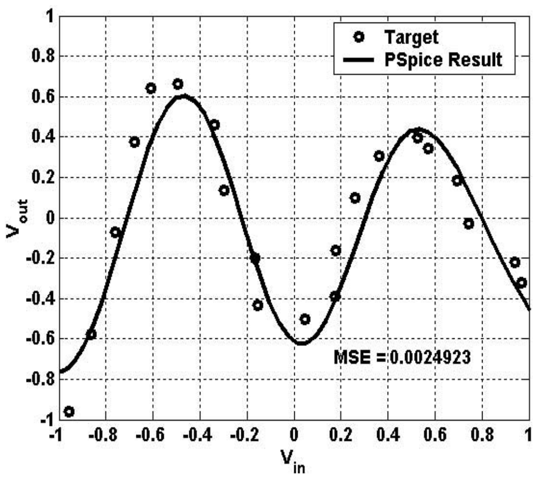

Fig. 6. Result of approximation for Example 1.

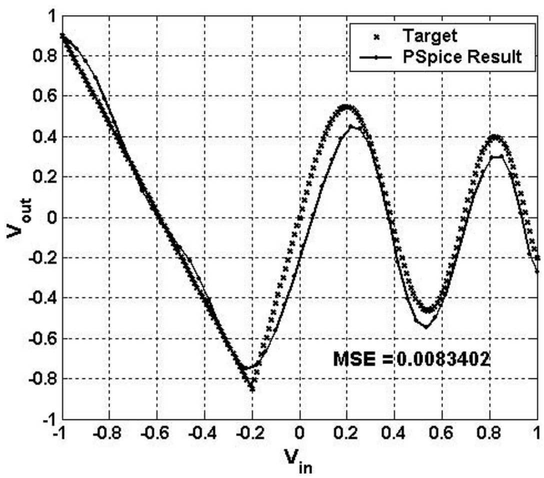

Fig. 7. Result of approximation for Example 2.

Example 1. The target function is given by the 21 data points and the approximation is done by 3 wavelons. The result of the approximation is shown in Fig. 6. The obtained mean squared error (MSE) is 0.0024923.

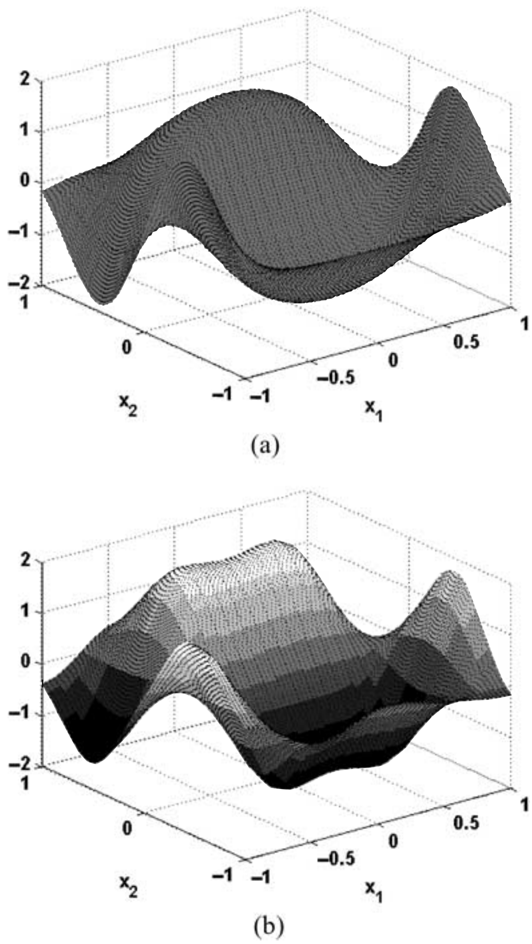

Fig. 8. (a) The 2D target function, (b) the result of approximation.

Example 2. The function to be approximated is given by

$$y = \begin{cases} -2.186x - 12.864 & -1 < x < -0.2 \\ 4.246x & -0.2 < x < 0 \\ e^{-0.5x-0.5}\sin((3x+7)x) & 0 < x < 1 \end{cases} \tag{14}$$

The approximation is implemented with 6 wavelons and a linear term. The obtained MSE is 0.0083402. The approximation result is demonstrated in Fig. 7.

Example 3. The 2-dimensional function to be approximated defined as $y = f(x_1, x_2)$

$$y = \begin{cases} 2e^{(x_1^2 - x_2^2)}\cos(2x_1)\cdot\sin(3x_2) \\ -1 < x_1 < 1 \quad -1 < x_2 < 1 \\ \quad 0 \quad \text{elsewhere} \end{cases} \tag{15}$$

The function in equation (15) is approximated using 10 2-dimensional wavelons. The target function and the result of PSpice simulation is shown in Fig. 8.

V. Conclusions and Future Work

The analog synthesis of a wavelet function approximator has been described in this study. The proposed analog circuit can approximate any multivariable finite energy function. The building blocks include bipolar junction transistors, operational amplifiers and linear passive circuit elements. The analog synthesis of the nonlinear dynamical systems is the subject of the future studies.

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