52nd IEEE Conference on Decision and Control
December 10-13, 2013. Florence, Italy

A New Robust *'Integral of Sign of Error'* Feedback Controller with Adaptive Compensation Gain

Baris Bidikli, Enver Tatlicioglu, Alper Bayrak, and Erkan Zergeroglu

Abstract— **In this paper, a new robust integral of signum of error (RISE) feedback type controller is designed for a class of uncertain nonlinear systems. Unlike the previous versions of RISE feedback type controllers, the proposed controller does not require prior knowledge of upper bounds of the vector containing the uncertainties of the dynamical system plus desired system dynamics (and their derivatives) for the control gain selection. The aforementioned enhancement is made possible via the design of a time–varying compensation gain as opposed to a constant gain used in previous RISE feedback type controllers. Asymptotic stability of the error signals and the boundedness of the closed–loop system signals are ensured via Lyapunov based arguments. Numerical simulation studies are presented to illustrate the viability of the proposed method.**

I. INTRODUCTION

The tracking control problem for nonlinear systems subject to uncertainties in their dynamics have attracted extensive attention from the control community for decades. Successful achievements have been made on general classes of nonlinear systems both by designing adaptive [1] and robust [2] controllers. If it is possible to separate the vector containing the uncertain system dynamics in a linearly parametrizable manner, (*i.e.*, as the multiplication of a regression matrix of known elements with a vector containing the constant or slowly–varying system uncertainties) adaptive controllers [3] are considered to be the commonly preferred choice. On the other hand, when the vector containing the uncertainties of the system or the desired trajectory is periodic with a known period, learning controllers [4] can be applied. When none of the above is possible and the only knowledge about the nonlinear system is that the uncertainties of the system are upper bounded (by either a known constant or a known function), robust controllers, such as variable structure controllers, are commonly considered. However, most robust controllers, due to the use of the signum function in their design, are discontinuous. Also with most robust controller designs, convergence of the error signal to an ultimate bound can be guaranteed, and over–shrinking this ultimate bound causes chattering which, for most mechanical systems, is undesirable.

To our best knowledge, the first continuous and asymptotically stable robust controller was presented in [5] and [6]. In [5], motivated by the work of [2], authors presented a continuous robust controller for a class of nonlinear systems with continuously differentiable dynamics. As the methodology utilized the integral of the signum of the error as opposed to the signum of error used in standard sliding mode controllers, it is referred as RISE (short for 'Robust Integral of Sign of Error') feedback [7]. Controller formulations fused with RISE feedback have then been applied to a wide variety of systems, including autonomous flight control [8], underwater vehicle control [9], control of special classes of multiple input multiple output (MIMO) nonlinear systems [10], [11], and even time delay compensation [12]. One major drawback of the RISE feedback, however, the formulation requires a sufficient high gain condition on the constant uncertainty compensation gain. Specifically, the constant uncertainty compensation gain in RISE feedback formulations require the knowledge of the upper bounds of vectors (functions of the desired system trajectories) containing system uncertainties. To reduce the heavy control effort enforced to the system by this high gain, researchers used adaptive [7] and neural network (NN) based [13], [14] feedforward compensation techniques in conjunction with RISE feedback. Recently, in [15], Jagannathan *et. al* proposed a controller formulation that utilized RISE feedback multiplied with an adaptive gain fused with NN feedforward compensation for a class of uncertain nonlinear systems. However, the formulation failed to prove $\mathcal{L}_1$ boundedness of the error term utilized in the design of the adaptive gain, as a result, there is no guarantee that the proposed time–varying adaptive gain would remain bounded under the closed–loop operation.

The increasing interest to RISE feedback in the robust control community motivated the authors to research possible extensions/modifications to RISE feedback methodology. In this paper, significant research has been aimed to extend the RISE controller formulation given in [5] by removing the need for prior knowledge of the upper bounds of the vector containing the desired system dynamics plus uncertainties (and their derivatives) for the control gain selection. The main motivation behind pursueing to extend the results in [5] is due to the fact that using a time–varying adaptive compensation gain reduces the heavy control effort and therefore eliminates the need for extra feedforward compensation methods to be fused with the RISE controller formulation. In this study, instead of the constant uncertainty compensation gain, a time–varying gain is designed in the controller. The stability of the designed controller is investigated via novel Lyapunov based analysis. The asymptotic convergence of the error signals and the boundedness of all the signals under the

B. Bidikli is with the Department of Electrical & Electronics Engineering, Celal Bayar University, Manisa, Turkey (E-mail: barisbidikli@cbu.edu.tr).
E. Tatlicioglu and A. Bayrak are with the Department of Electrical & Electronics Engineering, Izmir Institute of Technology, Izmir, 35430 Turkey (Phone: +90 (232) 7506536; Fax: +90 (232) 7506599; E-mail: [envertatlicioglu,alperbayrak]@iyte.edu.tr).
E. Zergeroglu is with the Department of Computer Engineering, Gebze Institute of Technology, 41400, Gebze, Kocaeli, Turkey (Email: ezerger@bilmuh.gyte.edu.tr).

978-1-4673-5717-3/13/$31.00 ©2013 IEEE

closed–loop operation is guaranteed. Simulation studies are conducted to demonstrate the proof of concept numerically. When compared to the previous controllers that utilized RISE feedback (including [5]), the proposed method relies on a time–varying compensation gain which does not impose heavy control effort to the system. When compared to the controller of [15], the proposed controller does not require a feedforward compensation method and the analysis provides the $\mathcal{L}_1$ boundedness of the error term utilized in the design of the time–varying gain, and thus proving the boundedness of all the signals under the closed–loop operation including the time–varying adaptive gain.

II. Error System Development

In this section[1], for the compactness of the presentation the following single input single output (SISO) nonlinear system is considered [5]

$$mx^{(n)} + f = u \tag{1}$$

where $x^{(i)}(t) \in \mathbb{R}$ $i = 0, ..., n$ are the system states, $m\left(x, \dot{x}, \cdots, x^{(n-1)}\right)$, $f\left(x, \dot{x}, \cdots, x^{(n-1)}\right) \in \mathbb{R}$ are uncertain nonlinear functions, and $u(t) \in \mathbb{R}$ is the control input. The standard assumption that the uncertain function $m(\cdot)$ being positive (*i.e.*, $m(\cdot) > 0$) is utilized in the subsequent development. Therefore, following bounds are assumed

$$\underline{m} \leq m(x) \leq \overline{m}\left(|x|, |\dot{x}|, \cdots, \left|x^{(n-1)}\right|\right) \tag{2}$$

where $\underline{m} \in \mathbb{R}$ is a positive constant and $\overline{m}(\cdot)$ is some positive non–decreasing function of its arguments. The uncertain functions $m(\cdot)$ and $f(\cdot)$ are assumed to be continuously differentiable up to their second order time derivatives. It is highlighted that while the development in this paper is for the SISO system model in (1), extension to MIMO systems is straightforward

To quantify the tracking control objective, the output tracking error, denoted by $e_1(t) \in \mathbb{R}$, is defined as

$$e_1 \triangleq x_r - x \tag{3}$$

where $x_r(t) \in \mathbb{R}$ represents the reference trajectory which is assumed to be bounded with bounded continuous time derivatives (*i.e.*, $x_r^{(i)}(t) \in \mathcal{L}_\infty$ for $i = 0, \cdots, (n+2)$). The main control objective is to ensure that the output tracking error in (3) converge asymptotically to zero, that is $|e_1(t)| \to 0$ as $t \to \infty$ by designing a continuous robust control law under full–state feedback (*i.e.*, $x^{(i)}$, $i = 0, \cdots, (n-1)$ are measurable).

To facilitate the control design, auxiliary error signals, denoted by $e_i(t) \in \mathbb{R}$, $i = 2, \cdots, n$, are defined in the following manner

$$e_2 \triangleq \dot{e}_1 + e_1 \tag{4}$$
$$\vdots$$
$$e_n \triangleq \dot{e}_{n-1} + e_{n-1} + e_{n-2}. \tag{5}$$

[1]As the proposed work aims to extend the results in [5], the notation in [5] is borrowed for a better comparison with the results in this paper.

It is noted that a general expression for $e_i(t)$ $i = 2, \cdots, n$ in terms of $e_1(t)$ and its time derivatives can be obtained as

$$e_i = \sum_{j=0}^{i-1} a_{i,j} e_1^{(j)} \tag{6}$$

where $a_{i,j} \in \mathbb{R}$ are known positive constant coefficients with $a_{n,(n-1)} = 1$. To ease the presentation of the subsequent stability analysis, another auxiliary error, denoted by $r(t) \in \mathbb{R}$, is defined to have the following form

$$r \triangleq \dot{e}_n + \alpha e_n \tag{7}$$

where $\alpha \in \mathbb{R}$ is a positive constant gain. It is noted that, the definition of $r(t)$ has $\dot{e}_n(t)$ which requires unmeasurable $x^{(n)}(t)$ then it is clear that $r(t)$ is not measurable and thus cannot be utilized in the control design.

After multiplying both sides of the time derivative of (7) with $m(\cdot)$, substituting the second time derivative of (6) for $i = n$, and the time derivative of (1), the following open–loop dynamics for $r(t)$ can be obtained

$$m\dot{r} = -\frac{1}{2}\dot{m}r - e_n - \dot{u} + N \tag{8}$$

where $N\left(x, \cdots, x^{(n)}, e_1, \cdots, e_n, r, x_r^{(n+1)}\right) \in \mathbb{R}$ is an auxiliary function defined as

$$\begin{aligned} N &\triangleq m\left[x_r^{(n+1)} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + \alpha \dot{e}_n\right] \\ &+ \dot{m}\left(\frac{1}{2}r + x^{(n)}\right) + \dot{f} + e_n. \end{aligned} \tag{9}$$

The above auxiliary function is partitioned as sum of two auxiliary signals which are denoted by $N_r\left(x_r, \cdots, x_r^{(n)}\right)$, $\tilde{N}\left(x, \cdots, x^{(n)}, e_1, \cdots, e_n, r, x_r^{(n+1)}\right) \in \mathbb{R}$ and are defined as

$$N_r \triangleq N|_{x=x_r, \cdots, x^{(n)}=x_r^{(n)}} \tag{10}$$
$$\tilde{N} \triangleq N - N_r. \tag{11}$$

It should be noted that since both $N_r(t)$ and $\dot{N}_r(t)$ are functions of the desired trajectory and its time derivatives, they are bounded functions of time (*i.e.*, $N_r(t)$, $\dot{N}_r(t) \in \mathcal{L}_\infty$).

Remark 1: Since the auxiliary signal $N(\cdot)$ defined in (9) is continuously differentiable, Mean Value Theorem [16] can be utilized to show that $\tilde{N}(\cdot)$ can be upper bounded as

$$\left|\tilde{N}(\cdot)\right| \leq \rho(\|z\|)\|z\| \tag{12}$$

where $\|\cdot\|$ denotes the standard Euclidean norm, $\rho : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is some globally invertible, non–decreasing function of its argument and $z(t) \in \mathbb{R}^{(n+1)}$ is the combined error signal defined as

$$z \triangleq [e_1, \cdots, e_n, r]^T. \tag{13}$$

Based on the subsequent stability analysis, the following continuous robust controller is proposed

$$u(t) = (k+1)\left[e_n(t) - e_n(t_0) + \alpha \int_{t_0}^{t} e_n(\sigma)\, d\sigma\right] + \int_{t_0}^{t} \hat{\beta}(\sigma)\,\mathrm{sgn}(e_n(\sigma))\, d\sigma \quad (14)$$

where $k \in \mathbb{R}$ is a constant positive control gain, $\hat{\beta}(t) \in \mathbb{R}$ is a subsequently designed time–varying (uncertainty compensation) control gain, α was introduced in (7) and $\mathrm{sgn}(\cdot)$ is the standard signum function. The constant term $e_n(t_0)$ is added to the controller to ensure $u(t_0) = 0$.

Remark 2: The controller in (14) can alternatively be considered as a modified linear controller [by treating the first line in (14) as a proportional integral (PI) controller in terms of $e_n(t)$] fused with a continuous self–updating nonlinear component for uncertainty compensation [*i.e.*, the second line in (14)].

Remark 3: A comparison of the development thus far and the corresponding part of [5] is now given. While the error system development and the open–loop error dynamics are similar, the controller in (14) is fundamentally different than that of the controller in [5]. Specifically, the control gain of the $\mathrm{sgn}(e_n)$ term in (11) of [5] is constant while in (14), the control gain of the $\mathrm{sgn}(e_n)$ term is time–varying. This is a novel departure from the existing controllers utilizing RISE feedback.

The time–varying control gain $\hat{\beta}(t)$ is decomposed as

$$\hat{\beta}(t) = \hat{\beta}_1(t) + \beta_2 \quad (15)$$

where $\hat{\beta}_1(t) \in \mathbb{R}$ is its time–varying part and $\beta_2 \in \mathbb{R}$ is its positive constant part (*i.e.*, $\beta_2 > 0$). The time–varying part of the control gain is designed as

$$\hat{\beta}_1 = \begin{cases} e_n(t) - |e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma & \text{if } e_n > 0 \\ -|e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma & \text{if } e_n = 0 \\ -e_n(t) - |e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma & \text{if } e_n < 0 \end{cases} \quad (16)$$

and taking its time derivative results in

$$\dot{\hat{\beta}}_1 = \begin{cases} \dot{e}_n(t) + \alpha e_n \mathrm{sgn}(e_n) & \text{if } e_n > 0 \\ \alpha e_n \mathrm{sgn}(e_n) & \text{if } e_n = 0 \\ -\dot{e}_n(t) + \alpha e_n \mathrm{sgn}(e_n) & \text{if } e_n < 0. \end{cases} \quad (17)$$

Alternatively, in a more compact form, the time–varying gain $\hat{\beta}_1(t)$ in (16) can be rewritten as

$$\hat{\beta}_1(t) = |e_n(t)| - |e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma \quad (18)$$

from which its time derivative is obtained as

$$\dot{\hat{\beta}}_1 = \dot{e}_n \mathrm{sgn}(e_n) + \alpha |e_n| = r\mathrm{sgn}(e_n) \quad (19)$$

where the definition of $r(t)$ in (7) was utilized. Notice from (18) that $\hat{\beta}_1(t_0) = 0$. The definitions (18) and (19) will be preferred in the subsequent stability analysis.

At this stage, to substitute into (8), the time derivative of the control input in (14) is calculated

$$\dot{u} = (k+1)\, r + \left(\hat{\beta}_1 + \beta_2\right) \mathrm{sgn}(e_n) \quad (20)$$

where (7) and (15) were utilized, and thus the closed–loop error system for $r(t)$ is obtained as

$$m\dot{r} = -\frac{1}{2}\dot{m}r - e_n - (k+1)\, r - \left(\hat{\beta}_1 + \beta_2\right) \mathrm{sgn}(e_n) + N_r + \tilde{N}. \quad (21)$$

III. Stability Analysis

Before presenting the main result of this section, two lemmas are stated where both of which will later be utilized in the proof of the theorem.

Lemma 1: The auxiliary function, denoted by $L_1(t) \in \mathbb{R}$, is defined as

$$L_1 \triangleq r\,(N_r - \beta_1 \mathrm{sgn}(e_n)) \quad (22)$$

where $\beta_1 \in \mathbb{R}$ is a positive constant. Provided that β_1 satisfy

$$\beta_1 \geq \|N_r(t)\|_{L_\infty} + \frac{1}{\alpha}\left\|\dot{N}_r(t)\right\|_{L_\infty} \quad (23)$$

where $\|\cdot\|_{L_\infty}$ denotes infinity norm, then

$$\int_{t_0}^{t} L_1(\sigma)\, d\sigma \leq \zeta_{b_1} \quad (24)$$

where $\zeta_{b_1} \in \mathbb{R}$ is a positive constant.

Proof: See Appendix I. ■

Lemma 2: The auxiliary function, denoted by $L_2(t) \in \mathbb{R}$, is defined as

$$L_2 \triangleq -\beta_2 \dot{e}_n \mathrm{sgn}(e_n). \quad (25)$$

Provided that $\beta_2 > 0$ then

$$\int_{t_0}^{t} L_2(\sigma)\, d\sigma \leq \zeta_{b_2} \quad (26)$$

where $\zeta_{b_2} \in \mathbb{R}$ is a positive constant.

Proof: See Appendix II. ■

Remark 4: A comparison of the stability analysis thus far and the corresponding part of [5] is now given. Lemma 1 in this paper is similar to Lemma 1 in [5], and Lemma 2 in this paper was not in [5]. In this paper, in Lemma 1, a constant parameter namely β_1 is introduced. This constant parameter is required to satisfy the condition in (23) (*i.e.*, it must be greater than the sum of the upper bound of the uncertain function N_r with the upper bound of its time derivative scaled by $\frac{1}{\alpha}$) but it is not utilized in the controller in (14). On the other hand, in [5], the similar constant parameter was utilized in the controller in (11). This difference highlights the main novelty of our work when compared to [5] which is removing the need for the knowledge of the upper bounds of the uncertain function and its time derivative. In our paper, different from [5], Lemma 2 is presented. In the proof of Lemma 2, the constant β_2 is only required to be positive and no additional constraints are imposed. While β_2 is in the controller in (14) [via being the positive constant part of the time–varying control gain $\hat{\beta}(t)$ as introduced in (15)], it being positive is sufficient.

The tracking result will now be proven via the following theorem.

Theorem 1: The controller in (14) with the time–varying gain in (15) and (18) ensures semi–global asymptotic convergence of the tracking error and its time derivatives in the sense that $\left|e_1^{(i)}(t)\right| \to 0$ as $t \to \infty$ provided that α is selected to satisfy $\alpha > \frac{1}{2}$, the control gain k is chosen large enough when compared to the initial conditions of the system and β_2 is chosen to be positive.

Proof: Following Lyapunov function candidate, denoted by $V(y,t) \in \mathbb{R}$, is defined as

$$V \triangleq \frac{1}{2}\sum_{j=1}^{n} e_j^2 + \frac{1}{2}mr^2 + \frac{1}{2}\tilde{\beta}_1^2 + P_1 + P_2 \tag{27}$$

where $P_1(t)$, $P_2(t) \in \mathbb{R}$ are defined as

$$P_1 \triangleq \zeta_{b_1} - \int_{t_0}^{t} L_1(\sigma)\,d\sigma \tag{28}$$

$$P_2 \triangleq \zeta_{b_2} - \int_{t_0}^{t} L_2(\sigma)\,d\sigma \tag{29}$$

and $\tilde{\beta}_1(t) \in \mathbb{R}$ is defined as

$$\tilde{\beta}_1 \triangleq \beta_1 - \hat{\beta}_1 \tag{30}$$

and $y(t) \in \mathbb{R}^{(n+4)\times 1}$ is defined as

$$y \triangleq \left[z^T, \tilde{\beta}_1, \sqrt{P_1}, \sqrt{P_2}\right]^T \tag{31}$$

where $z(t)$ was defined in (13).

From the proofs of Lemmas 1 and 2, it is clear that $P_1(t)$ and $P_2(t)$ are non–negative and thus $V(y,t)$ is also non–negative. The Lyapunov function in (27) can be bounded as

$$\frac{1}{2}\min\{1,\underline{m}\}\|y\|^2 \le V \le \max\left\{\frac{1}{2}\overline{m}(\|y\|),1\right\}\|y\|^2 \tag{32}$$

where (2) was utilized.

Remark 5: When compared with the Lyapunov function in (33) of [5], (27) includes two additional terms [*i.e.*, $\frac{1}{2}\tilde{\beta}_1^2(t)$ and $P_2(t)$]. The first new term is added as a direct consequence of the time–varying nature of the uncertainty compensation gain $\hat{\beta}(t)$. On the other hand, the $P_2(t)$ term is introduced to prove $\mathcal{L}_1$ boundedness of $e_n(t)$ (as will be demonstrated subsequently). This is required to prove the boundedness of the time–varying gain $\hat{\beta}(t)$. Proving the boundedness of $\hat{\beta}(t)$ is a significant improvement over the similar results in [15] where boundedness was not ensured.

After taking the time derivative of (27) and substituting (5), (7) and (21), following expression can be obtained

$$\begin{aligned}\dot{V} = & -\sum_{j=1}^{n-1} e_j^2 - \alpha e_n^2 + e_{n-1}e_n - r^2 - kr^2 \\ & + r\tilde{N} - \alpha\beta_2|e_n|\end{aligned} \tag{33}$$

where (22) and (25) were also utilized. By using the fact that $e_{n-1}e_n \le \frac{1}{2}\left(e_{n-1}^2 + e_n^2\right)$, an upper bound on (33) can be obtained as

$$\begin{aligned}\dot{V} \le & -\min\left\{\frac{1}{2},\alpha-\frac{1}{2}\right\}\|z\|^2 + \frac{\rho^2(\|z\|)}{4k}\|z\|^2 \\ & -\alpha\beta_2|e_n|\end{aligned} \tag{34}$$

where (12) was utilized. Provided that α is selected to satisfy $\alpha > \frac{1}{2}$ and the control gain k is selected according to

$$k > \frac{1}{4\min\left\{\frac{1}{2},\alpha-\frac{1}{2}\right\}}\rho^2(\|z\|), \tag{35}$$

from (34), following expression is stated

$$\dot{V} \le -\gamma\|z\|^2 - \alpha\beta_2|e_n| \tag{36}$$

where $\gamma \in \mathbb{R}$ is some positive constant.

Given the non–negative Lyapunov function in (27), its bounds in (32), and its non–positive time derivative in (36), a more conservative bound on the control gain k can be obtained, specifically, when k is chosen to satisfy

$$k > \frac{1}{4\min\left\{\frac{1}{2},\alpha-\frac{1}{2}\right\}}\rho^2\left[\sqrt{\frac{\max\{\overline{m}(\|y(t_0)\|),2\}}{\min\{1,\underline{m}\}}}\|y(t_0)\|\right] \tag{37}$$

with

$$\|y(t_0)\|^2 = \sum_{j=1}^{n}|e_j(t_0)|^2 + |r(t_0)|^2 + \beta_1^2 + \zeta_{b_1} + \zeta_{b_2} \tag{38}$$

then (35) is ensured.

From (27), (32) and (36), it is clear that $V(y,t) \in \mathcal{L}_\infty$ and thus $e_1(t), \cdots, e_n(t)$, $r(t)$, $\tilde{\beta}_1(t)$, $P_1(t)$, $P_2(t) \in \mathcal{L}_\infty$. Boundedness of $e_n(t)$ and $r(t)$ can be utilized along with (7) to show that $\dot{e}_n(t) \in \mathcal{L}_\infty$. These boundedness statements can be utilized along with (4)–(6) to prove that $\dot{e}_1(t), \cdots, \dot{e}_{n-1}(t) \in \mathcal{L}_\infty$. From (20), it can easily be concluded that $\dot{u}(t) \in \mathcal{L}_\infty$. The boundedness of the auxiliary error signals and their time derivatives can be utilized along with (6) to conclude that $e_1^{(i)}(t) \in \mathcal{L}_\infty$ $i = 1,\cdots,n$, which can then be utilized along with (3) and its time derivatives to prove that $x^{(i)}(t) \in \mathcal{L}_\infty$ $i = 1,\cdots,n$. The above boundedness statements can be utilized along with $m(\cdot)$, $f(\cdot) \in \mathcal{C}_2$, to prove that $m(\cdot)$, $f(\cdot)$, $\dot{m}(\cdot)$, $\dot{f}(\cdot) \in \mathcal{L}_\infty$. From (21), it is concluded that $\dot{r}(t) \in \mathcal{L}_\infty$.

After integrating (36) in time, following expression can be obtained

$$\gamma\int_{t_0}^{\infty}\|z(\sigma)\|^2 d\sigma + \alpha\beta_2\int_{t_0}^{\infty}|e_n(\sigma)|\,d\sigma \le V(t_0) - V(\infty) \tag{39}$$

and since $V(\infty) \ge 0$ following expressions are obtained

$$\int_{t_0}^{\infty}\|z(\sigma)\|^2 d\sigma \le \frac{V(t_0)}{\gamma} \tag{40}$$

$$\int_{t_0}^{\infty}|e_n(\sigma)|\,d\sigma \le \frac{V(t_0)}{\alpha\beta_2}. \tag{41}$$

From (40) and (41), it is clear that $z(t) \in \mathcal{L}_2$ and $e_n(t) \in \mathcal{L}_1$. Since $e_n(t) \in \mathcal{L}_1 \cap \mathcal{L}_\infty$, from (18), it is concluded that $\hat{\beta}_1(t) \in \mathcal{L}_\infty$, and since $r(t) \in \mathcal{L}_\infty$, then from (19), it is clear that $\dot{\hat{\beta}}_1(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can be utilized to prove that all the remaining signals remain bounded under the closed–loop operation. Since $z(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{z}(t) \in \mathcal{L}_\infty$, Barbalat's Lemma [1] can be utilized to prove that $\|z(t)\| \to 0$ as $t \to \infty$, and from its definition in (13), it is clear that the tracking error and its time derivatives asymptotically converge to zero. ∎

IV. Simulation Results

In order to substantiate the theoretical results, the proposed nonlinear controller has been tested on a generalized first order system that contains scalar variables. The equation of motion is given as [17]

$$\dot{x} = -x + \delta_0 + u \tag{42}$$

where $x(t)$, $u(t) \in \mathbb{R}$ are the state variable and the control input, respectively, and the unknown scalar time–varying function $\delta_0(t) \in \mathbb{R}$ is set to be

$$\delta_0(t) = \sin(t) + \cos(\pi t). \tag{43}$$

The initial value of the state is $x(0) = 2$. The control objective is to make the state variable $x(t)$ track the following sinusoidal reference trajectory

$$x_r(t) = \sin(t). \tag{44}$$

Since this example system is first order, then in view of Remark 2, the control input is considered as a PI controller in terms of $e_1(t)$ with a self–updating nonlinear component for uncertainty compensation. As a result, the control gains α and k are treated as PI control gains, and β_2 can arbitrarily be chosen as positive. Following set of control gains delivered satisfactory tracking performance

$$\alpha = 2,\ k = 10,\ \beta_2 = 5. \tag{45}$$

The results are shown in Figures 1–3. The tracking error, control input and the time–varying control gain $\hat{\beta}(t)$ are shown in Figures 1, 2 and 3, respectively. From Figure 1, it is clear that tracking control objective is met.

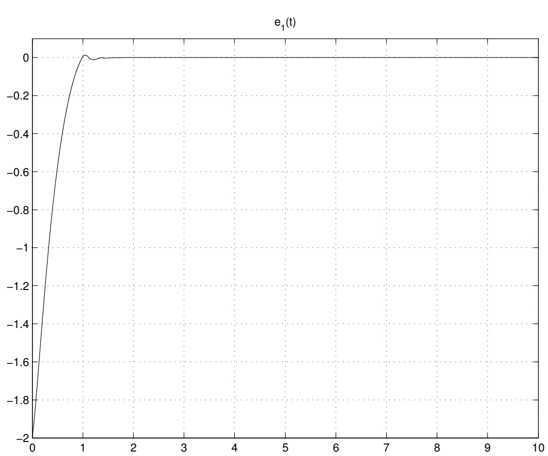

Fig. 1. Tracking error $e_1(t)$

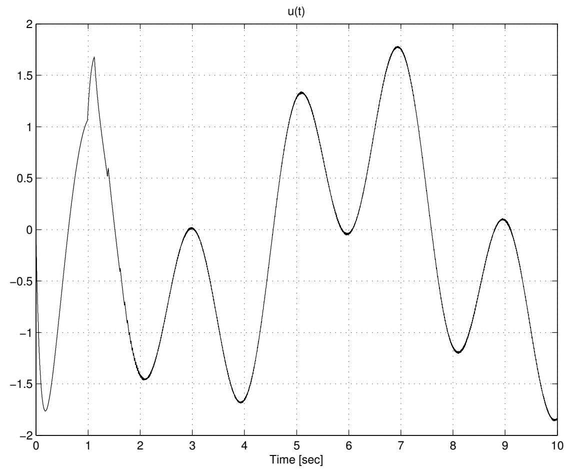

Fig. 2. Control input $u(t)$

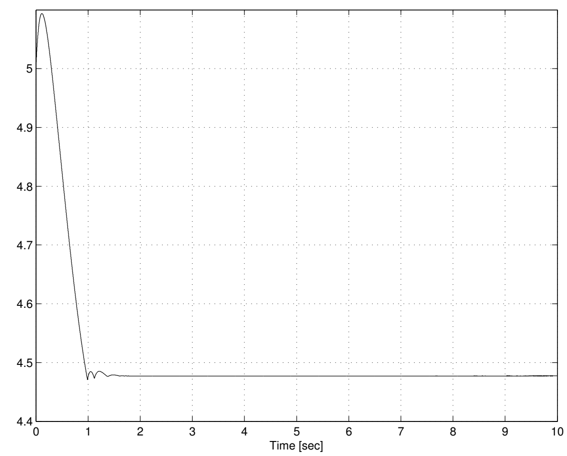

Fig. 3. The time–varying control gain $\hat{\beta}(t)$

V. Conclusions

In this paper, a new RISE feedback controller with a time–varying adaptive compensation control gain is designed. Different from the existing RISE feedback type controllers in the literature, in the proposed formulation, the control gain selection does not require prior knowledge of the upper bounds of the vector containing the desired system dynamics plus functions containing uncertainties. The use of the time–varying gain instead of constant compensation gain used in previous formulations aimed to reduce the heavy control effort and therefore to eliminate the need of extra feedforward compensation methods for RISE feedback controllers. The controller achieved semi–global tracking via a novel Lyapunov–type analysis. Numerical simulation studies are presented to illustrate the tracking performance of the proposed method for a first order scalar system.

Having designed a self–updating time–varying control gain for uncertainty compensation, a possible future work may be performing a similar modification for the other control gains (*i.e.*, k and α). Additionally, experimental verification is also aimed.

APPENDIX I
PROOF OF LEMMA 1

After substituting (7) into (22) and then integrating in time, the following expression is obtained

$$\begin{aligned}\int_{t_0}^{t} L_1(\sigma)\, d\sigma &= \alpha \int_{t_0}^{t} e_n(\sigma)\left[N_r(\sigma) - \beta_1 \mathrm{sgn}(e_n(\sigma))\right] d\sigma \\ &+ \int_{t_0}^{t} \frac{de_n(\sigma)}{d\sigma} N_r(\sigma)\, d\sigma \\ &- \beta_1 \int_{t_0}^{t} \frac{de_n(\sigma)}{d\sigma} \mathrm{sgn}(e_n(\sigma))\, d\sigma. \qquad (46)\end{aligned}$$

After integrating the second integral on the right–hand side by parts, following expression is obtained

$$\begin{aligned}\int_{t_0}^{t} L_1(\sigma)\, d\sigma &= \alpha \int_{t_0}^{t} e_n(\sigma)\left[N_r(\sigma) - \beta_1 \mathrm{sgn}(e_n(\sigma))\right] d\sigma \\ &+ e_n(\sigma) N_r(\sigma)\big|_{t_0}^{t} \\ &- \int_{t_0}^{t} e_n(\sigma) \frac{dN_r(\sigma)}{d\sigma} - \beta_1 \left|e_n(\sigma)\right| \big|_{t_0}^{t} \\ &= \alpha \int_{t_0}^{t} e_n(\sigma) \left[N_r(\sigma) - \frac{1}{\alpha}\frac{dN_r(\sigma)}{d\sigma}\right. \\ &- \left.\beta_1 \mathrm{sgn}(e_n(\sigma))\right] d\sigma \\ &+ e_n(t) N_r(t) - e_n(t_0) N_r(t_0) \\ &- \beta_1 \left|e_n(t)\right| + \beta_1 \left|e_n(t_0)\right|. \qquad (47)\end{aligned}$$

The right–hand side of (47) can be upper bounded as

$$\begin{aligned}\int_{t_0}^{t} L_1(\sigma)\, d\sigma &\leq \alpha \int_{t_0}^{t} \left|e_n(\sigma)\right| \\ &\times \left(\left|N_r(\sigma)\right| + \frac{1}{\alpha}\left|\frac{dN_r(\sigma)}{d\sigma}\right| - \beta_1\right) d\sigma \\ &+ \left|e_n(t)\right|\left(\left|N_r(t)\right| - \beta_1\right) \\ &+ \beta_1 \left|e_n(t_0)\right| - e_n(t_0) N_r(t_0). \qquad (48)\end{aligned}$$

From (48), it is easy to see that if β_1 satisfies (23), then (24) holds with

$$\xi_{b1} \triangleq \beta_1 \left|e_n(t_0)\right| - e_n(t_0) N_r(t_0). \qquad (49)$$

APPENDIX II
PROOF OF LEMMA 2

After integrating (25) in time, following steps can be obtained [18]

$$\begin{aligned}\int_{t_0}^{t} L_2(\sigma)\, d\sigma &= -\beta_2 \int_{t_0}^{t} \dot{e}_n(\sigma) \,\mathrm{sgn}(e_n(\sigma))\, d\sigma \\ &= -\beta_2 \int_{t_0}^{t} \mathrm{sgn}(e_n)\, d(e_n) \\ &= -\beta_2 \int_{t_0}^{t} d(\left|e_n\right|) \\ &= -\beta_2 \left(\left|e_n(t)\right| - \left|e_n(t_0)\right|\right) \\ &\leq \beta_2 \left|e_n(t_0)\right|. \qquad (50)\end{aligned}$$

From (50), it is easy to see that if β_2 is chosen as positive, then (26) holds with

$$\xi_{b2} \triangleq \beta_2 \left|e_n(t_0)\right|. \qquad (51)$$

REFERENCES

[1] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*. New York, NY, USA: John Wiley & Sons, 1995.
[2] Z. Qu, *Robust Control of Nonlinear Uncertain Systems*. New York, NY, USA: Wiley, 1998.
[3] S. S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*. Englewood Cliffs, NJ, USA: Prentice Hall, 1989.
[4] W. Messner, R. Horowitz, W. Kao, and M. Boals, "A new adaptive learning rule," *IEEE Trans. Autom. Control*, vol. 38, pp. 188–197, 1993.
[5] B. Xian, D. M. Dawson, M. S. de Queiroz, and J. Chen, "A continuous asymptotic tracking control strategy for uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 49, no. 7, pp. 1206–1211, 2004.
[6] B. Xian, M. S. de Queiroz, and D. M. Dawson, "A continuous control mechanism for uncertain nonlinear systems," in *Control, Stabilization, and Nonsmooth Analysis, Lecture Notes in Control and Information Sciences*. Heidelberg, Germany: Springer-Verlag, 2004, pp. 251–262.
[7] P. M. Patre, W. Mackunis, C. Makkar, and W. E. Dixon, "Asymptotic tracking for systems with structured and unstructured uncertainties," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 2, pp. 373–379, 2008.
[8] J. Shin, H. J. Kim, Y. Kim, and W. E. Dixon, "Autonomous flight of the rotorcraft-based UAV using RISE feedback and NN feedforward terms," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 5, pp. 1392–1399, 2012.
[9] N. Fischer, S. Bhasin, and W. E. Dixon, "Nonlinear control of an autonomous underwater vehicle: A RISE-based approach," in *Proc. American Control Conf.*, San Francisco, CA, USA, 2011, pp. 3972–3977.
[10] Z. Wang and A. Behal, "Continuous robust control for a class of uncertain MIMO nonlinear systems," in *Proc. IEEE Int. Conf. on Decision and Control and European Control Conf.*, Orlando, FL, USA, 2011, pp. 7561–7566.
[11] Z. Wang, J. Chen, and A. Behal, "Robust adaptive control design for a class of MIMO nonlinear systems," in *Proc. of Int. Symposium on Intelligent Control Part of IEEE Multi–Conf. on Systems and Control*, Yokohama, Japan, 2010, pp. 2284–2289.
[12] N. Sharma, S. Bhasin, Q. Wang, and W. E. Dixon, "RISE-based adaptive control of an uncertain nonlinear system with unknown state delays," in *Proc. IEEE Int. Conf. on Decision and Control*, Atlanta, GA, USA, 2010, pp. 1773–1778.
[13] P. Patre, W. E. Dixon, K. Kaiser, and W. Mackunis, "Asymptotic tracking for uncertain dynamic systems via a multilayer NN feedforward and RISE feedback control structure," in *Proc. American Control Conf.*, New York, NY, USA, 2007, pp. 5989–5994.
[14] T. Dierks and S. Jagannathan, "Neural network control of mobile robot formations using RISE feedback," *IEEE Tr. on Systems, Man, and Cybernetics, Part B*, vol. 39, no. 2, pp. 332–347, 2009.
[15] Q. Yang, S. Jagannathan, and Y. Sun, "NN/RISE-based asymptotic tracking control of uncertain nonlinear systems," in *Proc. of Int. Symposium on Intelligent Control Part of IEEE Multi–Conf. on Systems and Control*, Denver, CO, USA, 2011, pp. 1361–1366.
[16] H. K. Khalil, *Nonlinear Systems, 3rd Edition*. New York, NY, USA: Prentice Hall, 2002.
[17] Z. Qu and J.-X. Xu, "Model-based learning controls and their comparisons using Lyapunov direct method," *Asian J. Control*, vol. 4, no. 1, pp. 99–110, 2002.
[18] M. Krstic, *Delay Compensation for Nonlinear, Adaptive, and PDE Systems*. Boston, MA, USA: Birkhauser, 2009.