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A Comment on Change of Nusselt Number Sign in a Channel Flow Filled by a Fluid-Saturated Porous Medium with Constant Heat Flux Boundary Conditions

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Abstract The aim of this Letter is to show that, the Nusselt number sign might be changed without changing of heat transfer direction at the wall of channels, even for flows without viscous dissipation.

Keywords Porous medium · Channel flow · Constant heat flux · Boundary conditions

The sign of the Nusselt number is important for deciding on heat transfer direction at a solid wall. The change of the Nusselt number sign may be interpreted as the change of the direction of the heat transfer at a wall. There are studies, such as internal heat and fluid flow in a channel with viscous dissipation (Hung and Tso 2008, 2009; Mitrovic and Maletic 2007; Mobedi et al. 2010) or with an asymmetric heat flux boundary conditions (Cekmer et al. 2011) in which the sign of the wall Nusselt number changes. Nield and Kuznetsov (2008) studied in a very interesting paper the counter flow in a channel whose boundaries are asymmetrically heated and is consisted of two porous layers with different permeability values. These authors showed that even the sign of an overall Nusselt number defined based on the average wall temperatures and heat fluxes, and the mean permeability values of the two porous layers can also be changed and it can take negative values when a strong asymmetry heat flux is imposed to the boundaries. The change of Nusselt number sign at the walls are also observed in other studies of Kuznetsov (Kuznetsov and Nield 2010; Xiong and Kuznetsov 2000).

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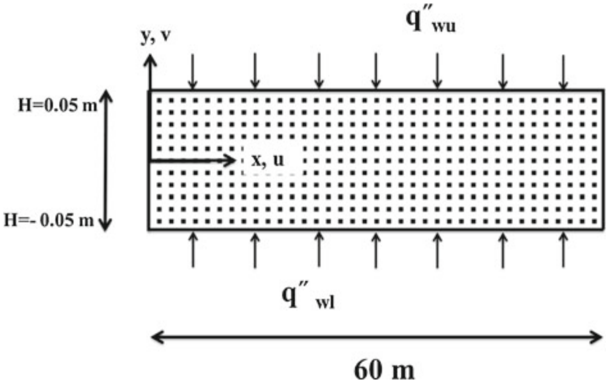


Fig. 1 The considered channel with asymmetric heat flux boundary condition

The above brief literature survey shows that there are several heat and fluid flow problems in which the sign of the Nusselt number at wall is changed. This change of the sign of the Nusselt number may be due to the change of the heat flux direction at the wall or it might be due to inappropriate definition of the temperature difference for convective heat transfer coefficient.

The aim of this Letter is to show that the sign of the Nusselt number can be changed without changing of the direction of the heat transfer at the walls of a channel. This Letter intends to draw the attention of researchers on the importance of temperature difference used to define a convective heat transfer coefficient, and to show that an inappropriate temperature difference may cause wrong comments on the direction of heat transfer. It is shown how the mean temperature can be greater or smaller than the wall temperatures for a channel in which heat is transferred from both walls to the fluid. In order to achieve this aim, a channel filled with a fluid-saturated porous medium is considered (Fig. 1). The temperature and velocity profiles in the channel for fully developed flow are obtained. The height of the channel is 0.1 m, while its length is considered 60 m. An asymmetrical heat flux is imposed at the upper and lower walls. Air enters into the channel at 300 K with the mean velocity of 0.38 m/s. The flow is laminar, and the viscous dissipation is neglected. The channel is filled with air saturated porous medium whose effective thermal conductivity and permeability are 1.1 W/mK and 3.23×10^{-6} m^2, respectively. The kinematic viscosity and thermal capacity of air are 1.983×10^{-5} m^2/s and 1.162 kJ/m^3 K, respectively. It is assumed that the porous medium has very high porosity value as 0.99 in order to have schematically understandable temperature profiles in the channel. It is also assumed that the fluid flows through the whole of the channel as fully developed and the velocity profile can be obtained by solving the following differential equation

$$\mu_{\mathrm{eff}} \frac{\mathrm{d}^2 u(y)}{\mathrm{d}y^2} - \frac{\mu}{K} u(y) - \frac{\mathrm{d}P}{\mathrm{d}x} = 0 \tag{1}$$

with the boundary conditions

$$u = 0 \quad \text{at} \quad y = \pm\, 0.05 \text{ m} \tag{2}$$

where μ_{eff} and μ are the effective and fluid dynamic viscosity. The value of the effective dynamic viscosity is expressed by $\mu_{\mathrm{eff}}/\mu_{\mathrm{f}} = 1/\varepsilon$, where ε is the porosity of the porous medium. Since the dimensional heat transfer equation can not be written in the form of

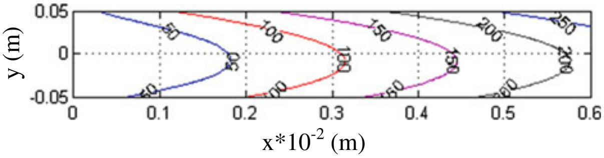


Fig. 2 Temperature profiles in the studied channel for $q_{\mathrm{wu}} = 100$ and $q_{\mathrm{wl}} = 70\ \mathrm{W/m^2}$

ordinary differential equation, the temperature field in the channel can be obtained by solving the following partial differential equation for the channel shown in Fig. 1,

$$\left(\rho C_p\right)_{\mathrm{f}} u(y) \frac{\partial T}{\partial x} = k_{\mathrm{eff}} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

where ρ is the fluid density, C_p is the specific heat of the fluid at constant pressure and k_{eff} is the effective thermal conductivity of the porous medium. The following thermal boundary conditions are valid for the problem,

$$\begin{aligned}
&q''_{\mathrm{wl}} = -k_{\mathrm{eff}} \left.\frac{\partial T}{\partial y}\right|_{y=-0.05} \quad \text{at} \quad y = -0.05\ \mathrm{m} \\
&q''_{\mathrm{wu}} = -k_{\mathrm{eff}} \left.\frac{\partial T}{\partial y}\right|_{y=0.05} \quad \text{at} \quad y = +0.05\ \mathrm{m} \\
&T = 300\ \mathrm{K} \quad \text{at} \quad x = 0
\end{aligned} \tag{4}$$

where q'' is the heat flux from the lower and upper walls. As it is known, the convective heat flux (i.e., q'') and the Nusselt number (i.e., Nu) for a flow in a channel with constant heat flux are defined as

$$q''_{\mathrm{w}} = -k_{\mathrm{eff}} \left.\frac{\partial T}{\partial y}\right|_{\mathrm{wall}} \tag{5}$$

and

$$Nu = \frac{2Hq''_{\mathrm{w}}}{k_{\mathrm{eff}}(T_{\mathrm{w}} - T_{\mathrm{m}})} \tag{6}$$

where T_{w} is the wall temperature. It should be mentioned that T_{w} should be replaced by T_{wl} or T_{wu} in order to obtain values of the Nusselt numbers for lower or upper walls, respectively. Here, $T_{\mathrm{m}}(x)$ is the mean temperature and it can be defined as

$$T_{\mathrm{m}}(x) = \frac{\int_{-H}^{+H} u(y) T(x, y) \mathrm{d}y}{\int_{-H}^{+H} u(y) \mathrm{d}y} \tag{7}$$

Equations (1) and (3) with the given boundary conditions (4) are solved numerically. Figure 2 shows the obtained temperature profiles inside the considered channel for the heat fluxes of $q_{\mathrm{wu}} = 100$ and $q_{\mathrm{wl}} = 70\ \mathrm{W/m^2}$. An asymmetrical behaviour in the temperature field is observed due to asymmetrical heating at the boundaries. For a cross-section of the channel, the temperature at the upper wall is greater than at the lower wall due to the higher imposed heat flux. The changes with x of the temperatures at the upper and lower walls, and the mean temperature along the channel are shown in Fig. 3. It is noticed that after a distance, the temperature differences between the walls and mean temperature are not changed. It also shows that the temperature profile at the exit of channel is thermally fully developed. Since this study is performed on the fully developed heat and fluid flow, our attention focuses on the outlet temperature profiles. The outlet temperature profiles for four different heat flux

Fig. 3 The variation with x of the wall temperatures at the lower and upper walls, and the mean temperature along the channel

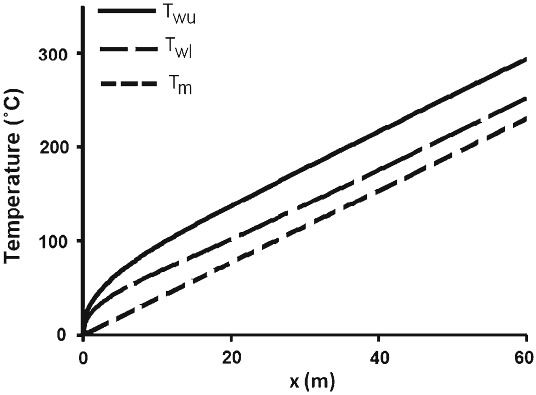


Fig. 4 Temperature profiles for different values of heat flux ratio

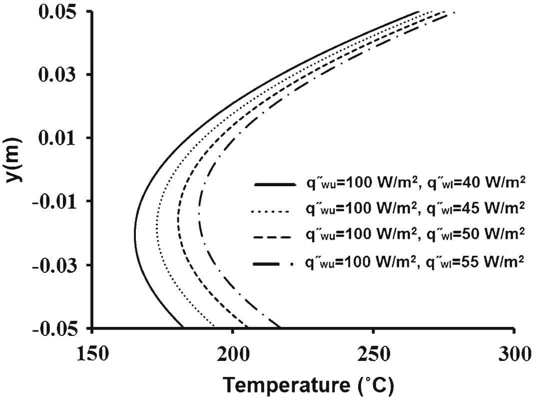


Table 1 Nusselt Numbers for specified heat flux ratios

q_{wu}	q_{wl}	$q_{\mathrm{wu}}/q_{\mathrm{wl}}$	Nu_{u}	Nu_{l}
100	40	2.5	3.64799	−16.556
100	45	2.23	3.75716	−61.91
100	50	2	3.87306	51.9553
100	55	1.81	3.99634	20.7423

ratios of $q_{\mathrm{wu}}/q_{\mathrm{wl}} = 100/40$, $100/45$, $100/50$, $100/55$ are plotted in Fig. 4. As it seen, the temperature profiles at the upper and lower walls are greater than the fluid temperature adjacent to the wall, showing transferring of heat from the both upper and lower walls to the fluid. The values of the Nusselt numbers for the plotted profiles are calculated by using Eqs. (5) and (6). The results are shown in Table 1. For $q_{\mathrm{wu}}/q_{\mathrm{wl}} = 100/40$ and $100/45$, the Nusselt numbers for the upper wall is positive while it is negative for the lower plate. Then, the values of the Nusselt number at the lower wall become positive for $q_{\mathrm{wu}}/q_{\mathrm{wl}} = 100/50$ and $100/55$, respectively. The sign of the Nusselt number is changed without changing the temperature slope at the lower wall, as it can be seen in Fig. 4.

The variation of the mean, upper and lower walls temperatures at the exit of the channel with the imposed heat flux the ratio $q_{\mathrm{wu}}/q_{\mathrm{wl}}$ are shown in Fig. 5. It is noticed that the differences between the mean temperature, and temperatures of the lower and upper walls

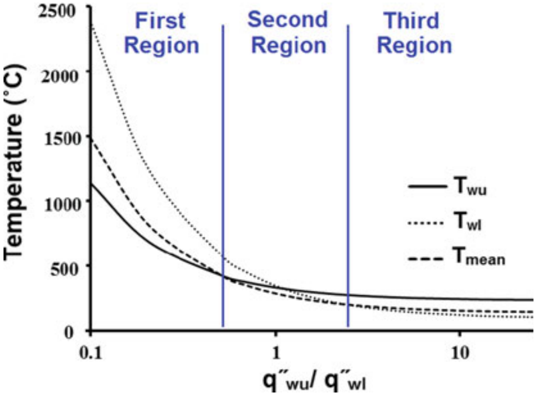


Fig. 5 The variation of the mean, upper and lower walls temperatures with the heat flux ratio at the outlet of the channel

vary with the change of $q_{\mathrm{wu}}/q_{\mathrm{wl}}$. The diagram can be divided into three regions. In the first region ($q''_{\mathrm{wu}}/q''_{\mathrm{wl}} < 0.48$), the mean temperature is greater than the temperature of upper wall (T_{wu}) and smaller than the temperature of lower wall (T_{wl}), hence $(T_{\mathrm{wu}} - T_{\mathrm{m}}) < 0$ while $(T_{\mathrm{wl}} - T_{\mathrm{m}}) > 0$. In the second region ($0.48 < q''_{\mathrm{wu}}/q''_{\mathrm{wl}} < 2$), the mean temperature is smaller than the both temperatures of the upper and lower walls and consequently $(T_{\mathrm{wu}} - T_{\mathrm{m}}) > 0$ and $(T_{\mathrm{wl}} - T_{\mathrm{m}}) > 0$. In the third region ($q''_{\mathrm{wu}}/q''_{\mathrm{wl}} > 2$), the mean temperature is smaller than the temperature of the upper wall (T_{wu}) and greater than the temperature of the lower wall (T_{wl}), therefore $(T_{\mathrm{wu}} - T_{\mathrm{m}}) > 0$ while $(T_{\mathrm{wl}} - T_{\mathrm{m}}) < 0$.

The values of the Nusselt numbers at the upper and lower walls are calculated for different values of the ratio of imposed heat flux at boundaries (i.e., $q''_{\mathrm{wu}}/q''_{\mathrm{wl}}$). Considering Fig. 4, it should be reminded that heat is transferred from the walls to fluid for all values of the ratios of imposed heat flux. The variation of the Nusselt numbers of the upper and lower walls for fully developed condition with heat flux ratio $q''_{\mathrm{wu}}/q''_{\mathrm{wl}}$ is shown in Fig. 6. It can be seen that for the first region ($q''_{\mathrm{wu}}/q''_{\mathrm{wl}} < 0.48$), the Nusselt number of the upper plate is negative, while the Nusselt number of the lower plate is positive. The negative sign of the Nusselt number at the upper wall is due to negative values of $(T_{\mathrm{wu}} - T_{\mathrm{m}})$ that is, $(T_{\mathrm{wu}} - T_{\mathrm{m}}) < 0$. In the second region ($0.48 < q''_{\mathrm{wu}}/q''_{\mathrm{wl}} < 2$), both the upper and lower values of the Nusselt numbers are positive since the wall temperature of the both walls are greater than the mean temperature. For the case of $q''_{\mathrm{wu}} = q''_{\mathrm{wl}}$, the Nusselt numbers of the upper and lower walls are equal as 5.6 and it is positive for the both walls. For the third region ($q''_{\mathrm{wu}}/q''_{\mathrm{wl}} > 2$), the trend is vice versa of the first region and the Nusselt numbers of the upper region is positive and in the lower region the Nusselt number is negative since $(T_{\mathrm{wl}} - T_{\mathrm{m}}) < 0$. The changes of the sign of the Nusselt number at the upper and lower walls are due to the improper temperature difference in the definition of heat transfer coefficient. Bejan (2004) defined the mean constant temperature T_{m} for the internal flow in a channel filled with a porous medium as follows:

$$T_{\mathrm{m}} = \frac{\int_{-H}^{+H} T(y)\mathrm{d}y}{2H} \tag{8}$$

Figure 7 shows the variation of the Nusselt number with the heat flux ratio of the upper and lower walls based on the mean temperature of Eq. (8). The same behavior for the change of the Nusselt number is observed. The only difference is the value of the heat flux ratio for which the sign of the Nusselt number is changed. If the Nusselt number is defined based on Eq. (8), the changes of the sign of the Nusselt number occurs at $q''_{\mathrm{wu}}/q''_{\mathrm{wl}} = 0.47$ and 1.99, respectively.

Fig. 6 The changes of upper and lower wall Nusselt number with heat flux ratio. **a** upper wall **b** lower wall

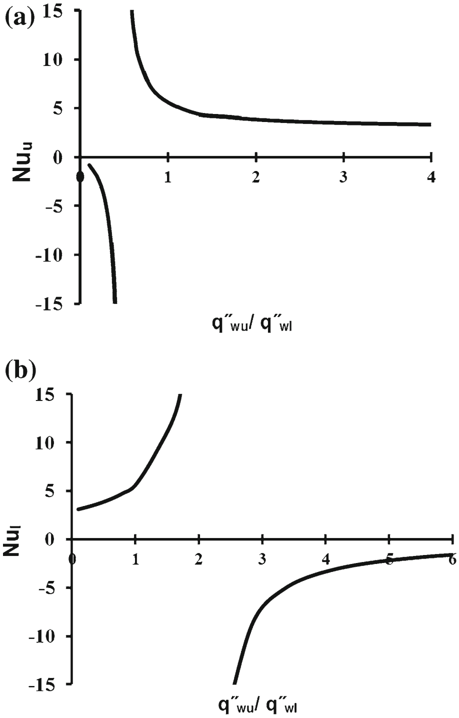
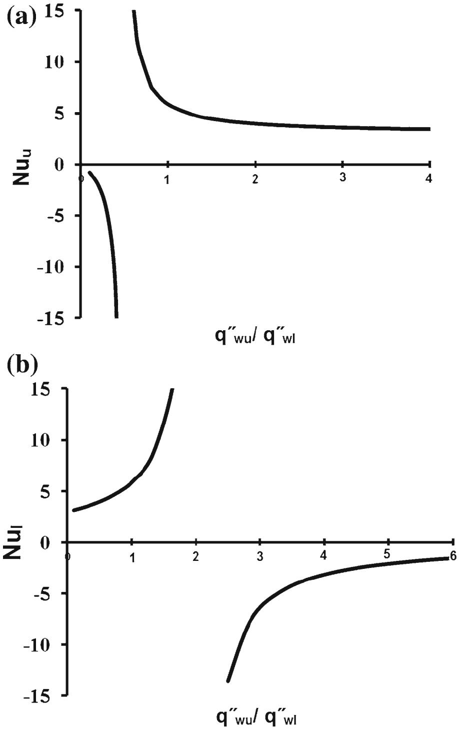


Fig. 7 The changes of the Nusselt number with heat flux ratio at the upper and lower walls by using of Eq. (8): **a** upper wall; **b** lower wall



In conclusion, it found that an appropriate temperature difference should be defined for determination of the Nusselt numbers of a wall for the channels with asymmetric heat flux boundary conditions. Researchers should be very careful on the interpretation of the changes of the sign of the wall Nusselt number for this kind of internal heat and fluid flow problems.

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