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Constructal vascularized structures

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Abstract: Smart features such as self-healing and self-cooling require bathing the entire volume with a coolant or/and healing agent. Bathing the entire volume is an example of point to area (or volume) flows. Point to area flows cover all the distributing and collecting kinds of flows, i.e. inhaling and exhaling, mining, river deltas, energy distribution, distribution of products on the landscape and so on. The flow resistances of a point to area flow can be decreased by changing the design with the guidance of the constructal law, which is the law of the design evolution in time. In this paper, how the flow resistances (heat, fluid and stress) can be decreased by using the constructal law is shown with examples. First, the validity of two assumptions is surveyed: using temperature independent Hess-Murray rule and using constant diameter ducts where the duct discharges fluid along its edge. Then, point to area types of flows are explained by illustrating the results of two examples: fluid networks and heating an area. Last, how the structures should be vascularized for cooling and mechanical strength is documented. This paper shows that flow resistances can be decreased by morphing the shape freely without any restrictions or generic algorithms.

Keywords: constructal law; vascularization; point to area flows; distributing flows; smart materials

1 Emergence of vascularization

Advanced capabilities such as self-healing and self-cooling require bathing the entire volume with coolant fluid or healing agent [1–5], which can be achieved by vascularization. In addition of being necessary for the advanced capabilities, vascularization is also essential to decrease the resistances of the distribution of energy, goods and water [1, 6, 7]. In smart materials, the structure is bathed with coolant or healing agent which is supplied from a reservoir to obtain the smart features. Similarly, a factory distributes all its products to the cities located around the world. All these flows are examples of flow from a point to an area (or volume).

In addition, the peak temperature and the maximum stress of a heated and mechanically loaded structure can be kept under an allowable limit as its weight decreases with vascularization [8, 9]. Therefore, the penalty of moving the structure decreases which is crucial for avionics. Decreasing this penalty decreases the fuel consumption which is also essential due to finite source of fuel supplies. Vascularization is a necessity for the advanced aircrafts and space shuttles because it promises to protect the vehicles under great heat fluxes and great mechanical loads with being capable of repairing itself and being light and robust at the same time.

There are two kinds of cooling requirements for a structure: deterministic and random. The deterministic are due to heat sources that are known which are steady. However, random cooling requirements are unsteady and diverse. Vascularization also protects the structure from random heat sources [10]. Because random cooling requirements are unpredictable, they are responsible of damaging the structure which is designed to work under an allowable temperature level.

Constructal law is the law of the design evolution in time, and it was stated in 1996 by Adrian Bejan as “For a finite size system to persist in time (to live), it must evolve such a way that it provides easier access to the imposed currents that flow through it” [11]. Constructal law is a law because it is applicable for both animate and inanimate, i.e. it is valid for everything. Because it is a law, constructal law can be found in diverse fields such as biology, geophysics, engineering, social dynamics and evolution of sports [1, 12–43]. Constructal law is also significant because it fills the gap of design parameter in thermal sciences which is overlooked in the field.

In this paper constructal vascularized structures are used to enable advanced capabilities such as self-healing and self-cooling while decreasing the resistances of flow (heat, fluid and stresses). Constructal designs can be defined as the designs that perform the best in their environment (boundary conditions, initial conditions) with the existence of constraints (size, shape, volume of material). The power of constructal designs comes from pursu-

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ing the designs best fits to their environment without any assumptions, design constraints and generic algorithms. Constructal designs are free to vary and there is no optimal design.

2 Validity of flow assumptions

The accuracy of the assumptions is crucial for the validity of the results. Therefore, this section discusses the validity of two assumptions in fluid flow problems. First, it is questioned how a duct should be shaped to discharge fluid along its length while the flow resistance is the smallest, Figure 1. Second, the effect of temperature on Hess-Murray rule is uncovered by assuming the properties of the fluid is temperature dependent.

2.1 Tapered ducts

Imagine a duct which discharge fluid along its length which can be a model of a duct connected to a number of users receiving the fluid along its length, Figure 1 [44]. The flow rate in the duct varies linearly $\dot{m}(x) = \dot{m}x/L$. The pressure drops for laminar and turbulent flows are

$$\Delta P_1 = C_1 \frac{\dot{m}}{L} \int_0^L \frac{x}{D^4} dx$$
$$\Delta P_t = C_t \frac{\dot{m}^2}{L^2} \int_0^L \frac{x^2}{D^5} dx, \tag{1}$$

where $C_1 = 128/\nu\pi$ and $C_t = 32f/\pi^2\rho$, and ν, ρ and f are the kinematic viscosity, the density and the constant friction factor for turbulent flow in the fully developed and fully rough regime,respectively [1]. m, L, x and D are the mass flow rate that enters the duct, the length, the distance from the closed end of the duct, and the diameter, respectively. The volume constraint is

$$Vol = \int_0^L \frac{\pi}{4} D^2 dx. \tag{2}$$

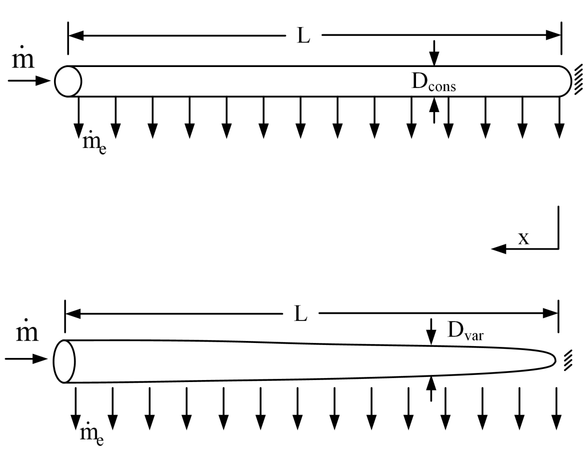

Figure 1: Supply duct with longitudinally distributed discharge: duct with constant diameter (top) and tapered duct (bottom).

The pressure drops for laminar and turbulent flow regimes along the duct with constant diameter are

$$\Delta P_{0,1} = C_1 \frac{\dot{m}L}{2D_0^4}$$
$$\Delta P_{0,t} = C_t \frac{\dot{m}^2 L}{3D_0^5}. \tag{3}$$

Now consider the duct shape is free to vary. By using variational calculus, pressure drops of Equation (1) can be minimized subject to the volume constraint of Equation (2). The tapered channel diameters and their pressure drops for laminar and turbulent regimes are

$$\Delta P_{min,1} = \frac{3^3\pi^2}{4^5} \frac{C_1 \dot{m} L^3}{Vol^2}$$
$$D_{min,1} = \left(\frac{16}{3\pi}\frac{Vol}{L}\right)^{1/2} \left(\frac{x}{L}\right)^{1/6} \tag{4}$$

$$\Delta P_{min,t} = C_t \dot{m}^2 \pi^{5/2} \left(\frac{7L}{44Vol}\right)^{7/2}$$
$$D_{min,t} = \left(\frac{44}{7\pi}\frac{Vol}{L}\right)^{1/2} \left(\frac{x}{L}\right)^{2/7}. \tag{5}$$

Dividing the minimized pressure drops of Equations (4) and (5) by the pressure drops of Equation (1), for turbulent and laminar flows respectively, shows the reduction in the flow resistance by tapering the ducts in laminar and turbulent flow regimes

$$\frac{\Delta P_{min,1}}{\Delta P_{0,1}} = \frac{3^3 2}{4^3} = 0.84$$
$$\frac{\Delta P_{min,t}}{\Delta P_{0,t}} = 3\left(\frac{7}{11}\right)^{7/2} = 0.62. \tag{6}$$

The results of Equation (6) show that the flow resistance of the tapered ducts is smaller when the duct discharges fluid along its length. In laminar flow regime, modeling the channel as a constant diameter duct would not affect the results as much as in turbulent flow regime.

2.2 Temperature dependence

Dendritic flow structures offer less resistance when bifurcations are accompanied with the Hess-Murray rule for the diameter ratio of the mother and daughter channels. The concept is reviewed by assuming the general case of temperature dependent properties [8]. In addition, the mother tube is connected to n identical daughter tubes, and the flow regime can be laminar or turbulent. The fluid volume is fixed. The pressure drop formula for laminar flow is

$$\Delta P = C_1 \nu_i \dot{m}_i \frac{L_i}{D_i^4}, \tag{7}$$

where ν_i is the kinematic viscosity corresponding to the mean temperature $T_{me} = \int T dVol / \int dVol$.

The total flow volume and pressure drop area

$$\begin{aligned} Vol &= \pi \frac{D_1^2}{4} L_1 + n\pi \frac{D_2^2}{4} L_2 \\ \Delta P &= C\nu_1 \dot{m}_1 \frac{L_1}{D_1^4} + C\nu_2 \dot{m}_2 \frac{L_2}{D_2^4}, \end{aligned} \tag{8}$$

where $\dot{m}_2 = \dot{m}_1/n$. The diameter ratio for minimum ΔP in laminar flow regime is

$$\frac{D_1}{D_2} = n^{1/3} \left(\frac{\nu_1}{\nu_2}\right)^{1/6}. \tag{9}$$

The pressure drop formula for fully developed and fully rough turbulent flow is

$$\Delta P = \frac{C_t \dot{m}_i^2}{\rho_i} \frac{L_i}{D_i^5}, \tag{10}$$

where ρ_i is the density of the fluid corresponding to T_{me}. The diameter ratio corresponding to the smallest flow resistance in turbulent flow is

$$\frac{D_1}{D_2} = n^{3/7} \left(\frac{\rho_2}{\rho_1}\right)^{1/7}. \tag{11}$$

The effect of variable properties is felt through the ratios $(\nu_1/\nu_2)^{1/6}$ and $(\rho_2/\rho_1)^{1/7}$ for laminar flow and turbulent flow, respectively. Assumption of the fluid with temperature independent properties is valid when the temperature variations are small enough.

3 Point to area flows: distribution on the landscape

The distribution of fluids, energy and products from a source to the individuals is essential for life. Its importance has increased with globalization because the production efficiency has increased with the mass production. Similarly, the efficiency of heaters increases as their size increases [7, 45, 46]. Therefore, the distribution of hot water to a number of users from a central heater, i.e. urban heating, became more efficient than individual heating. The materials gathered around the world (area to point flow) become end products in a factory, and these products are distributed to millions of people every day (point to area flow). These distributing and collecting types of flows can perform with greater efficiency if their flow resistances are reduced [1]. The flow resistances cannot be eliminated but they can be reduced to a limit. In heat engines this limit is known as Carnot limit (Carnot efficiency).

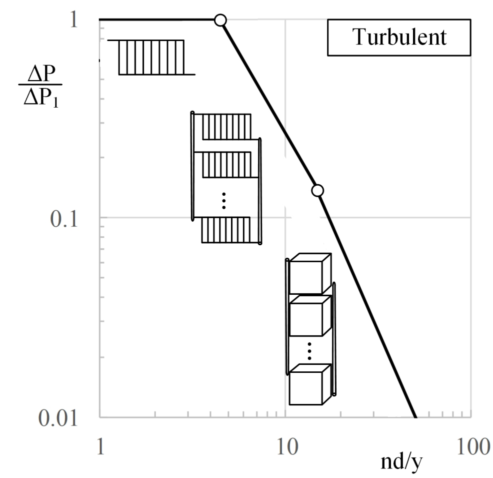

Figure 2: The pressure drop of three competing designs divided by the pressure drop of the design of stack of n elemental flow volumes for turbulent flow regime.

3.1 Fluid networks

In the case of distributing fluid to a number of elemental volumes or users there are infinite design possibilities. However, the objective is to find the design that corresponds to the smallest resistance for a known size (number of elemental volumes or users) [44, 47]. Depending on the flow regime and other conditions there is a design which achieves the smallest flow resistance, Figure 2 [44]. Figure 2 shows that as the number of elemental volumes increase the flow resistance can be decreased by changing the design. If the size of the elemental volume is fixed, this result means that as the size of the structure increases, the design should be changed, i.e. the design of the miniature structure should be different than the normal scale structure to achieve the smallest flow resistance possible.

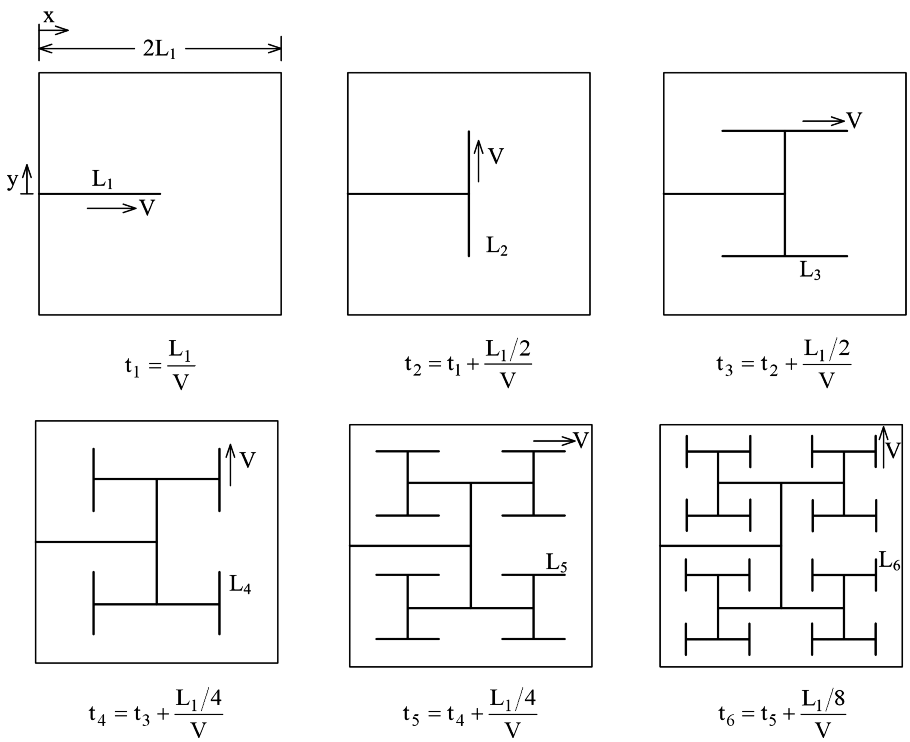

Figure 3: Tree-shaped line invasion of a conducting domain with assumed T- shaped bifurcations.

3.2 Heating of an area

Similar to distributing fluid to an area and bathing the area with fluid, imagine that a conducting area is invaded by heating, for instance, is bathed with a hot stream. The conducting domain is two dimensional with the uniform initial temperature (T_0), conductivity (k) and thermal diffusivity (α). The boundaries of the square are insulated. Beginning with the time t = 0, lines of uniform temperature (T_1) invade the conducting domain with the constant speed V, Figure 3 [48]. Heat is transferred by thermal diffusion from the invading lines to the conducting material. The details of the solution method can be seen in Ref. [48]. The temperature averaged over the square area is

$$T_{avg} = \frac{1}{A} \iint_A T dx dy \tag{12}$$

and it rises from T_0 to T_1.

Figure 4 shows that S curve of the history of invading becomes steeper as the invading tree morphed freely . The overall resistance to the heat flow is decreased just by changing the shape of the invading tree as all the other parameters (total length of the tree, invading speed, boundary conditions and initial conditions) are the same. Figure 4 emphasizes the importance of design parameter in thermal sciences.

4 Vascularization for cooling and mechanical strength

Vascularization increases the cooling performance of a structure, and it decreases the mechanical strength of the structure if the material is removed for the cooling channels. However, vascularization increases the mechanical strength of the structure if the material of the structure is fixed, i.e. the material removed for the channels are placed around the cooling channels. Constructal law states that the material should be placed where it is needed the most to decrease the resistance to its flow. This flow can be flow of stresses [1, 49–51] as well as the flow of fluid and heat. Greater mechanical strength is promised by placing the material away from the center where the material is stressed the least in the case of a plate loaded with a uniform force from its below and the edge of this plate is a no displacement boundary condition. A beam is an example to how mechanical strength can be increased by vary-

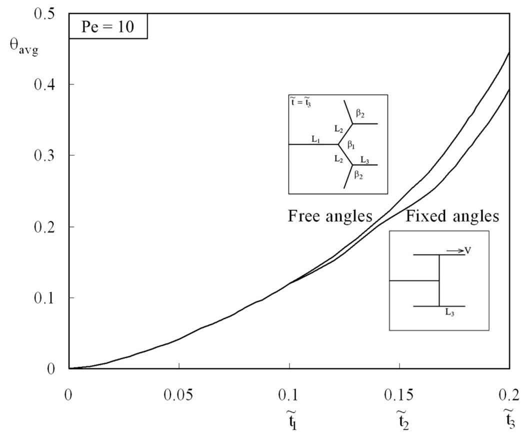

Figure 4: The S curve of tree invasion with free angles is steeper (faster) than the S curve of tree invasion with fixed branching angles.

ing the shape of the beam when the amount of material is fixed.

4.1 Radial and tree-shaped channel configurations

Here the thermal and mechanical performances of a heated and mechanically loaded circular plate have increased by embedding vascular structures in it. The diameter and thickness of the plate are D and H, and their ratio is fixed $D/H = 10$, Figure 5a [8]. The total volume and the volume of the channels are fixed. The plate with radial cooling channels is subjected to uniformly distributed force and uniform heat flux, both acting from below, Figure 5a. The dimensionless governing equations (the mass conservation, the conservation of the momentum for the fluid domain, the energy equation, the generalized Hooke's law and the conservation of momentum equations for solid domain) were solved in a finite element software[1]. Mesh test was also performed to confirm mesh independency of the results [8].

The heat flux and mechanical load which are subjected from the bottom of the plate as shown in Figure 5a, are fixed. The pressure difference between inlet and outlet is nondimensinalized as [52, 53].

$$\tilde{P}_{max} = \frac{\left(P_{in} - P_{ref}\right) D^2}{\mu\alpha}, \tag{13}$$

where μ and α are dynamic viscosity and thermal diffusivity. The value of $\tilde{P}_{max}$ represents their dimensionless overall pressure difference. The flow is laminar in all the channels.

The purpose of morphing the shape is to decrease the peak temperature and stress. Figure 5b shows the relation between the temperature, stress and number of ducts when $\tilde{P}_{max}$ is 10^7 and 10^8. The maximum stress decreases when the number of the cooling channels increases from 6 to 8 and then increases when the number of the cooling ducts increases. The reason of this behavior is that the maximum stress increases in the vicinity of the junctions of the cooling ducts. Even though σ_{max} is the minimum when the number of the channels is 8, neighboring designs (design of 6 cooling channels when $\tilde{P}_{max} = 10^7$ and design of 12 cooling channels when $\tilde{P}_{max} = 10^8$) offer minimum peak temperatures. In summary, when $\tilde{P}_{max}$ is specified, it is possible to identify one design (or a group of similar designs) that provides low peak stress and peak temperature. However, there is no optimal design for all the conditions.

4.2 Hybrid channel configurations

Consider a square plate with length L, thickness $H = 0.1L$, and embedded cooling channels, Figure 6a [9]. The plate is subjected to a uniformly distributed force acting from below, and it is heated uniformly. The volume of the structure and the flow volume are fixed. L_g is the side of the square area in which the grid cooling channels are embedded. The grid channels are connected to the periphery with radial channels. Coolant enters or exits from the center of the grid, and it is driven by the pressure difference maintained between the inlet and outlet. The results were obtained by solving the governing equations numerically as discussed in the previous subsection.

Figure 6b shows the minimum peak temperatures plotted against the peak stresses as L_g/L varies. The effect of the flow direction is weak. The smaller T_{peak} and σ_{peak} values occur when $L_g/L < 0.25$. Peak stress is the minimum when the design is a hybrid of grid and trees. However, the peak temperature is the minimum when the channels are configured as radial channels.

4.3 Concentrated heating

An important aspect to consider is the concentrated heating in the vascularized solid. Until now the heat generation effect was uniform. Here the effect of concentrating the heat generation in a small area is documented. The area of the heated spot is 1/16 of the square area of length

1 See www.comsol.com for information about comsol multiphysics.

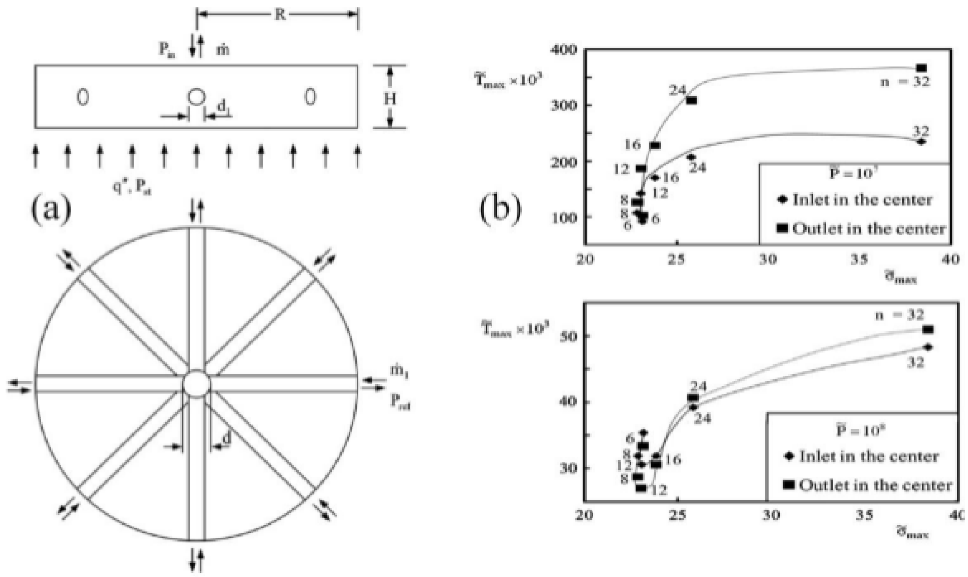

Figure 5: (a) Radial cooling channel configuration embedded in the circular plate. (b) The effect of the number of cooling ducts on the maximum temperature and stress.

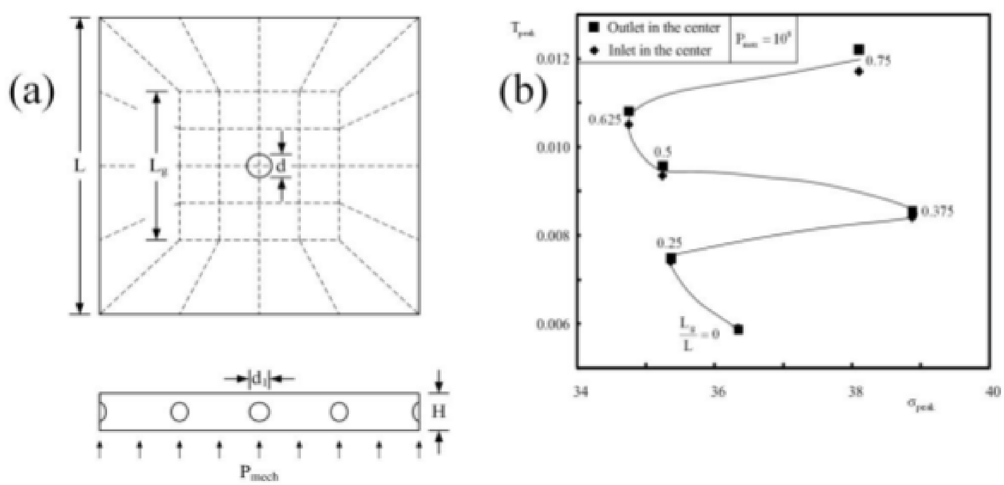

Figure 6: (a) Grid structure connected to the perimeter with radial channels, hybrid structure of a square slab. (b) Minimum peak temperatures relative to their peak stresses as L_g/L varies.

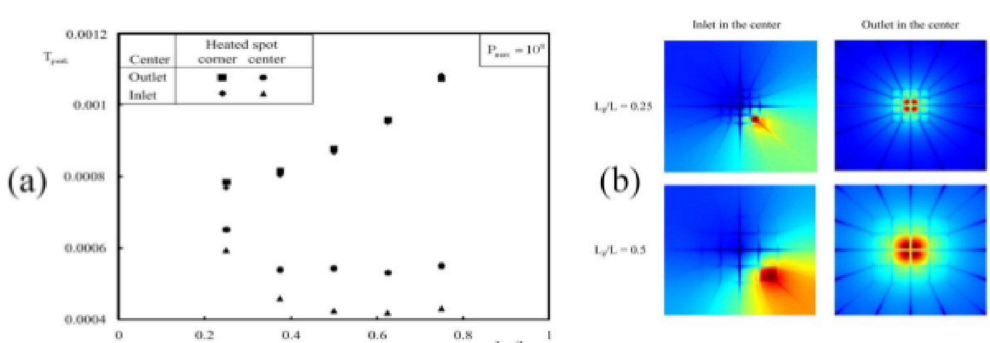

Figure 7: (a) Peak temperature relative to L_g/L when the flow direction and the concentrated heat generation location change. (b) The temperature distribution in the mid-plane of the slab.

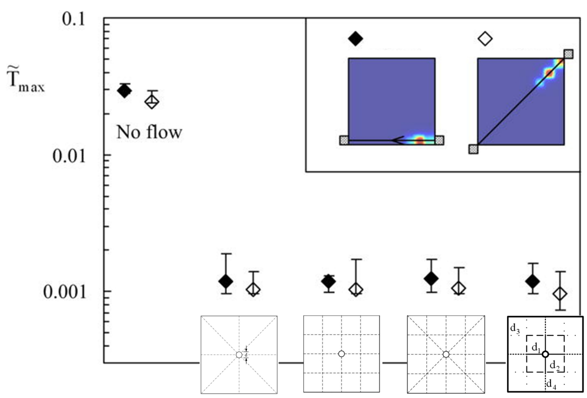

Figure 8: The average peak temperature in foure competing designs.

L_g, Figure 7 [9]. The heating rate of the concentrated heat generation is fixed.

Figure 7 shows the temperature distribution when the heat generation is concentrated in the center of the slab and in the corner of the grid. Two designs are illustrated, $L_g/L = 0.25$ and 0.5. The flow direction changes from inlet in the center to outlet in the center. Figure 7 also shows the evolution of the peak temperature as L_g/L increases. When the concentrated heating is located in the center of the slab, T_{peak} decreases as L_g/L. When the concentrated heating is located in the corner of the grid, T_{peak} increases as L_g/L increases. The T_{peak} value is the lowest with $L_g/L = 0.25$ when the concentrated heating is in the corner, and with $L_g/L = 0.625$ when the concentrated heating is in the center. In addition, when $L_g/L = 0.375$ the peak temperature becomes almost as low as the lowest peak temperature obtained when the concentrated heating is located in the center or in the corner.

4.4 Moving hot spot

Consider a square plate of width and length L, and with thickness of $H = 0.1L$, Figure 8 [10]. A structure of cooling channels is embedded in this plate to keep it under its maximum allowable temperature while the plate is heated with a concentrated and moving heat flux spot. The length scale of the square footprint of the heating spot is $0.1L$ and it moves with the constant speed of W from one edge of the plate to its other edge. Reference 8 documents the change in the temperature for four possible beam paths. The volume of the solid structure and the volume of the coolant fluid are fixed. Coolant enters or exits from the center of the slab while the pressure difference between the entrance and exit is fixed. The flow is incompressible with constant properties and it is time dependent. The governing equations were solved as in the previous subsections.

Figure 8 shows the average peak temperature in four competing designs. The error bars indicate the maximum and minimum peak temperatures when the dimensionless time is greater than 0.1, i.e. after the entire beam enters the plate surface. The peak temperatures for four different possible beam paths are documented. Figure 8 also shows that a plate heated by a moving beam with an unpredictable path can be cooled to under an allowable temperature level by embedding vascular cooling channels in the plate. The effect of changing from no cooling to vascular cooling is dramatic.

5 Conclusions

The main conclusion of this paper is that the flow resistances (heat, fluid and stress) can be decreased with vascularization by using the constructal law. However, there is no design that is optimal. The design should be changed to the new phase of the constructal design for the greatest performance as the conditions (i.e., boundary conditions, objectives and assumptions) change. This paper shows that the design is live and should vary freely for the greatest performance.

First, the assumptions of using temperature independent Hess-Murray rule and using constant diameter ducts where the duct discharges fluid along its edge are valid in which limits are documented. Then, point to area types of flows are explained by illustrating the results of two examples: fluid networks and heating of an area. These examples showed that the design should be varied freely for the smallest flow resistances as the conditions change.

This paper shows that the cooling performance and mechanical strength of a system which is heated and loaded with a distributed force can be increased by embedding vascular structures into it. The embedded vascular structure can be designed such that its cooling performance and mechanical strength is the greatest for the given conditions. The cooling of the system is also documented when the system is heated locally.

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