# DYNAMIC ANALYSIS OF NON-CIRCULAR CURVED BEAM SUBJECTED TO MOVING LOADS

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# ABSTRACT


## DYNAMIC ANALYSIS OF NON-CIRCULAR CURVED BEAM SUBJECTED TO MOVING LOADS

In this thesis, analysis of the dynamic responses of non-circular curved beams subjected to moving loads is studied by using ANSYS which is Finite Element software. An APDL (ANSYS Parametric Design Language) code is developed for a parabolic curved beams having fixed-fixed boundary conditions. The moving load is acted on the curved beam as a single load with constant speed during the movement of the load.

First of all, the proper number of finite element used in the developed APDL code for curved beam is determined by convergence test. In order to verify the mass and stiffness matrices of the curved beam, natural frequencies are found and compared with the results available in the literature. Then, moving load algorithm used in the developed APDL code is validated by using a straight beam model which has exact solution. After validations, static deflections of curved beam under slowly moving load and dynamic deflections under moving load are presented. Finally, discussion of numerical results are given.

# ÖZET

## HAREKETLİ YÜKLERE MARUZ DAİRESEL OLMAYAN EĞRİ ÇUBUĞUN DİNAMİK ANALİZİ

Bu tezde, Sonlu Elemanlar programı olan ANSYS kullanılarak hareketli yüke maruz dairesel olmayan eğri çubukların dinamik cevapları incelenmiştir. Sabit sabit sınır koşullarına sahip parabolik eğri kirişler için bir APDL (ANSYS Parametrik Tasarım Dili) kodu geliştirilmiştir. Yük eğri kiriş üzerindeki hareketi sırasında, sabit bir hızda ve tek bir yük olarak etkitilmiştir.

Her şeyden önce, eğri kiriş için gelişmiş APDL kodunda kullanılan sonlu elemanın uygunluk sayısı, yakınsama testi ile belirlenmiştir. Eğri kirişin kütle ve direngenlik matrislerini doğrulamak için, doğal frekanslar bulunmuş ve literatürde bulunan sonuçlar ile karşılaştırılmıştır. Daha sonra, geliştirilen APDL kodunda kullanılan hareketli yük algoritması, tam çözüme sahip düz bir kiriş modeli kullanılarak doğrulanmıştır. Doğrulama işlemlerinden sonra, yavaş hareket eden yük altında eğri kirişin statik çökmeleri ve hareketli yük altında dinamik çökmeleri sunulmuştur.. Son olarak, sayısal sonuçların tartışılması verilmiştir.

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| | |
|---|---|
| $b$ | width of the beam |
| $c_F$ | the distance from the left end of the span |
| $[C]$ | damping matrix |
| $e$ | finite element number |
| $E$ | modulus of elasticity |
| $f_1$ | first natural frequency |
| $f_n(t)$ | time function |
| $F$ | force |
| $G$ | shear modulus |
| $h$ | depth of the beam |
| $i$ | mass polar moment inertia per unit length |
| $I_{xx}$ | second moment of area about $xx$ axis |
| $J$ | polar moment of area |
| $[K]$ | stiffness matrix |
| $L$ | length of the beam |
| $m$ | mass per unit length |
| $[M]$ | mass matrix |
| $M_x$ | internal bending moment about $x$-axis |
| $M_z$ | internal twisting moment about $z$-axis |
| $N$ | internal normal force, number of element |
| $\{q_o\}$ | initial displacement vector |
| $\{q(t)\}$ | displacement vector |
| $\{q_1\}$ | displacement vector at time $t_1$ |
| $\{q_{j+1}\}$ | displacement vector at time $t_{j+1}$ |
| $R(s)$ | radius function |
| $s$ | circumferential coordinate |
| $s_L$ | length of the beam |
| $T$ | time |
| $v(s, t)$ | displacement in $y$ direction |
| $v$ | velocity of moving load |
| $v_{cr}$ | critical velocity of moving load |

| | |
|---|---|
| $V_y$ | shear force about *y* axis |
| $x_i$ | mode shapes associated with $i^{th}$ natural frequency |
| $Y_n$ | displacement in $n^{th}$ mode |
| $Y_{nst}$ | static displacement in $n^{th}$ mode |

| | |
|---|---|
| $\beta(s, t)$ | angular displacement about *z* axis |
| $\Delta$ | displacement ratio |
| $\Phi_n$ | $n^{th}$ modal shape function |
| $\theta$ | slope |
| $\kappa(s)$ | curvature function |
| $\rho$ | density |
| $\rho_o$ | variable radius of curvature |
| $\tau$ | twisting |
| $\omega_i, \omega_n$ | $i^{th}$ and $n^{th}$ natural frequency |
| $\Omega_n$ | $n^{th}$ modal frequency |

# CHAPTER 1

# GENERAL INTRODUCTION

Some structures in rest or in a motion are under moving loads due to their functions. In practice, transport engineering is in the first place for this type of applications. Dynamic stress in structure and vibrations due to moving loads are main interest in this topic.

Curved beams in different shape and size can be seen in several structures. The common curved beams are planar form due to the simple production process. In a few practical applications, curved beams have spatial forms. Therefore, in this study, planar curved beam are considered.

Planar curved beams (for simplicity, planar curved beam is hereafter called by curved beam) may have in-plane or out-of plane deformations due to the load action on them. If it is under in-plane load, it has in-plane bending and axial deformations. However, if it under out-of-plane load, namely, load is perpendicular the plane of curved beam; it has out-of-plane bending and torsion. In both types of loading, two deformations are not independent. Out-of plane loading is selected for this study.

A fixed-fixed non-circular curved beam under moving load is shown Figure 1.1.

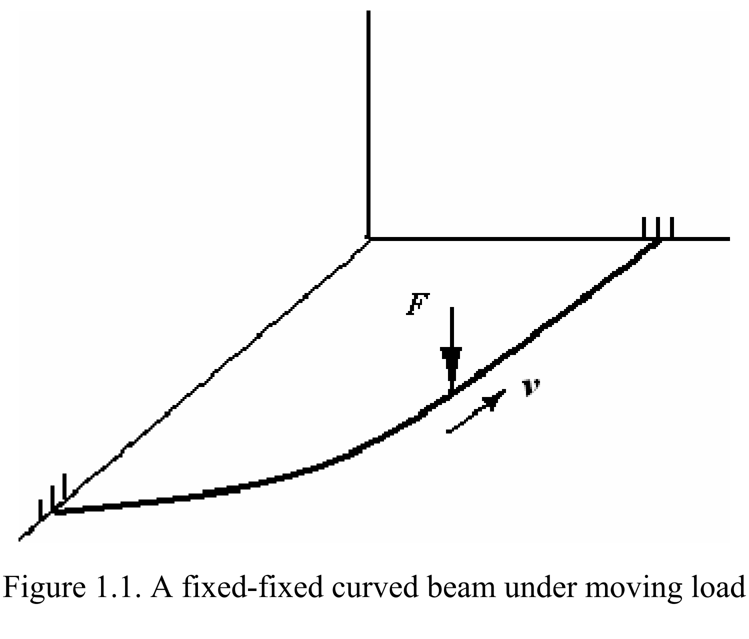


Figure 1.1. A fixed-fixed curved beam under moving load

A numerous studies can be found for straight beams under moving loads. For this type of beam, different elastic foundations and different boundary conditions are also considered in the studies available in the literature. However, the studies on curved beam under moving load are comparatively less than studies on the straight beams under moving loads. In the following paragraphs, for the sake of completeness, the selected papers on straight beams under moving loads are presented first, and then curved beams subjected to moving loads are summarized.

Kim (2004) investigated the vibration and stability of an infinite Bernoulli–Euler beam resting on a Winkler-type elastic foundation and subjected to a static axial force and a moving load.

Kargarnovin and Younesian (2004) studied on the response of a Timoshenko beam with infinite length and subjected to an arbitrary-distributed harmonic moving load. They used Pasternak-type viscoelastic foundation in their study.

Kim (2005) discussed the stability and dynamic response of an infinite Rayleigh beam–column resting on an elastic foundation and subjected to moving loads. He modeled the elastic foundation by a Winkler-type and a two-parameter-type.

Kim and Cho (2006) extended the study of Kim (2005) by considering the effect of shear in beams.

Lou et al (2007) presented the dynamic analysis of Timoshenko beam with various boundary conditions and subjected to moving concentrated forces.

Uzzal et al (2012) investigated the dynamic responses of an Euler-Bernoulli beam under constant moving load as well as moving mass. They considered the elastic supports based on the two-parameter Pasternak foundation.

Zrnić et al (2013) discussed the most significant published papers on cranes structures development theoretically and practically.

Chen (2014) presented a model reduction method for the dynamic response of a beam structure under a moving load or a moving body.

Yildirim and Esim (2015) analyzed over head crane systems with one and double bridges and having one or more cars by using Finite Element Method.

Now, the studies on curved beam structure under moving load are given below:

Huang et al (2000) proposed an accurate solution by using the dynamic stiffness matrix and numerical Laplace transform technique for the responses of circular curved beams subjected to a moving load.

Yang and Wu (2001) derived the analytical solutions by using Galerkin's method for a horizontal curved beam subjected to vertical and horizontal moving loads.

Wu and Chiang (2003) tried to determine the out-of-plane responses of a circular curved Timoshenko beam under a moving load by using the curved beam finite elements. They derived element stiffness and mass matrices from the energy expressions.

Wu and Chiang (2004) derived a new finite element to investigate the in-plane vibration responses of a circular arch under in-plane moving load. Their employed the simple implicit-form shape functions having the radial, tangential and rotational displacements.

Li et al (2013) proposed a closed-form out-of-plane dynamic displacement response of a curved track subjected to moving loads by using transfer matrix. They modeled the curved track by a planar curved Timoshenko beam periodically supported by the double-layer spring-damping elements.

Nikkhoo and Kananipour (2014) used the differential quadrature method (DQM) for deflections of curved beam structures under in-plane constant moving load. They considered the Euler-Bernoulli beam theory.

It is useful to address the studies on vibrations non-circular curved beams. The selected important ones are mentioned as follows.

Volterra and Morell (1961) used Rayleigh-Ritz method to find the vibrations of curved beams with non-circular axis such as a cycloid, a catenary and a parabola.

Wang (1975) investigated out-of-plane vibration of a clamped elliptic by using the Rayleigh-Ritz method, too.

Takahashi and Suzuki (1977) found the out-of-plane vibrations of elliptic arcs by using power series.

Suzuki et al. (1978) studied on vibration of more types of non-circular arcs including ellipse, sine catenary, hyperbola, parabola and cycloid. Similar to former studies, they used the Rayleigh-Ritz method.

Irie et al. (1980) obtained the out-of-plane motions of several non-circular Timoshenko beams by using the transfer matrix approach.

Suzuki et al. (1983) studied on free vibrations of non-circular curved bars having varying cross-section by series solution.

In this study, dynamic analysis of non-circular curved beam subjected to moving load is presented by using finite element method. As a practical application of this topic,

Figure 1.2 is given. For non-circular curved beam, the parabola is chosen as the shape of curved beam axis.

The computer code is developed in APDL (ANSYS Parametric Design Language) in ANSYS. The proper number of finite element used in model is determined by convergence test. The mass and stiffness matrices of the model are verified with the results available in the literature. Moving load algorithm used in the code is validated by using a straight beam model which has exact sulution. After validations, static and dynamic deflections under moving load are presented. The numerical results are discussed.

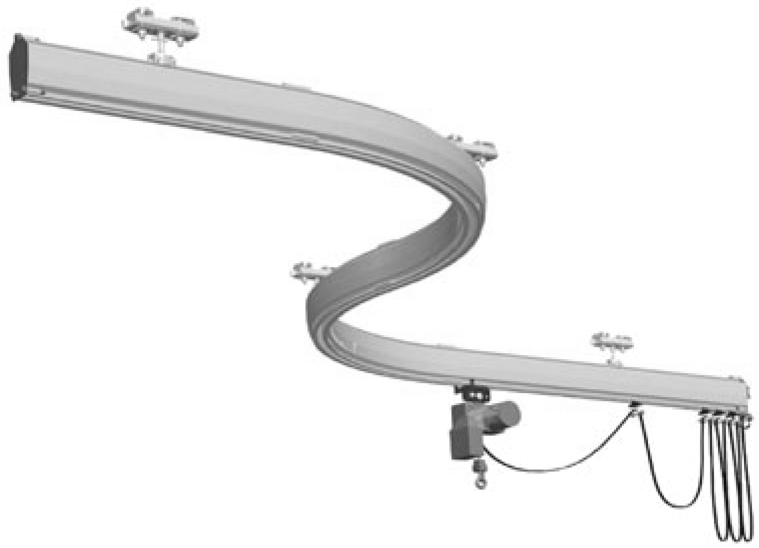

Figure 1.2. Monorail crane with curved beam (Source: www.insem.si, 2017)

# CHAPTER 2

# THEORETICAL STUDIES

## 2.1. Introduction

The aim of this chapter to present the theoretical backgrounds regarding the topics related with the thesis subject. First of all, the parabola is introduced for its parameters as selected non-circular form. The equations of motion of the non-circular curved beam under moving load are given to see the parametric interactions, although it is not used in this thesis. In second step, modal analysis and time response of multi-degree-of-freedom systems are summarized. Then, as third step which is main topic, response of a beam subjected to moving load is discussed. Finally, finite element model and analyses in ANSYS are detailed.

## 2.2. Geometry of Curved Beam Axis

The parabola given in Figure 2.1 is considered. In order to refer the equation of motion from literature, the same co-ordinate system with the literature is used. The arc length of a parabola part $s$ and curvature $\kappa$ of a point on the curve are expressed as follows:

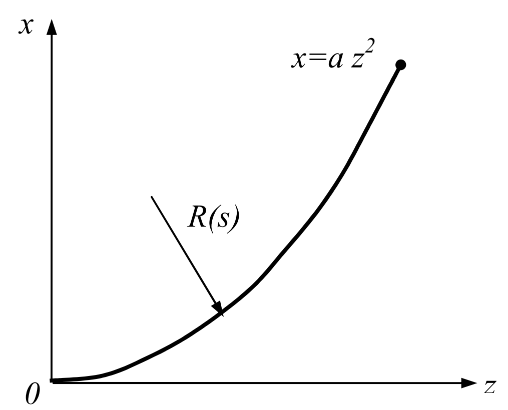


Figure 2.1. The parabola and its parameters.

$$s = \int_{s1}^{s2} ds = \int_{z1}^{z2} \sqrt{1 + \left(\frac{dx}{dz}\right)^2} dz \tag{2.1}$$

where $s_1$, $s_2$, $z_1$, and $z_2$ are the boundaries of the integrals,

$$\kappa = \frac{\frac{d^2x}{dz^2}}{\left[1 + (dx/dz)^2\right]^{3/2}} \tag{2.2}$$

## 2.3. Out-of-Plane Vibrations of Curved Beam

Equations of motion for out of plane displacement of a non-circular curved beam under the transverse moving load shown in Figure 2.2 can be obtained from Love (1944) as

$$\frac{\partial V_y(s,t)}{\partial s} + F_y(s,t) = m\frac{\partial^2 v(s,t)}{\partial t^2} \tag{2.3}$$

$$\frac{\partial M_x(s,t)}{\partial s} + \frac{M_z(s,t)}{\rho_0(s)} - V_y(s,t) = 0 \tag{2.4}$$

$$\frac{\partial M_z(s,t)}{\partial s} - \frac{M_x(s,t)}{\rho_0(s)} = i\frac{\partial^2 \beta(s,t)}{\partial t^2} \tag{2.5}$$

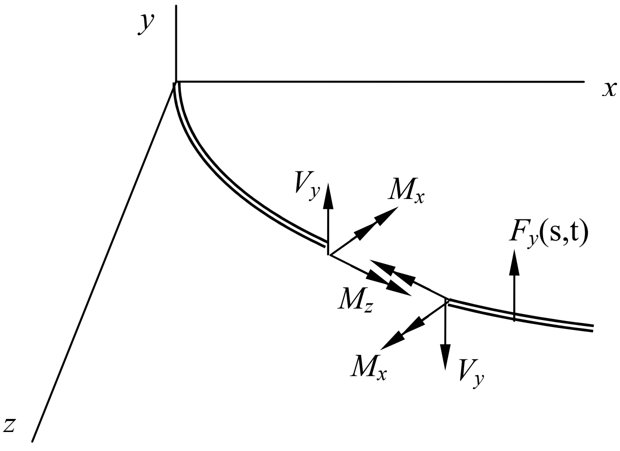


Figure 2.2. Internal reactions and external force of a curved beam

where $m = \rho A(s)$ is mass per unit length and $i = \rho J(s)$ is mass polar moment inertia of unit length. Other notations can be seen from Figure 2.2. Equation (2.4) is substituted into Equation (2.3) to get two copuled equation of motions instead of three. Thus,

$$\frac{\partial^2 M_x(s,t)}{\partial s^2} + \frac{\partial}{\partial s}\left(\frac{M_z(s,t)}{\rho_0(s)}\right) + F_y(s,t) = m\frac{\partial^2 v(s,t)}{\partial t^2} \tag{2.6}$$

$$\frac{\partial M_z(s,t)}{\partial s} - \frac{M_x(s,t)}{\rho_0(s)} = i\frac{\partial^2 \beta(s,t)}{\partial t^2} \tag{2.5}$$

Equation (2.5) is copied for the sake of completeness. Bending moment $M_x(s, t)$ and twisting moment $M_z(s, t)$ are written as

$$M_x(s,t) = EI_{xx}\,\kappa(s,t) \tag{2.7}$$

$$M_z(s,t) = GJ\,\tau(s,t) \tag{2.8}$$

where $I_{xx}$ and $J$ area moment of inertia about $xx$-axis and torsional constant of the cross-section, respectively. Also, $\kappa(s)$ and $\tau(s)$ are time and position dependent curvature and twisting functions. They are given by Love (1944) as

$$\kappa(s,t) = \left(\frac{\beta(s,t)}{\rho_0(s)} - \frac{\partial^2 v(s,t)}{\partial s^2}\right) \tag{2.9}$$

$$\tau(s,t) = \left(\frac{d\beta(s,t)}{ds} + \frac{1}{\rho_0(s)}\frac{\partial v(s,t)}{\partial s}\right) \tag{2.10}$$

where $v(s, t)$ and $\beta(s, t)$ are transverse and angular displacements. Also, $\rho_0(s)$ is radius of curvature of the non-circular curved beam at position $s$.

Substituting Equations (2.7) and (2.8) along with Equations (2.9) and (2.10) into Equations (2.5) and (2.6), the following equations of motions in terms of displacements are obtained:

$$\frac{\partial^2}{\partial s^2}\left(EI_{xx}\left(\frac{\beta(s,t)}{\rho_0(s)}-\frac{\partial^2 v(s,t)}{\partial s^2}\right)\right)$$
$$+\frac{\partial}{\partial s}\left(\frac{GJ}{\rho_0(s)}\left(\frac{\partial\beta(s,t)}{\partial s}+\frac{1}{\rho_0(s)}\frac{\partial v(s,t)}{\partial s}\right)\right) \tag{2.11}$$
$$+F_y(s)=m\frac{\partial^2 v(s,t)}{\partial t^2}$$

$$\frac{\partial}{\partial s}\left(GJ\left(\frac{\partial\beta(s,t)}{\partial s}+\frac{1}{\rho_0(s)}\frac{\partial v(s,t)}{\partial s}\right)\right)$$
$$-\frac{EI_{xx}}{\rho_0(s)}\left(\frac{\beta(s,t)}{\rho_0(s)}-\frac{\partial^2 v(s,t)}{\partial s^2}\right)=i\frac{\partial^2\beta(s,t)}{\partial t^2} \tag{2.12}$$

Since the cross-sectional properties are constant along the curved beam axis, Equations (2.11) and (2.12) can be expanded to the following forms:

$$EI_{xx}\frac{\partial^2}{\partial s^2}\left(\frac{\beta(s,t)}{\rho_0(s)}\right)-EI_{xx}\frac{\partial^4 v(s,t)}{\partial s^4}$$
$$+GJ\frac{\partial}{\partial s}\left(\frac{1}{\rho_0(s)}\frac{\partial\beta(s,t)}{\partial s}\right)+GJ\frac{\partial}{\partial s}\left(\frac{1}{\rho_0(s)^2}\frac{\partial v(s,t)}{\partial s}\right) \tag{2.13}$$
$$+F_y(s,t)=m\frac{\partial^2 v(s,t)}{\partial t^2}$$

$$GJ\frac{\partial^2\beta(s,t)}{\partial s^2}+GJ\frac{\partial}{\partial s}\left(\frac{1}{\rho_0(s)}\frac{\partial v(s,t)}{\partial s}\right)$$
$$-\frac{EI_{xx}\,\beta(s,t)}{\rho_0(s)^2}-\frac{EI_{xx}}{\rho_0(s)}\left(\frac{\partial^2 v(s,t)}{\partial s^2}\right)=i\frac{\partial^2\beta(s,t)}{\partial t^2} \tag{2.14}$$

In order to solve the equations of motions having two unknown functions $v(s, t)$ and $\beta(s, t)$ found in Equations (2.13) and (2.14) considering the boundary conditions, the proper restrictions given below are used:

- Free end: shear force $V_y$=0, bending moment $M_x$=0, and twisting moment $M_z$=0.
- Pinned end: displacement $v$=0, bending moment $M_x$=0, and twisting moment $M_z$=0.
- Fixed end: displacement $v$=0, slope $v'$=0, and rotation $\beta$=0.

## 2.4. Vibration of Multi-Degree-of-Freedom Systems

### 2.4.1. Modal Analysis

A multi-degree-of-freedom system has multi independent coordinates to specify the positions of the masses of the system (Seto, 1983). Equation of motion of multi-degree-of-freedom system can be written in matrix form as

$$[M]\{\ddot{q}(t)\}+[C]\{\dot{q}(t)\}+[K]\{q(t)\}=\{F(t)\} \tag{2.15}$$

where $[M]$, $[K]$, and $[C]$ are mass, stiffness, and damping matrices, respectively. Also, $\{q(t)\}$ and $\{F(t)\}$ are displacement and force vectors, respectively. Natural frequencies of the system having a few degrees-of-freedom can be found by using the characteristic equation described as

$$\det([K]-\omega^2[M])=0 \tag{2.16}$$

However, when the matrix size is large, Equation (2.16) is not practical. Thus, the following generalized discrete eigenvalue problem is solved

$$([K]-\omega_i{}^2[M])\{x_i\}=\{0\} \tag{2.17}$$

where $\omega_i$ is $i^{th}$ natural frequency and $\{x_i\}$ is the mode shapes associated with $i^{th}$ natural frequency.

### 2.4.2. Time Response

Finding $\{q(t)\}$ in Equation (2.15) is known as transient analysis and $\{q(t)\}$ is called and time response. Direct integration methods or step-by-step methods (Cook 1989) can be used to find time response.

To find the time response $\{q(t)\}$ accurately, the time interval $(0, T)$ can be divided into $N$ equal time intervals $\Delta t = T / N$, where $T$ represents the final time.

Determination of the value of $\Delta t$ is very important for numerical stability and accuracy. For good accuracy, it is selected as twenty times of natural period (Petyt 2010). Then, the time response is found at times $\Delta t$, $2\Delta t$, $3\Delta t$, . . . , $T$. The following solution procedure using central difference method is given by Petyt (2010)

1. Solve the following equation for the acceleration $\{\ddot{q}_0\}$,

$$[M]\{\ddot{q}_0\} + [C]\{\dot{q}_0\} + [K]\{q_0\} = \{F_0\} \tag{2.18}$$

2. Calculate $\{q_1\}$ by using the next equation,

$$\{q_1\} = \{q_0\} + \Delta t\,\{\dot{q}_0\} + ((\Delta t)^2 / 2)\{\ddot{q}_0\} \tag{2.19}$$

3. Calculate the $\{q_{j+1}\}$ starting with $j$=1, from the following equation

$$[A]\{q_{j+1}\} = \{F_j\} + [B]\{q_j\} - [D]\{q_{j-1}\} \tag{2.20}$$

where

$$[A] = 1/(\Delta t)^2[M] + 1/(2\Delta t)[C] \tag{2.21}$$

$$[B] = 2/(\Delta t)^2[M] - [K] \tag{2.22}$$

$$[D] = 1/(\Delta t)^2[M] - 1/(2\Delta t)[C] \tag{2.23}$$

until target time $T$. Equation (2.20) is arranged for $\{q_{j+1}\}$ in order to see clearly the calculations required in this steps as follows:

$$\{q_{j+1}\} = [A]^{-1}\{F_j\} + [A]^{-1}[B]\{q_j\} - [A]^{-1}[D]\{q_{j-1}\} \tag{2.24}$$

## 2.5. Response of a Beam Subjected to Moving Load

A simply supported straight beam of *m* mass/length under moving load shown in Figure 2.3 is considered (Source: Biggs, 1964).

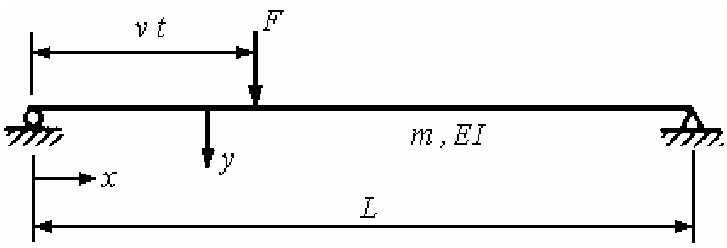


Figure 2.3. A simply supported straight beam under moving load
(Source: Biggs, 1964)

Modal equation of motion of the beam shown Figure 2.3 is written as

$$\ddot{Y}_n + \omega_n^2 Y_n = \frac{F\phi_n(c_F)}{\int_o^L m[\phi_n(x)]^2\, dx} \qquad (2.25)$$

where $\omega_n$ is the $n^{th}$ natural frequency and $\phi_n(x)$ is the $n^{th}$ modal-shape function. Since the beam is simply supported, $\omega_n$ is given as

$$\omega_n^2 = \frac{(n\pi)^4 EI}{mL^4} \qquad (2.26)$$

where *EI* is bending rigidity of the beam, and *L* is the length of the beam. Also, $n^{th}$ modal-shape function is written as

$$\phi_n(x) = \sin\frac{n\,\pi\,x}{L} \qquad (2.27)$$

$c_F$ in Equation (2.25) is the distance from the left end of the span to the force and written as

$$c_F = v\,t \tag{2.28}$$

where $t$ is measured from the instant at which the force entered the span. Substituting Equations (2.27) and (2.28) into Equation (2.25), modal equation of motion becomes

$$\ddot{Y}_n + \omega_n^2 Y_n = \frac{2F}{mL}\sin\frac{n\pi v t}{L} \tag{2.29}$$

It can be seen from the right side of Equation (2.29), the time function can be written as

$$f_n(t) = \sin\frac{n\pi vt}{L} = \sin\Omega_n t \tag{2.30}$$

It is apparent that the modal solution is the same as that for a one-degree system subjected to a sinusoidal force. Thus, the Dynamic Load Factor (DLF) is expressed as

$$(DLF)_n = \frac{1}{1-(\Omega_n/\omega_n)^2}(\sin\Omega_n t - \frac{\Omega_n}{\omega_n}\sin\omega_n t) \tag{2.31}$$

Therefore, the modal solution is found as

$$Y_n = \frac{2F}{mL\omega_n^2}(DLF)_n \tag{2.32}$$

The coefficient of $(DLF)_n$ in Equation (2.32) is corresponds to static displacement in n$^{th}$ mode, $Y_{nst} = 2F/(mL\omega_n^2)$. Combination of the $N$ modes is expresses as follows:

$$y = \sum_{n=1}^{N} Y_n \phi_n(x) \tag{2.33}$$

Substituting Equation (2.32) along with (2.31) and (2.27) into Equation (2.33), the final expression is obtained as

$$y = \frac{2F}{mL}\sum_{n=1}^{N}\frac{1}{\omega_n^2-\Omega_n^2}(\sin\Omega_n t - \frac{\Omega_n}{\omega_n}\sin\omega_n t)\sin(\frac{n\pi x}{L}) \tag{2.34}$$

## 2.6. Finite Element Method

If Rayleigh-Ritz Method is generalized, Finite Element Method is obtained (Reddy 1993, Petyt 2010). It has been developed after 1960s (Cook 1989). The method is based on the division of the geometrically complex shape into small geometrical shapes of which stiffness and mass properties can be obtained easily. As an examples for small geometrical shapes; bar, beam, plate, shell, tetrahedral solid, hexahedral solid can be said. These small shapes are called as finite element. The most critical concept in finite element modeling is continuity condition of finite elements. Finite elements are connected its neighbors by its nodes which are the points having freedoms such as translations and rotations in solid mechanics. Division process of the complex geometrical shape into finite elements is called as meshing. All finite elements are combined by using governing equations and this is known as assembling process. Mathematically, assembled finite element system representing the complex geometrical shape is described by systems of algebraic equations which can be written in matrix form. Determination and application of the boundary conditions are the most critical step. On the other hand, usage of symmetry and other reduction techniques can be used to reduce the computation time. In dynamics of elastic structure, the equation of motion given by Equation (2.15) is developed by using proper finite elements (Yardimoglu, 2015), and then solved by using numerical methods depending on the problem types.

Meshing of a curved beam with six finite elements is shown in Figure 2.4.

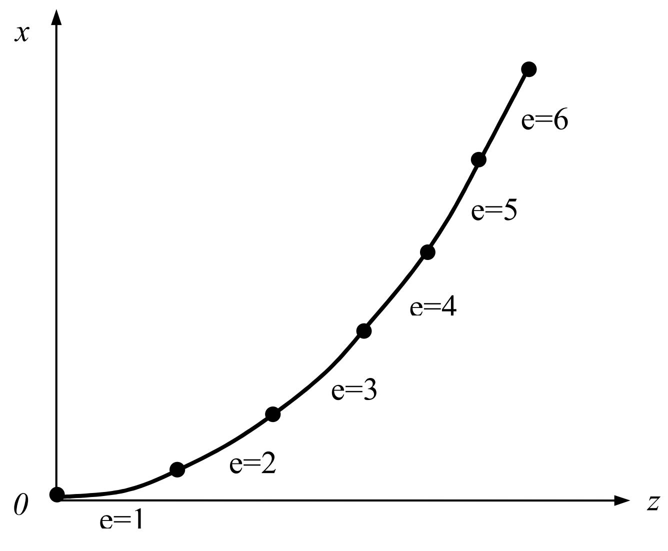


Figure 2.4. Meshing of a curved beam with six finite elements (e≡element).

## 2.7. Modeling and Analysis in ANSYS

### 2.7.1. Introduction

Modelling of non-circular curved beam can be accomplished by using straight beam finite element. In ANSYS, BEAM4 (ANSYS, 2007) is selected. BEAM4 is used for tension, compression, torsion, and bending due to the nodal freedoms which are three translations and three rotations. In addition to elastic stiffness, geometric stiffness is also included. Nodes of BEAM4 are shown in Figure 2.5. The shape functions of BEAM4 are given as follows (Kohnke 2004)

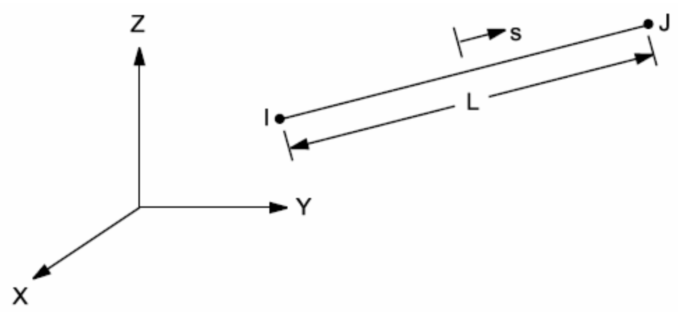


Figure 2.5. Nodes of BEAM4
(Source: Kohnke 2004)

$$u = 0.5[u_I(1-s) + u_J(1+s)] \tag{2.35}$$

$$\begin{aligned} v = {} & 0.5[v_I(1-0.5s(3-s^2)) + v_J(1+0.5s(3-s^2))] \\ & + 0.125L[\theta_{zI}(1-s^2)(1-s) - \theta_{zJ}(1-s^2)(1+s)] \end{aligned} \tag{2.36}$$

$$\begin{aligned} w = {} & 0.5[w_I(1-0.5s(3-s^2)) + w_J(1+0.5s(3-s^2))] \\ & - 0.125L[\theta_{yI}(1-s^2)(1-s) - \theta_{yJ}(1-s^2)(1+s)] \end{aligned} \tag{2.37}$$

$$\theta_x = 0.5[\theta_{xI}(1-s) + \theta_{xJ}(1+s)] \tag{2.38}$$

The geometry of the BEAM4 is taken from original source and shown in Figure 2.6. BEAM4 may have two or three nodes depending on the selection. Third node of this element is needed for orientation of the element. Real constants (ANSYS, 2005) are

AREA, IZZ, IYY, TKZ, TKY, THETA

ISTRN, IXX, SHEARZ, SHEARY, SPIN, ADDMAS

| | |
|---|---|
| where AREA | : Cross-sectional area |
| IZZ and IYY | : Area moment of inertia about z and y axis, respectively |
| TKZ, TKY | : Thickness in z and y directions |
| THETA | : Orientation angle about x axis |
| ISTRN | : Initial strain |
| IXX | : Torsional moment of inertia |
| SHEARZ, SHEARY | : Shear deflection constant |
| SPIN | : Rotational frequency |
| ADDMAS | : Added mass/unit length |

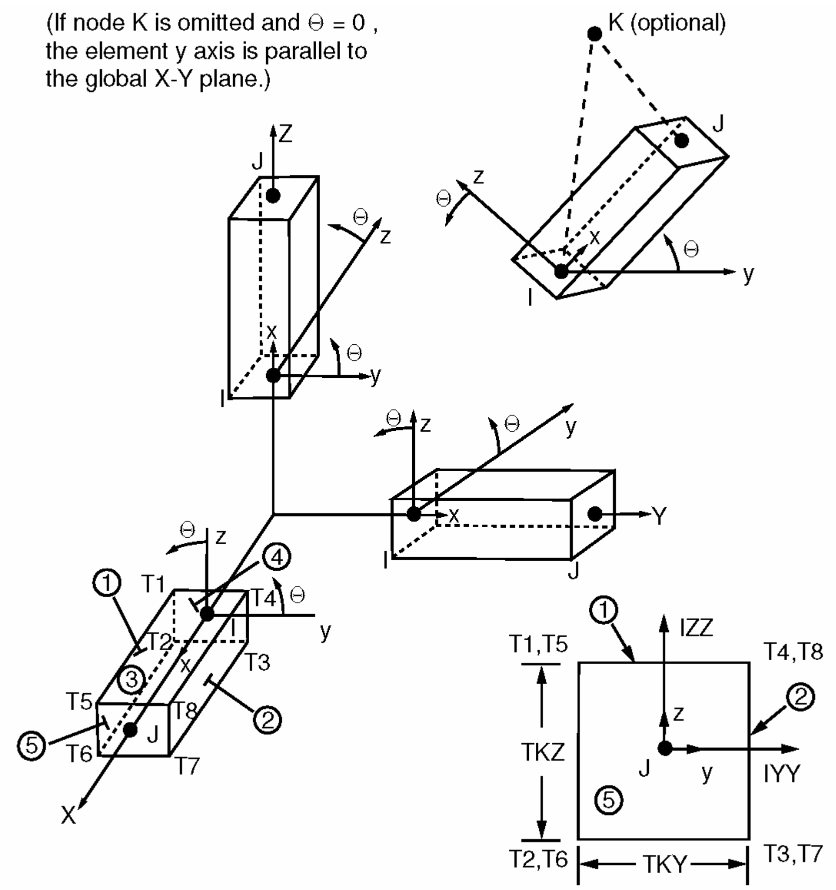


Figure 2.6. Geometry of BEAM4
(Source: ANSYS 2007)

### 2.7.2. Static Analysis

A static analysis is based on the calculations under the effects of statical loads acting on a structure. By using this option, static displacements, stresses, strains, and forces in structures are found. The following types of loading can be applied:

- Externally applied forces and pressures
- Steady-state inertial forces (such as gravity or rotational velocity)
- Imposed (nonzero) displacements
- Temperatures (for thermal strain)

### 2.7.3. Modal Analysis

Modal analysis is used to find the natural frequencies and associated mode shapes of a structure. As stated before, correct the time increment is necessary in transient analysis and it is determined after modal analysis. In the ANSYS modal analysis is a linear. The following mode-extraction methods can be chosen:

- Block Lanczos (default):
- Subspace:
- Power Dynamics:
- Reduced:
- Unsymmetric:
- Damped:
- QR damped: This allows for unsymmetrical damping and stiffness matrices.

### 2.7.4. Transient Analysis

Transient analysis is determination of the dynamic response of a structure under the general time-dependent effects. In dynamics of solid mechanics, the time-varying displacements under several types of time dependent loads are found, and then strains and stresses can be calculated.

Basically, transient dynamic analysis is interested in the solution of Equation (2.15).

The Newmark time integration method or an improved method called HHT is used in ANSYS to solve the mentioned equation. ANSYS can perform transient dynamic analysis by using the following three methods:

1. Full method: The full system matrices are used to calculate the transient response. All types of nonlinearities in the problem can be included.
2. Mode superposition method: Mode shapes obtained from a modal analysis are used to calculate the structure's response.
3. Reduced method: Master degrees of freedoms defined by users and reduced matrices are used. After finding the displacements at the master DOF, ANSYS expands the solution to the original full DOF set.

# CHAPTER 3

# NUMERICAL STUDIES

## 3.1. Convergence Test

The aim of this section is to determine the adequate number of element for dynamic analysis. Therefore, the best testing method is to find the natural frequencies of the finite element model which are based on the mass and stiffness matrices.

To do this, a computer code is developed by using APDL (ANSYS Parametric Design Language) in ANSYS for a fixed-fixed curved beam shown in Figure 3.1. In this program, parabolic curved beam is modelled parametrically and meshed by BEAM4.

The geometrical details of axis of the parabolic curved beam are given in Figure 3.1. For the convergence studies natural frequencies are found by using different number of finite elements.

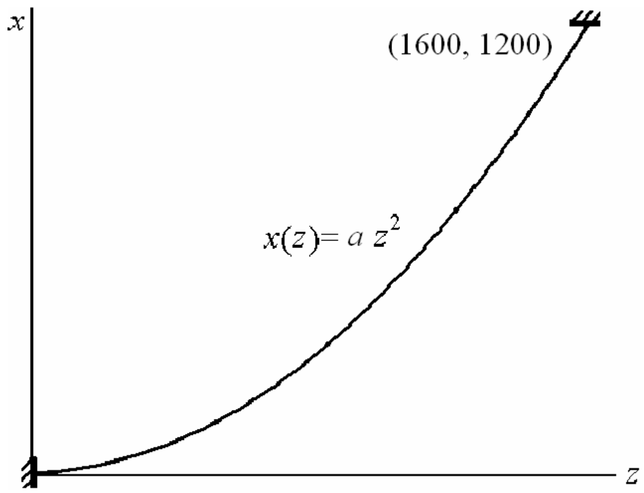


Figure 3.1. Axis of the present non-circular curved beam

Hollow box beam 60x60x6 mm is selected. Material of the beam is given below:

Modulus of elasticity *E*=200000 MPa,

Shear modulus *G*=80000 MPa,

Density $\rho=7.85.10^{-9}$ ton/mm$^3$.

The calculated natural frequencies are given in Table 3.1 and Figure 3.2. It is seen from the presented results that the reasonable number of element $N$=40.

Table 3.1. Convergence of first natural frequency

| $N$ | First natural frequency $f_1$ (Hz) |
|---|---|
| 8 | 89.641 |
| 12 | 89.527 |
| 16 | 89.486 |
| 20 | 89.467 |
| 24 | 89.457 |
| 28 | 89.451 |
| 32 | 89.447 |
| 36 | 89.444 |
| 40 | 89.442 |
| 44 | 89.440 |
| 48 | 89.439 |
| 52 | 89.438 |

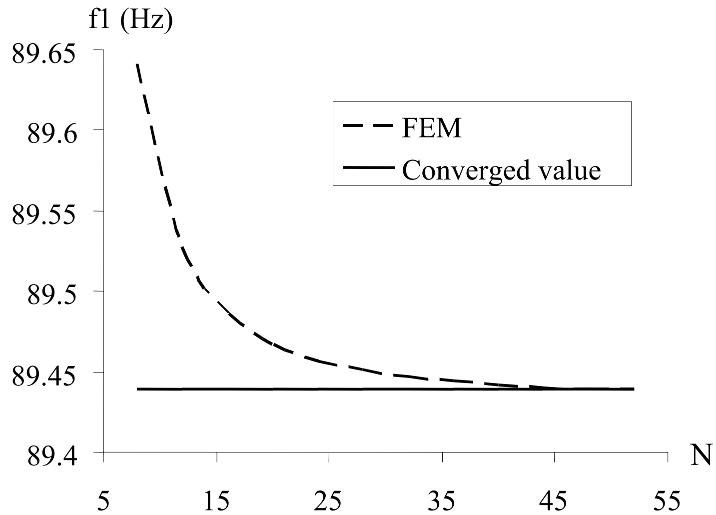

Figure 3.2. Convergence of first natural frequency

## 3.2. Validation of Mass and Stiffness Matrices

Validation of the mass and stiffness matrices in the developed code can be done by finding natural frequencies of a circular curved beam having fixed-fixed boundary conditions, since it has solution in the existing literature (Blevins, 1979 and Culver, 1967). By using the cross-sectional and material data given in former section, a quarter-circular curved beam with radius 1325 mm which corresponds to the same arch length of parabola tested formerly verified for the convergence is modeled in ANSYS by using again 40 BEAM4 elements. The first mode shape is illustrated in Figure 3.3.

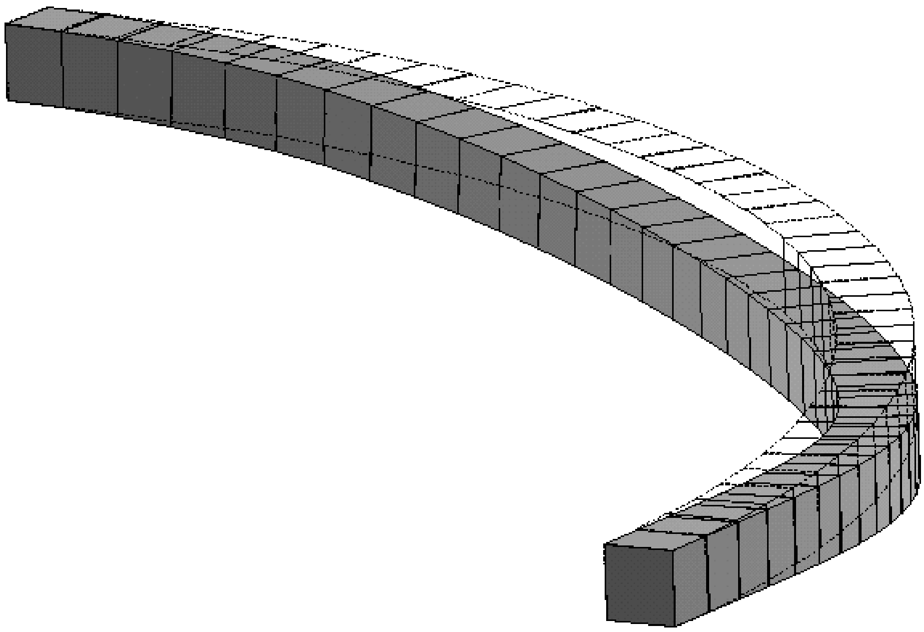

Figure 3.3. First mode shape of a quarter-circular fixed-fixed curved beam

The results of present ANSYS model and Rayleigh-Ritz Method given by Culver (1967) are given in Table 3.2. It is seen from Table 3.2 that the present model in ANSYS gives very close natural frequency value to another value.

Table 3.2. Comparison of first natural frequencies (Hz)

| Present result from ANSYS | Result by Rayleigh-Ritz Method (Culver, 1967) |
|---|---|
| 84.317 | 84.69 |

## 3.3. Validation of Moving Load Algorithm

In order to perform this step, a simply supported straight beam is considered. A steel hollow box beam 60x60x6 mm with length L=2080 mm is taken. It should be pointed out that the length of the beam is the same with the arch length of the parabolic curved beam considered in Section 3.1. Also, the material and cross-sectional properties are the same with the parabolic curved beam considered before in Section 3.1.

Theoretical time response of a simply supported straight beam under a moving load with constant velocity *v* as shown in Figure 2.3 is obtained by using the theory outlined in Section 2.5.

The first and second terms in the parentheses in Equation (2.31) which is the Dynamic Load Factor (DLF) are $\sin\Omega_n t$ and $-(\Omega_n / \omega_n)\sin\omega_n t$, respectively. It is noted that the first and second terms given above are related with the forced and free vibrations, respectively.

For the moving load with velocity *v*=6240 m/s acting on the simply supported straight beam, the time response plot obtained by using the Equation (2.34) is illustrated in Figure 3.4.

Validation of the moving load algorithm in developed APDL code is accomplished by using the considerations given above. The same plot shown in Figure 3.4 is obtained.

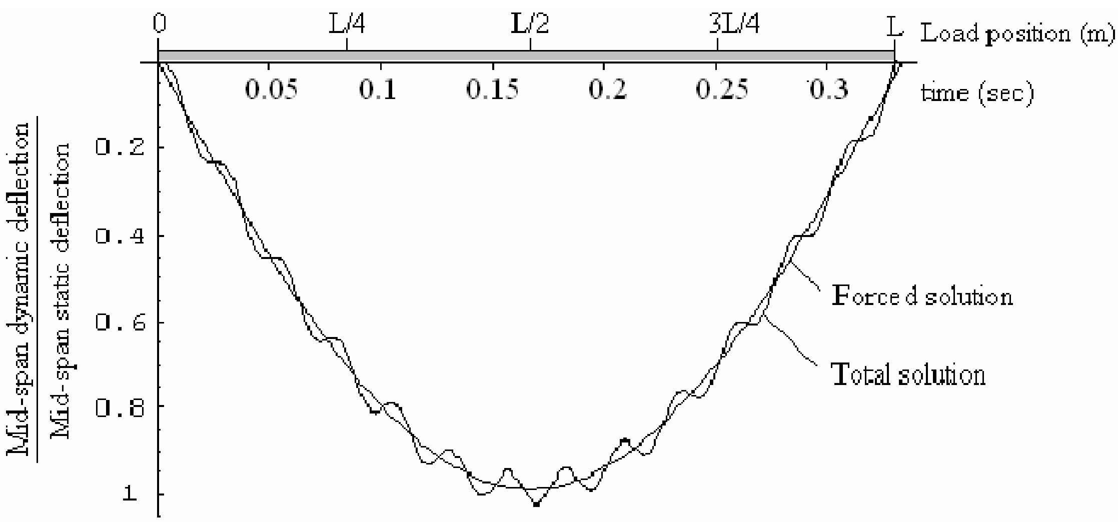


Figure 3.4. Mid-span displacements of simply supported straight beam

## 3.4. Static Deflections under Slowly Moving Load

The parabolic curved beam introduced in Section 3.1 is considered here to find the statical deflections under 10000 N applied different nodes. The node numbers of the finite element model of the parabolic curved beam are shown in Figure 3.5. The results are given in Figures 3.6- 3.14.

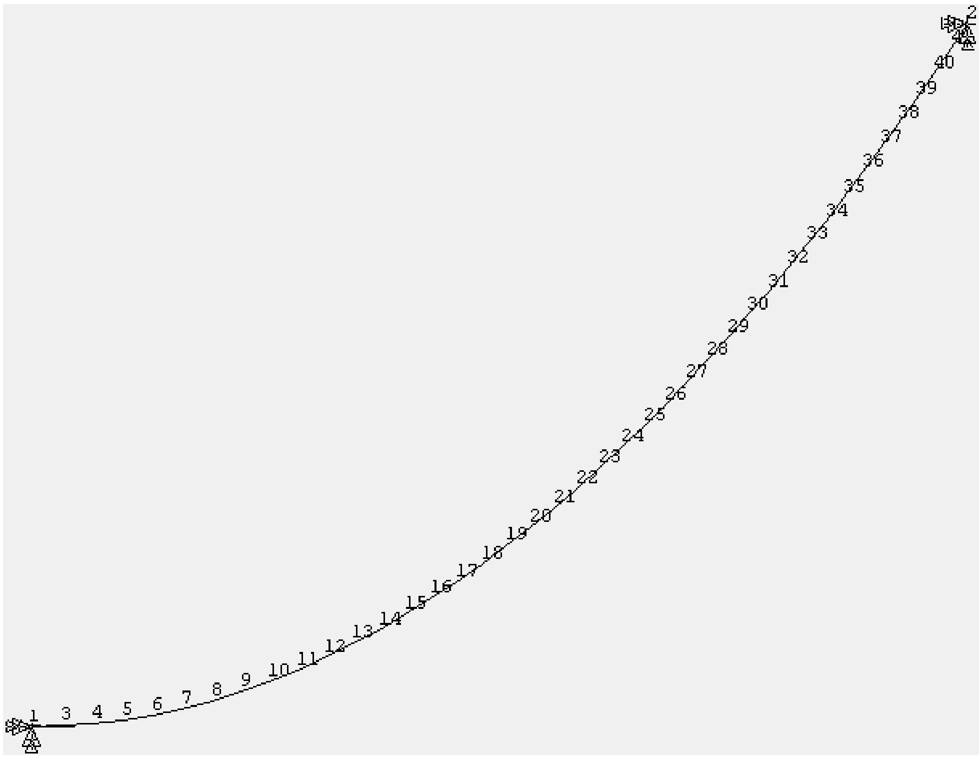


Figure 3.5. Node numbers of parabolic curved beam

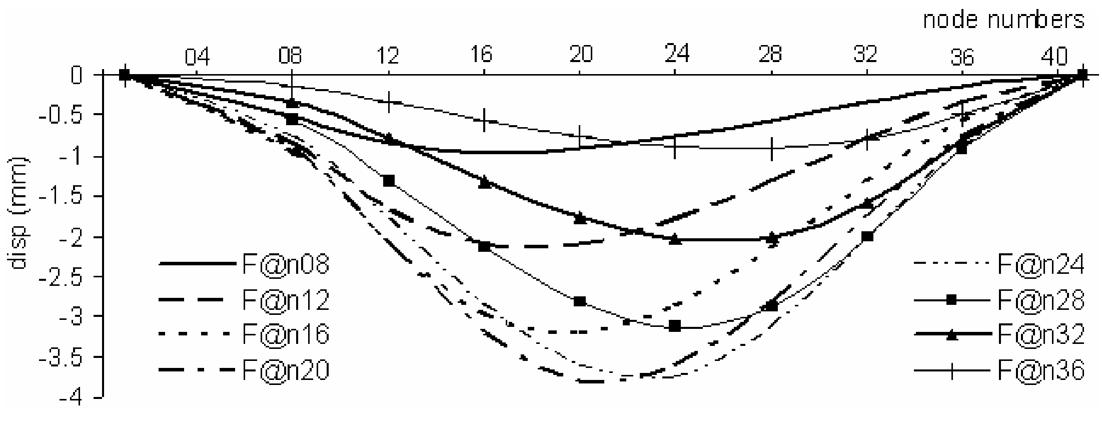


Figure 3.6. Static displacements of parabolic curved beam under several loadings

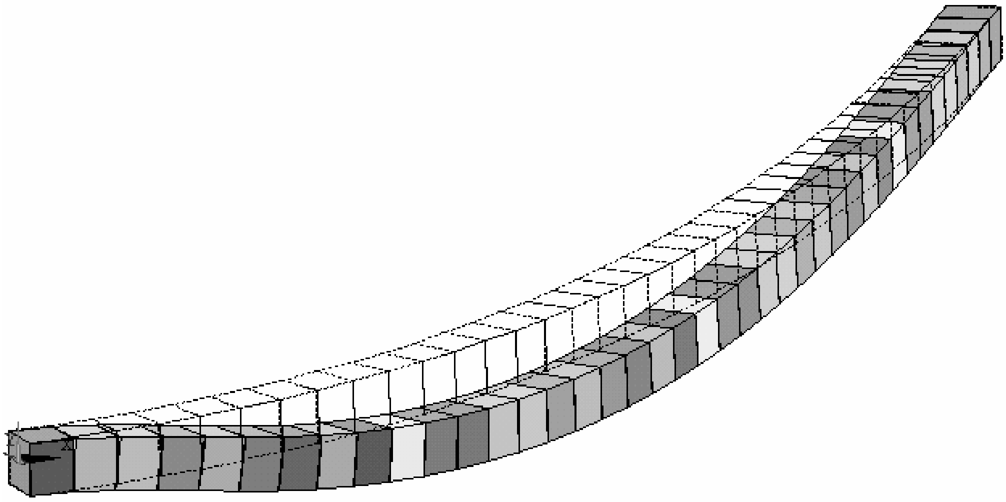

Figure 3.7. Statical displacement of parabolic curved beam due to force at node 8

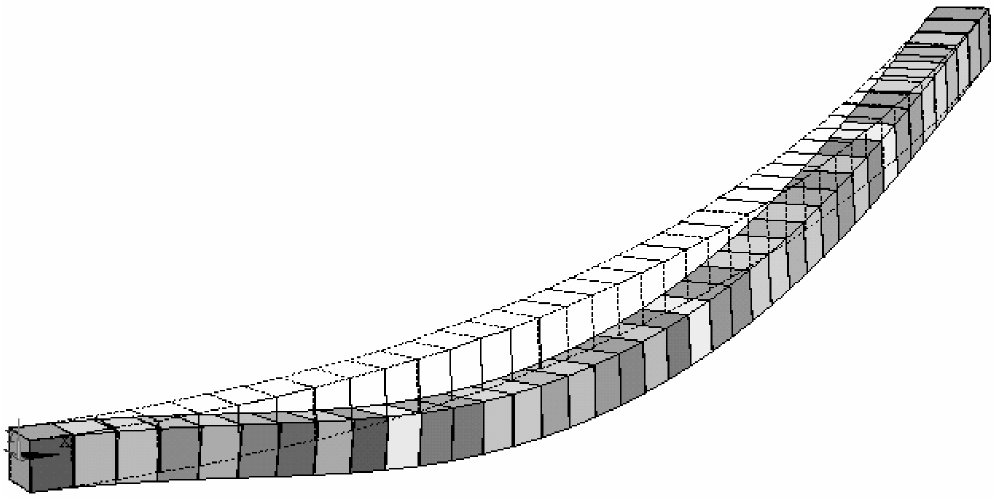

Figure 3.8. Statical displacement of parabolic curved beam due to force at node 12

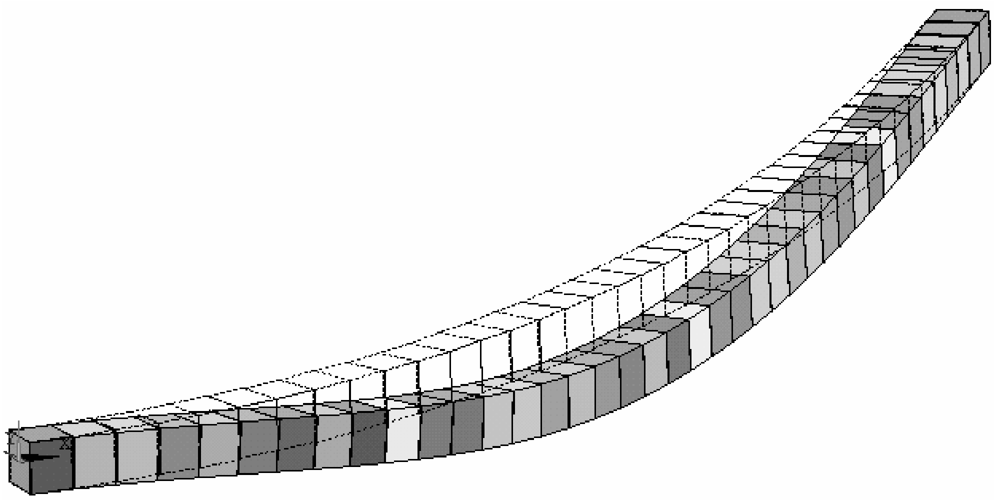

Figure 3.9. Statical displacement of parabolic curved beam due to force at node 16

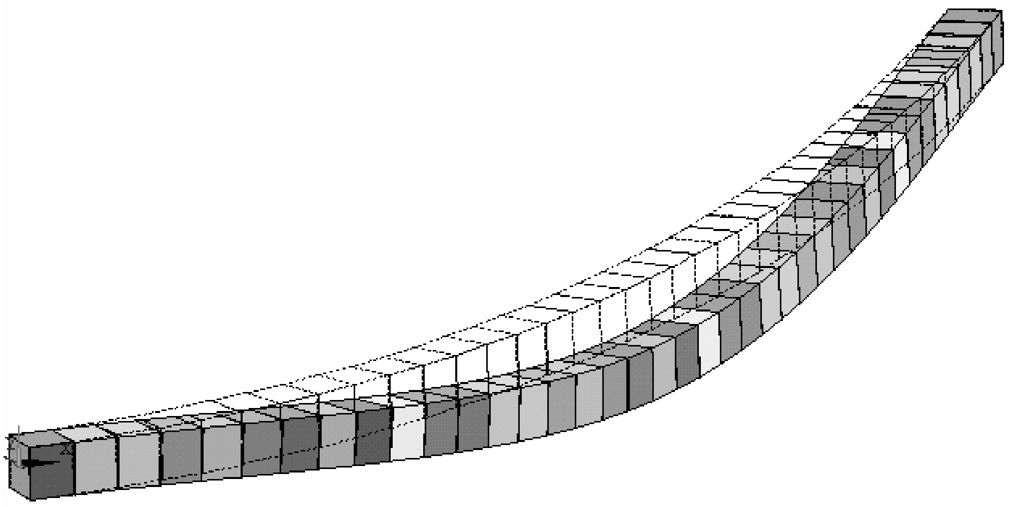

Figure 3.10. Statical displacement of parabolic curved beam due to force at node 20

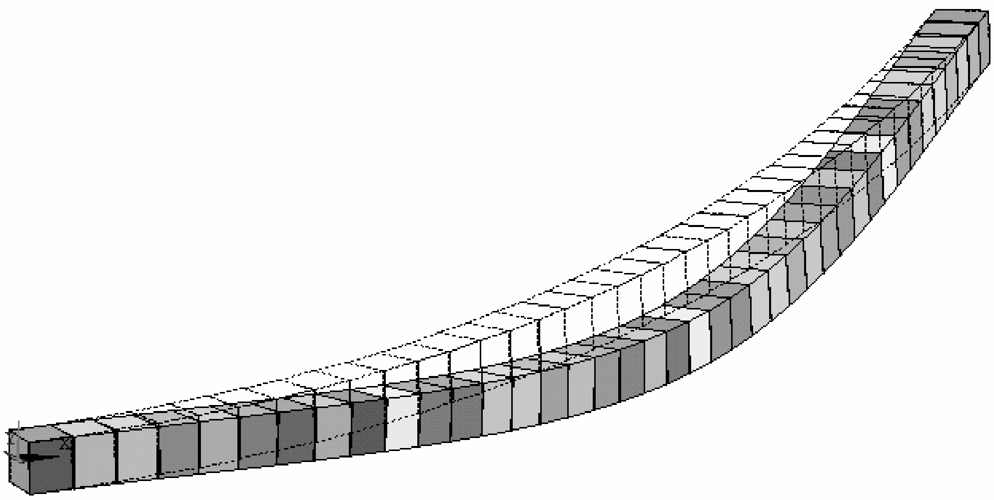

Figure 3.11. Statical displacement of parabolic curved beam due to force at node 24

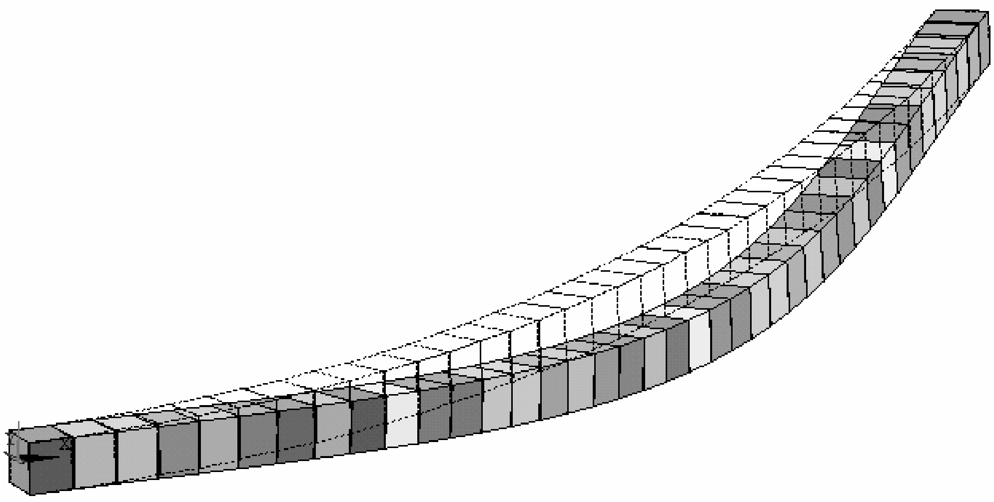

Figure 3.12. Statical displacement of parabolic curved beam due to force at node 28

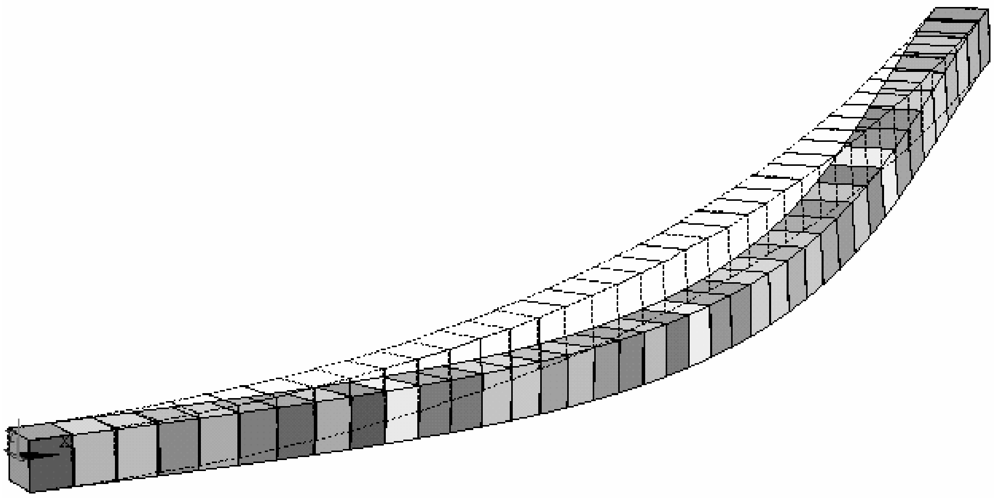

Figure 3.13. Statical displacement of parabolic curved beam due to force at node 32

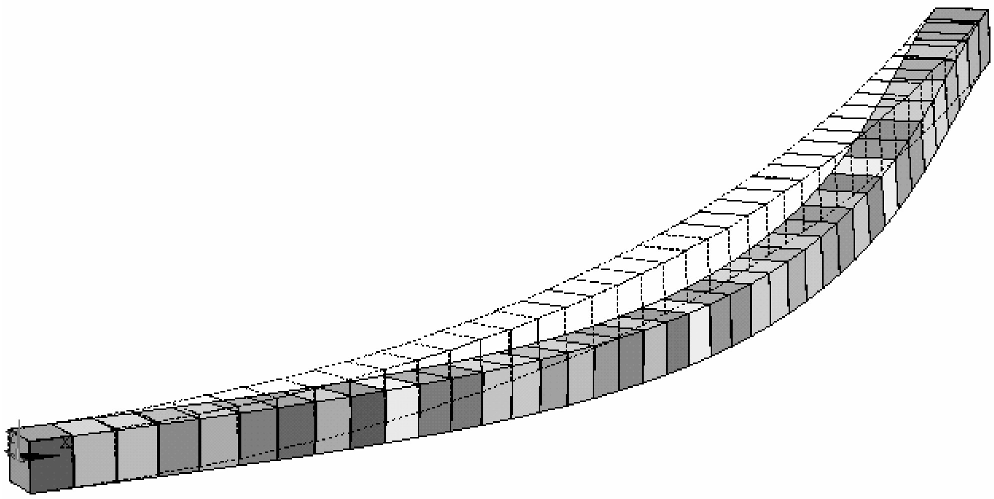

Figure 3.14. Statical displacement of parabolic curved beam due to force at node 36

## 3.5. Dynamic Deflections under Moving Load

As first application, the parabolic curved beam introduced in Section 3.4 is considered to find the dynamical deflections under moving load $F$=10000 N. Different moving load velocities are chosen to see its effect on time response.

Time responses for moving load velocities $v$={2080, 4160, 6240, 8320} mm/s of parabolic curved beam at nodes {8, 12, 16, 20, 24, 28, 32, 36} are obtained and the plots are shown in Figures 3.15-3.18.

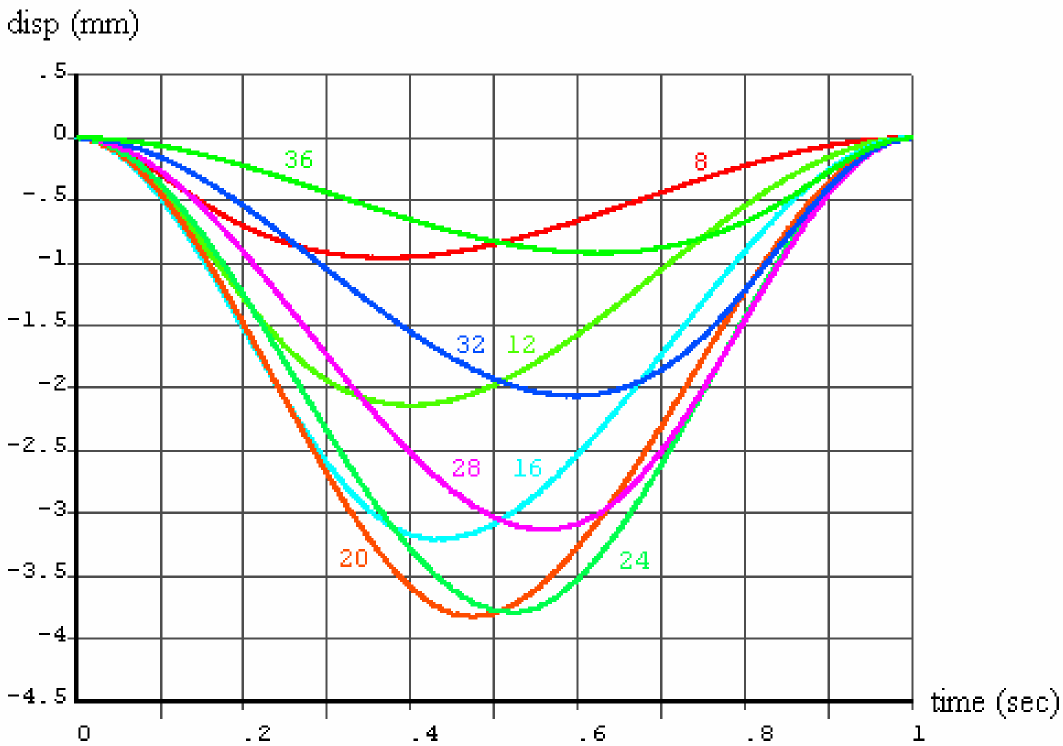


Figure 3.15. Displacements of different nodes for $v$=2080 mm/s

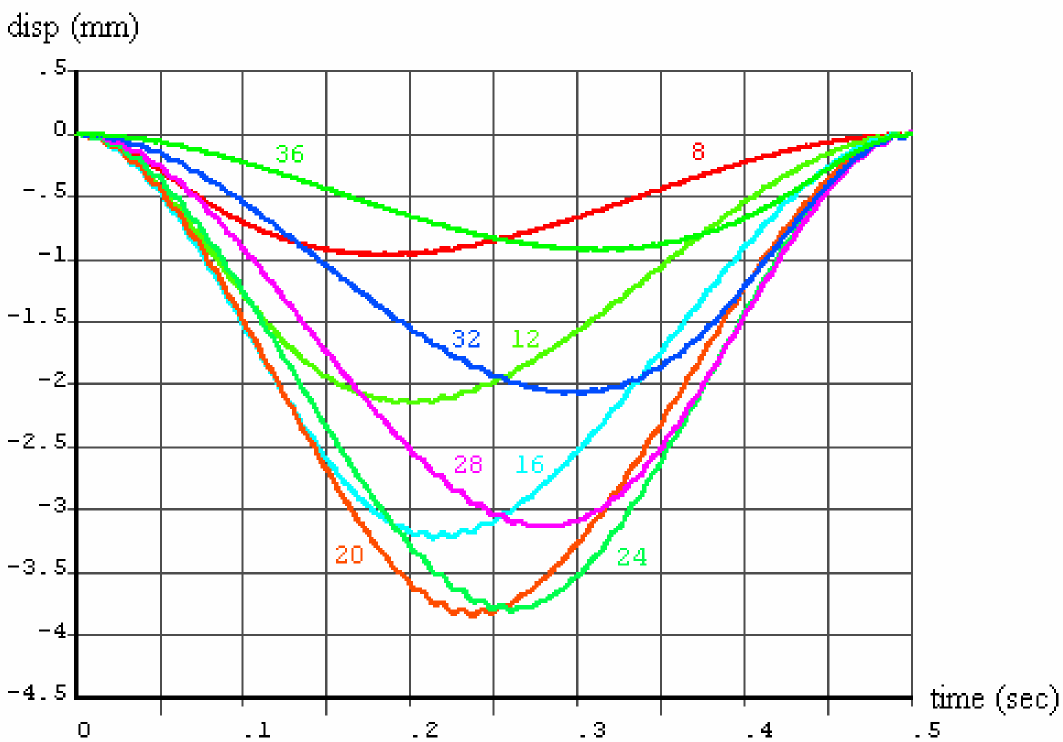


Figure 3.16. Displacements of different nodes for $v$=4160 mm/s

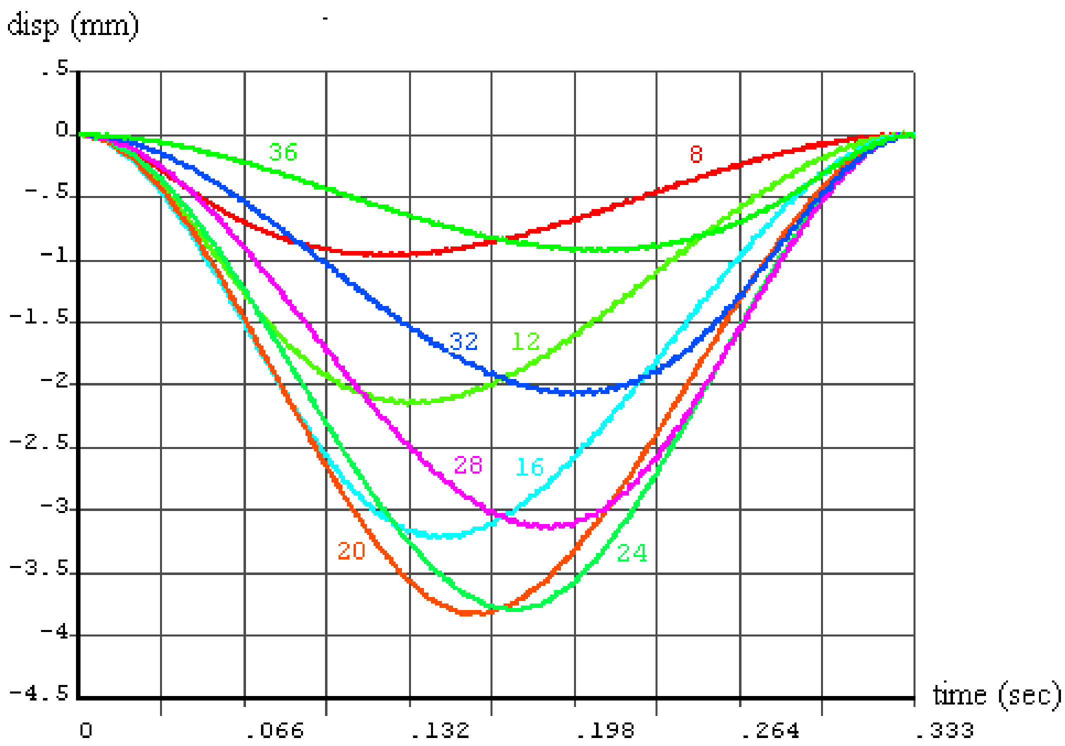


Figure 3.17. Displacements of different nodes for $v$=6240 mm/s

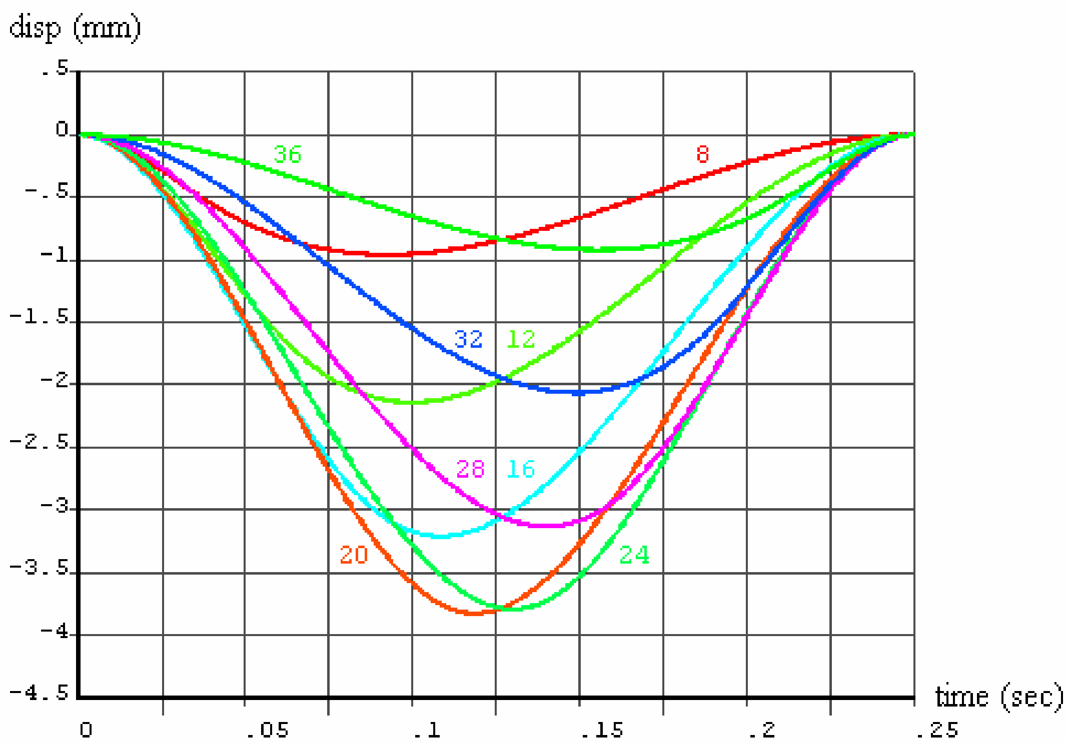


Figure 3.18. Displacements of different nodes for $v$=8320 mm/s

Secondly, boundary conditions of the parabolic curved beam in first application are changed to simply supported type. The same moving load is selected for the moving load velocities $v$={520, 1040, 2080, 4160, 6240, 8320} mm/s. Similarly, time response plots at nodes {8, 12, 16, 20, 24, 28, 32, 36} are shown in Figures 3.19-3.25.

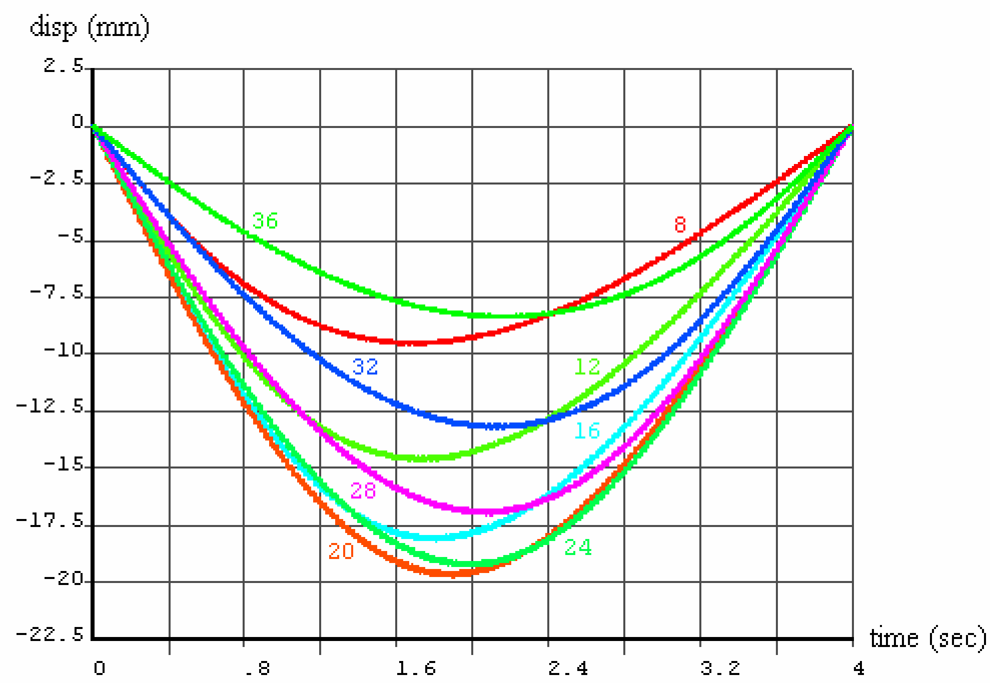


Figure 3.19. Displacements of different nodes for $v$=520 mm/s

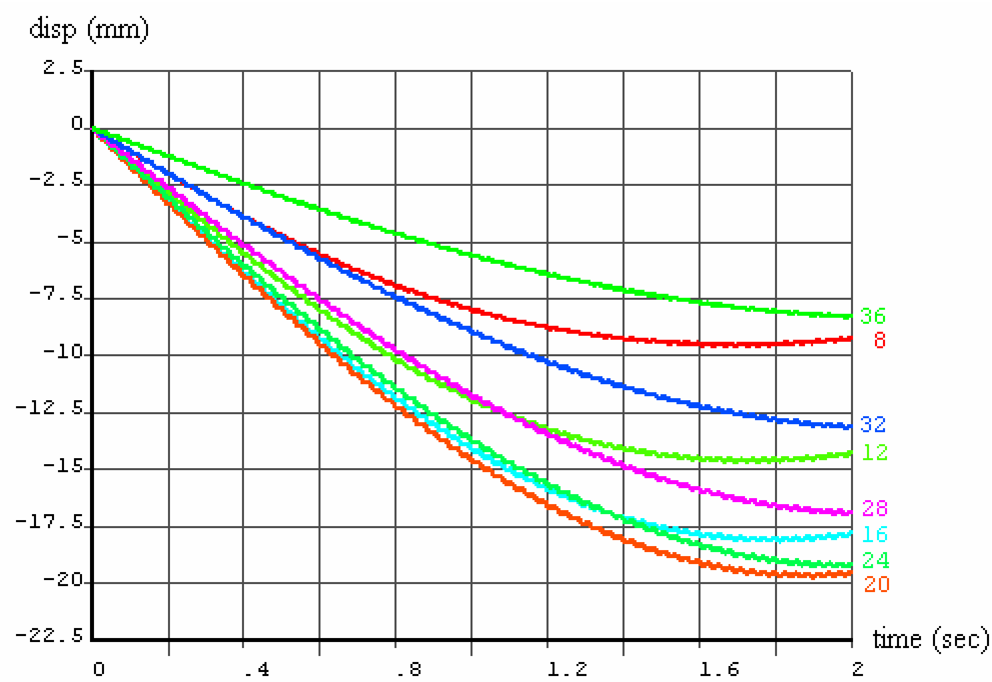


Figure 3.20. Left sides of the plots given in Figure 3.19

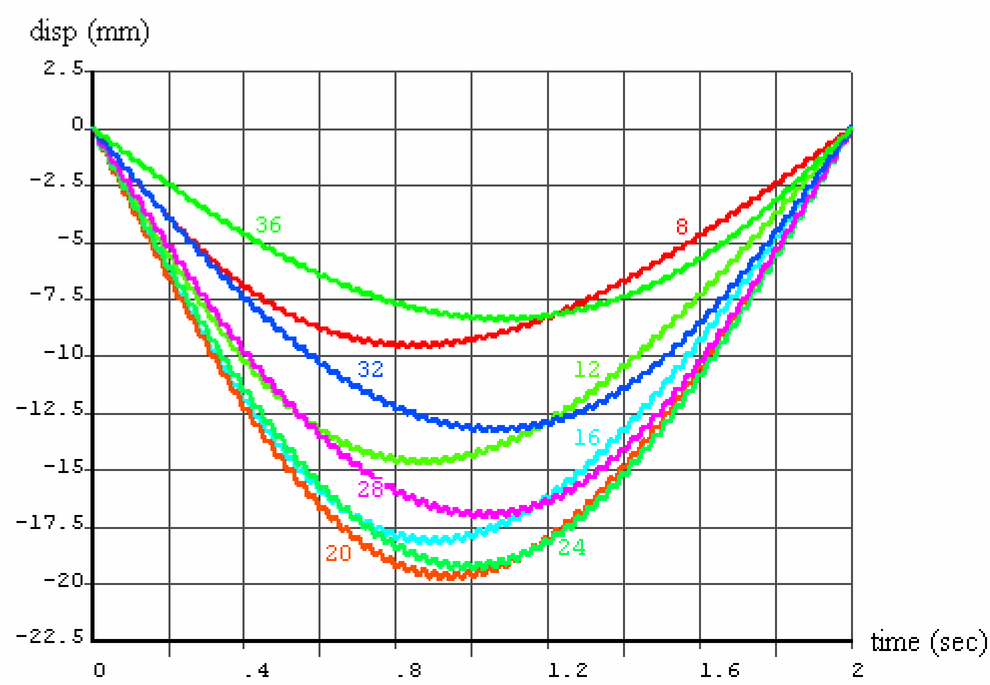


Figure 3.21. Displacements of different nodes for $v$=1040 mm/s

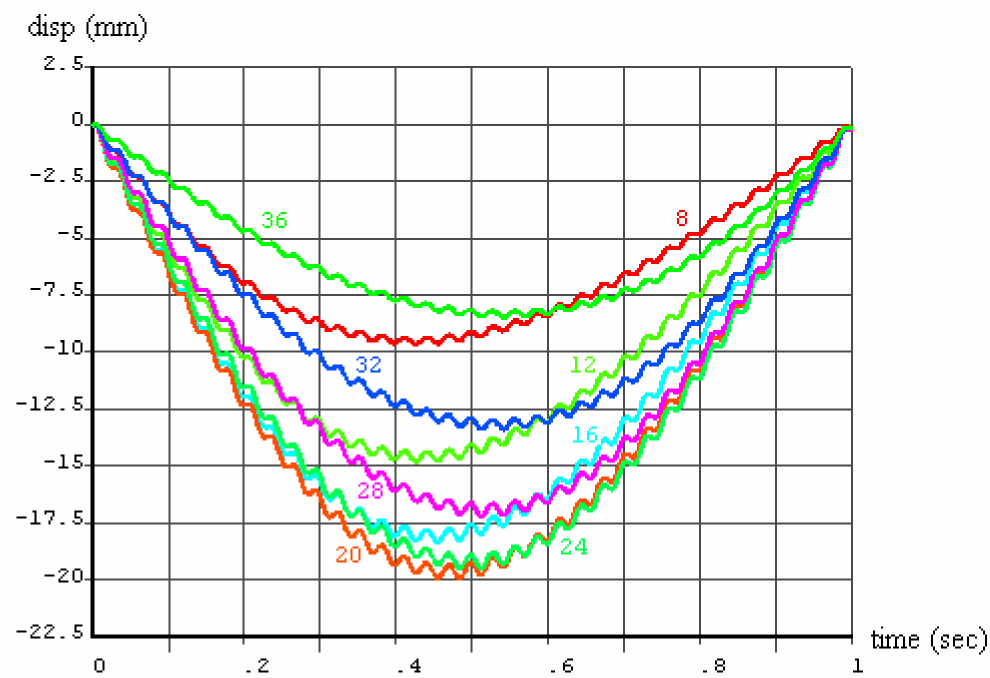


Figure 3.22. Displacements of different nodes for $v$=2080 mm/s

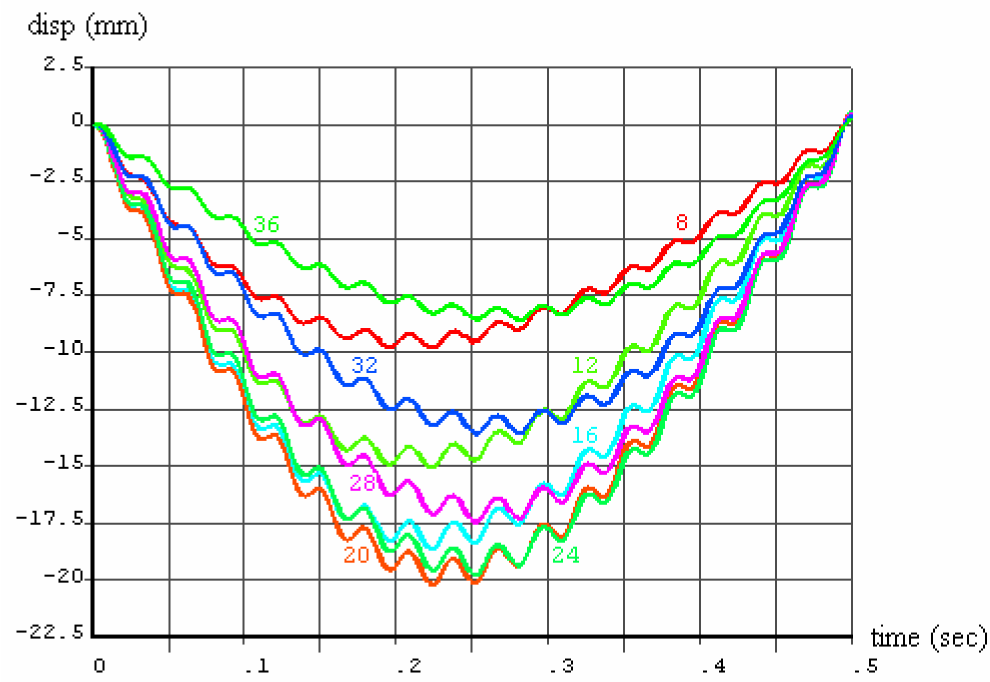


Figure 3.23. Displacements of different nodes for $v$=4160 mm/s

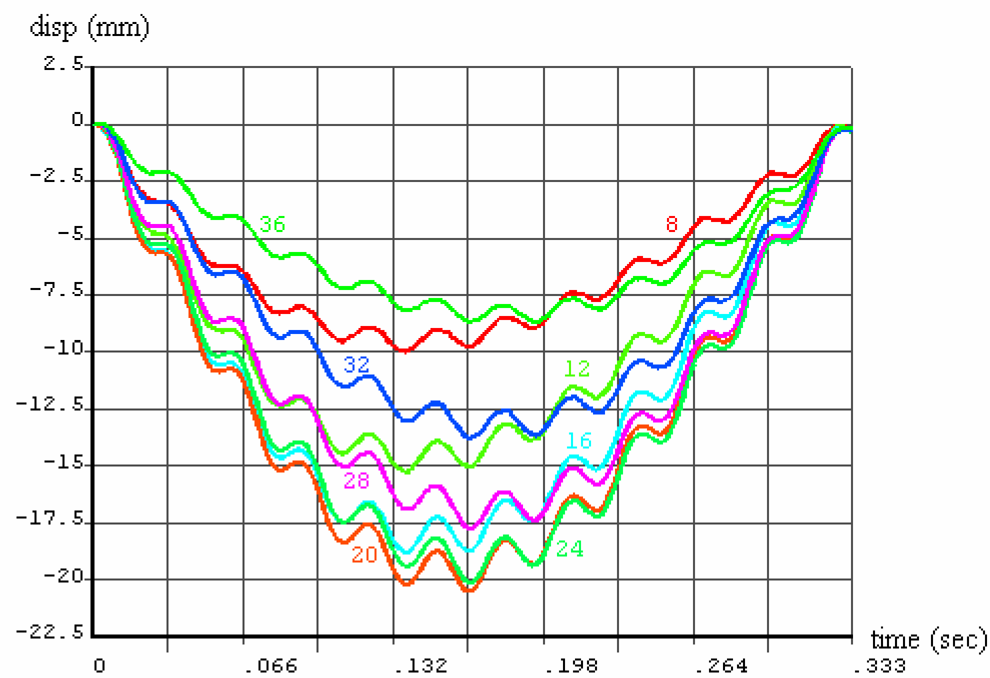


Figure 3.24. Displacements of different nodes for $v$=6240 mm/s

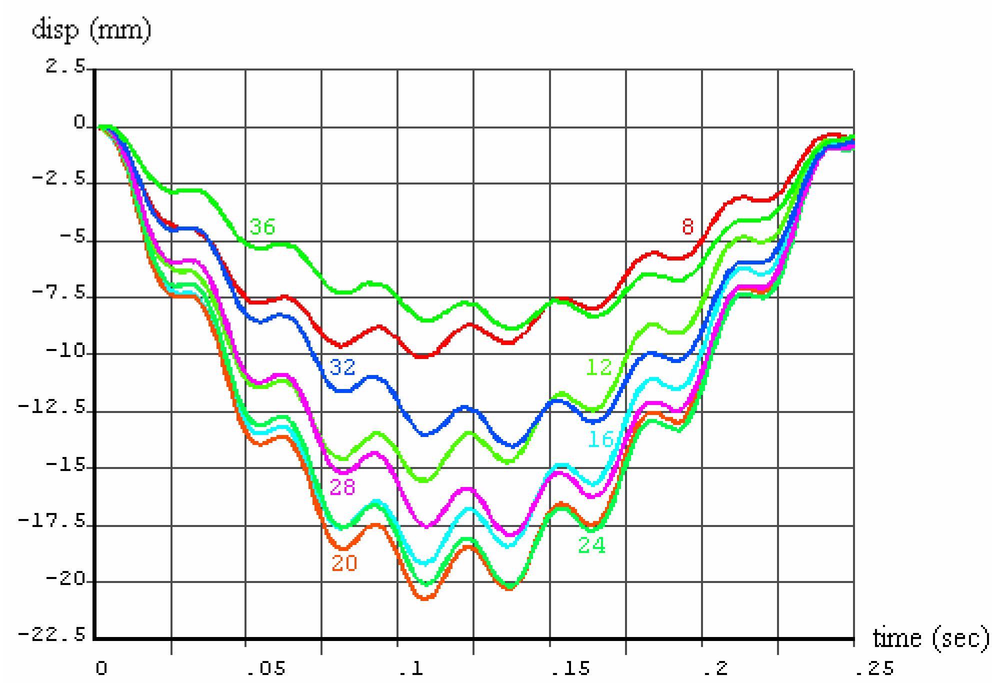


Figure 3.25. Displacements of different nodes for $v$=8320 mm/s

In order to determine the effect of the velocity of the moving load on dynamic displacements, a non-dimensional parameter Δ, which is defines as,

$$\Delta = \frac{\text{maximum dynamic disp. at L/2}}{\text{static disp. at L/2}} \tag{3.1}$$

can be used. It can be pointed out that the nominator of the Equation (3.1) can be obtained by taking the maximum displacement value at mid point of the length of the curved beam at any time.

For the presented two previous applications, Δ versus velocity plots can be plotted. Static displacements at mid point of the fixed-fixed and simply supported parabolic curved beams are found as 3.89375 mm and 19.8065 mm, respectively.

Δ-v plot for fixed-fixed parabolic curved beam is obtained by using a do loop from 20 m/s to 800 m/s with step 20 m/s considering critical velocity $v_{cr}=2Lf_1$.

Δ-v plot for simply supported parabolic curved beam is obtained by using a do loop from 10 m/s to 300 m/s with step 10 m/s considering critical velocity.

Δ-v plots for fixed-fixed and simply supported parabolic curved beams are given in Figures 3.26 and 3.27, respectively.

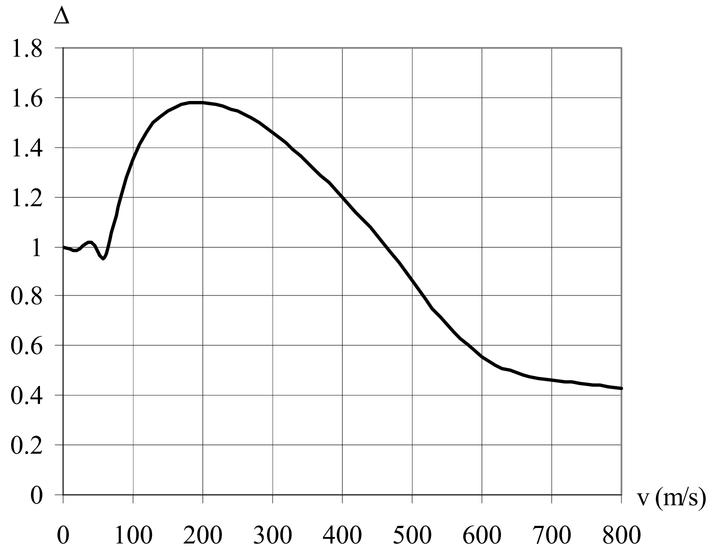


Figure 3.26. Δ-v plot for fixed-fixed parabolic curved beam

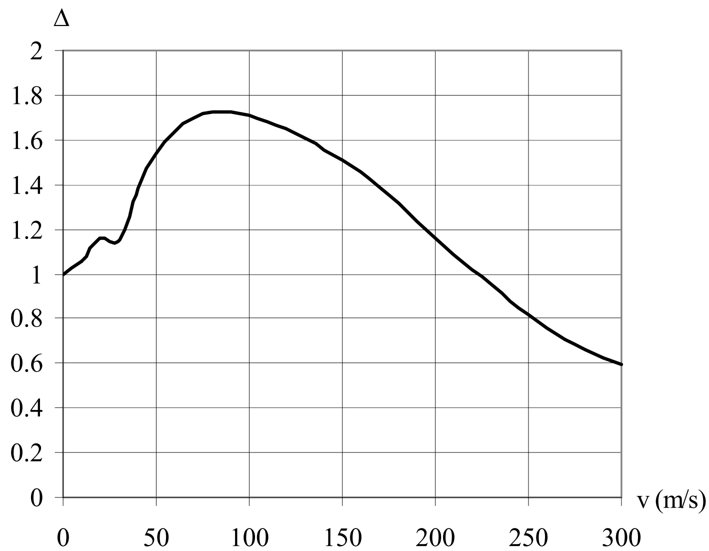


Figure 3.27. Δ-v plot for simply supported parabolic curved beam

## 3.6. Discussion of Numerical Results

The reasonable number of finite element $N$=40 is found for a parabolic curved beam with hollow box form 60x60x6 mm and arch length 2080 mm. By using the number of finite element N=40, stiffness and mass matrices of the curved beam modeled by BEAM4 in finite element code developed by APDL in ANSYS is validated by the theoretical results.

Moving load algorithm in the finite element code is verified by using a simply supported straight beam under moving load since it has exact solution.

Statical displacements of parabolic curved beam under slowly moving load are found for different nodes. Figure 3.6 shows non-symmetrical displacement distributions due to the variable curvature of the curved beam.

Dynamical displacements are effected by the velocity of the moving load and natural frequency of the beam as expressed by *DLF* in Equation (2.31). Two harmonic terms in Equation (2.31) can be written separately by omitting *n* as

$$(DLF)_\Omega = \frac{1}{1-(\Omega/\omega)^2}\sin\Omega t \tag{3.2}$$

$$(DLF)_\omega = \frac{1}{1-(\Omega/\omega)^2}\left(-\frac{\Omega}{\omega}\sin\omega t\right) = \frac{\Omega\omega}{\Omega^2-\omega^2}\sin\omega t \tag{3.3}$$

Now, the following two conclusions based Equations (3.2) and (3.3) on can be drawn for $\Omega << \omega$.

1. The amplitude of harmonic term with forcing frequency $\Omega$ is approximately unity.
2. The amplitude of harmonic term with natural frequency $\omega$ is about $\Omega/\omega$ which is small value.

It should be pointed out that the second conclusion drawn above is effected by the boundary conditions of the system.

When the Figures 3.15-3.18 regarding first application having fixed-fixed boundary conditions are examined, it can be seen that $(DLF)_\omega$ is very small.

On the other hand, when the Figures 3.19-3.25 related with the second application having simply supported boundary conditions are analyzed, it can be seen that $(DLF)_\omega$ is not very small as before.

Therefore, boundary conditions of the beam are very effective on time response of the beam under moving load.

Finally, it can be said by analyzing Figure 3.26 and 3.27 that, the dynamical displacements are very close to statical displacements for practical velocities (0.5 m/s or 2 m/s) for the selected examples with $L$=2.08 m due to the high natural frequencies.

# CHAPTER 4

# CONCLUSIONS

Dynamic responses of fixed-fixed parabolic curved beams subjected to moving loads are presented. An APDL (ANSYS Parametric Design Language) code is developed for the parabolic curved beams. The moving load having constant speed is acted upon the curved beam during the movement of the load. Static and dynamic responses for different cases are determined and presented. Fixed-fixed and simply supported boundary conditions are applied to parabolic curved beams to see their effects on the time responses. It is determined that boundary conditions of the beam are very effective on time response of the beam under moving load.

Finally, it can be said that critical moving load velocity which is based on the natural frequency of the structure plays very critical role. In practice, if small structure such as monorail crane shown in Figure 1.2 has high natural frequencies, practical velocities about 2 m/s can not cause large deflections.

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