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Corrigendum: On density theorems for rings of Krull type with zero divisors

Başak AY SAYLAM*

Department of Mathematics, Faculty of Science, İzmir Institute of Technology, İzmir, Turkey

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Abstract: This corrigendum is written to correct some parts of the paper "On density theorems for rings of Krull type with zero divisors". The proofs of Proposition 2.4 and Proposition 4.3 are incorrect and the current note makes the appropriate corrections.

Key words: Krull ring, ring of Krull type, additively regular rings

In [3], for the Marot ring provided in Example 2.5, [2, Proposition 2.4] does not hold. Thus, we change our hypothesis "Marot" to "additively regular" in [2, Proposition 2.4] and reprove it.

Proposition 2.4 *Let R be an additively regular ring and $P, P_1, \ldots, P_n$ a collection of prime regular ideals such that $P \nsubseteq P_i$ for any i. Then $Reg(P) \nsubseteq \bigcup_{i=1}^{n} P_i$.*

Proof We have that $P \nsubseteq P_i$ for any i, and hence, by [1, Proposition 1.11], $P \nsubseteq \bigcup_{i=1}^{n} P_i$. Thus, there is $a \in P - \bigcup_{i=1}^{n} P_i$. Since the product of regular elements is regular, there exists a regular element $b \in P \cap P_1 \cap P_2 \cap \ldots \cap P_n$. Thus, there exists a $u \in R$ such that $x = a + ub$ is regular in R, and $x \in Reg(P) - \bigcup_{i=1}^{n} P_i$. □

This change we make in [2, Proposition 2.4] affects only [2, Lemma 2.5] and [2, Proposition 3.1], where the hypothesis "Marot" is changed to "additively regular". Furthermore, we note that the 2-generated regular ideal A, found in [2, Theorem 4.2], may not be invertible.

Proposition 4.3 *Let R be an additively regular ring of Krull type. Denote by v_i the valuation associated with the valuation ring $R_{(P_i)}$, where P_i is the center of v_i for each i and by G_i the associated value group. For the 2-generated regular ideal A, found in Theorem [2, Theorem 4.2], there exists an ideal I such that $(AI)_{(P)} = R_{(P)}$ for each center P.*

Proof The ring R is of finite character, and there are at most finitely many centers $P_1, \ldots, P_n$ with corresponding valuations v_i, $1 \leqslant i \leqslant n$, at which A is positive. By [2, Theorem 4.1], we can choose a regular element $x \in R$ such that $v_i(x) = v_i(A)$ for all i. Let $Q_1, \ldots, Q_t$ with corresponding valuations w_j, $1 \leqslant j \leqslant t$, be the set of centers, other than P_i, at which $w_j(x)$ is positive. By [2, Theorem 4.1], we can choose a regular element $y \in R$ such that $v_i(y) = 0$, $1 \leq i \leqslant n$, and $w_j(y) = w_j(x)$, $1 \leqslant j \leqslant t$. Let $M_1, \ldots, M_l$ with the corresponding valuations u_k, $1 \leqslant k \leqslant l$, be the set of centers, other than P_i and Q_j, at which $u_k(y)$ is positive. Again by [2, Theorem 4.1], there exists a regular element $z \in R$ such that $v_i(z) = 0$, $1 \leqslant i \leqslant n$, $u_k(z) = 0$,

*Correspondence: basakay@iyte.edu.tr

2010 *AMS Mathematics Subject Classification:* 13F05, 13A18 and $w_j(z) = w_j(x)$, $1 \leqslant j \leqslant t$. Let $I = (x^{-1}y, x^{-1}z)$. Consider the ideal AI. We observe that, locally at each center P with the corresponding valuation v_P, $v_P((AI)_{(P)}) = 0$, implying that $(AI)_{(P)} = R_{(P)}$. □

Finally, the hypothesis that R be Prüfer should be added to [2, Corollary 4.4].

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