A CRITICAL REVIEW ON CLASSIFICATION AND TERMINOLOGY OF SCISSOR STRUCTURES

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ABSTRACT

When the existing literature on the research of scissor structures is thoroughly investigated, it is seen that different researchers use different terminologies and classifications especially for the definition of the primary units and the motion type. Some of the studies define the whole geometry based on the geometric properties of the primary scissor units and the unit lines while some other studies define it according to the loops. All these studies use different names for similar elements. This article aims to review the literature on the classification and terminology of scissor structures and represent the state of art on the studies. Tables are represented showing all approaches in the literature. In addition, the article criticizes the missing points of each terminology and definition, and proposes some new terminology. In order to arrive at this aim, different definitions of the primary scissor units and motion types used in key studies in the literature are investigated thoroughly. With several examples, it is demonstrated that naming the scissor units according to the resulting motion type might be misleading and it is better to specify the motion type for the whole structure. A classification for transformation of planar curves is presented.

Keywords: Deployable structures, scissor structures, unit-based method, loop-based method

1. INTRODUCTION

Scissor structures are commonly used in deployable and transformable structures. Beginning from the Greek and Roman era, these structures have been used in wide range of applications in architecture and engineering. In the literature, the academic studies on the use of scissor linkages as deployable structures date back to Piñero who developed reticulated single and double-layer domes and various spatial space grids [1]-[4]. Following Piñero, many designers, engineers and architects have enhanced the principles of Piñero and developed novel scissor units and scissor structures. In addition, many researchers have tried to explain the main geometric, kinematic and structural principles of different types of scissor structures.

This paper aims to review the above-mentioned studies and present the state of the art on scissor structures. Instead of a chronological or researcher based approach, the common points and differences of the studies are presented. This is necessary, because when the literature on the scissor structures is investigated, it can be seen that there are many different terminologies and classifications used by different researchers. The difference in terminology causes a complexity on the topic. In addition, some classifications do not meet some special geometries and conditions. The study also aims to reveal these problematic points. For this purpose, first, a classification for the scissor structures based on the units and the loops are presented. Then, the exceptional cases that are not demonstrated in any of the key studies in the literature are introduced.
Finally, detailed Tables (1 and 2) for the classification and the terminology of the scissor structures are given.

2. CLASSIFICATION AND TERMINOLOGY OF SCISSOR STRUCTURES

The existing classifications of deployable structures are generally based on their applications, morphologies, materials or kinematic behaviors. It is possible to find many studies about the classification of such structures. For instance, in 1996, Escrig [5] presented a classification based on their applications in which the scissor structures were defined as “bundles of struts hinged at the extremes in groups of three or more with a geometry compatible with a process of deploying.” In that study, he proposed three main types for the scissor structures that are collapsible grids, un-hitchen grids and X-frames. Apart from the classification based on the applications, Hanaor and Levy [6] classified the deployable structures based on their morphological aspects and kinematic properties. In their paper, scissors were reviewed under the lattice structures. According to the morphology, they presented five categories for scissor structures such as peripheral scissors, radial scissors, angulated scissors, masts and arches. Another classification for the deployable structures was proposed by Rivas-Adrover based on the materials and the geometry [7], [8]. Since the focus of this paper is only the scissor structures as a type of the deployable structures, the classifications related to the other types are not included.

The classifications of the scissor structures are mostly based on primary scissor units. However, there are some special cases having different geometric conditions, which have not been demonstrated in any categorization. When the studies in the literature are investigated, it is seen that the researchers working on the scissor structures have used different terminologies for the definitions of the elements and units although many proposed structures are composed of similar units and have similar deployment behaviors. They even have used different terms for the developed structures such as expandable, extendable, collapsible, foldable, movable, retractable, reconfigurable, transformable, adaptive, etc. Thus, a systematic classification is necessary to gain insight into the design methods and principles.

There are two geometric design methodologies for the scissor structures, which are unit-based method and loop-based method. A majority of the studies in the literature on the topic uses inductive unit-based method in order to reach the desired deployable form. However, deductive loop-based method is also very effective for the definition of the expected geometry. This section introduces these two methods and discusses the advantages and disadvantages of these approaches.

2.1. Unit-Based Method

In many studies, the scissor structures have been designed and formulated using the primary scissor units. A scissor unit is a pair of links hinged with an intermediate joint (Figure 1). Scissor units are connected to each other with two pairs of hinges, which we shall call terminal joints. The imaginary line joining the two terminal joints on one side is usually called a unit line. The unit lines are generally quite useful in design, but they might not have any geometrical meaning for the design task and may be useless for some types of scissor structures. There are alternative names used for the joints in different studies. Each link in general has three design parameters: two link lengths between the intermediate joint and the terminal joints, and the kink angle defined by these three joints. Typically, the two links constituting a scissor unit are selected to be identical and usually have a vertical symmetry.

As shown in Figure 1, in many studies the primary scissor units are categorized into three main groups: translational unit, polar unit and angulated unit, but these are not the only possible scissor units. There are also alternative classifications and different terminologies used in the literature as it is explained later in this section. Translational and polar units comprise identical straight links. For translational units, the intermediate joint is right at the middle of the links. In an angulated unit, the links are not straight and the kink angle is less than 180°.

![Figure 1: Primary scissor units a) translational unit; b) polar unit; c) angulated unit](image-url)
In the unit-based design method, the scissor structure is obtained by connecting the units to each other through the terminal joints. It is possible to generate the scissor structures by multiplying one of the primary units or by assembling several sub-linkages produced using different primary units. By this way, various scissor structures with different curvatures can be obtained.

In Table 1, a systematic review based on the scissor units is given. As depicted in the table, some researchers gave specific names to the scissor units while the others did not. Piñero [4] did not mention any specific name in his patent neither for the elements nor for the units. He defined his structure as “three-dimensional reticulate structure” that is composed of rods pivotally connected to each other by coupling. Although there is no specific name, he defined two parameters that affect the geometrical characteristics of the structure. The first one is the number of rods in the coupling. The second one is the distance between the intermediate coupling (i.e. joint) and the other two couplings (i.e. the terminal joints) of the rods. He stated that the structure does not present any curvature when the intermediate coupling is equidistant from the upper and lower couplings. In this case, the series of couplings (upper/lower terminal joints and intermediate joints) remain in parallel planes. On the other hand, a curved form is generated when the intermediate coupling is not equidistant from the other two. By changing the location of the couplings, either planar or curved scissor structures are obtained.

Piñero’s description based on the geometric characteristics of the structures is quite similar to most of his followers’ definitions since they have also defined the units according to the location of the intermediate joint. For instance, Zeigler [9] used the term “straight strut pairs” for translational unit and “curved strut pairs” for polar unit in his patent. In another patent, he presented the alternate sliding and fixed pivoting for the intermediate nodes and the flexible spring type connections for end nodes instead of using the simple pin pivots at the connections [10]. While he defined the scissor units as “crossed pairs” in [10], he used another terminology for his basic unit in [11]: “scissor-like chains/ladders”. Similar to Zeigler’s hexagonal basic unit in [10], Clarke [12] developed a scissor unit called “tri-scissor/trissor” composed of three scissor pairs.

Apart from the basic units composed of identical elements, some researchers have focused on other types of units with different bar lengths and eccentric joints. The leading researcher on this area is Escrig who developed different types of linkages and scissor structures. In [13], Escrig used the terms “crosses” and “struts” for the elements and did not give any specific names to the units and linkages. Rather, he defined them with their properties. The first type of linkage he developed is a plane assembly in which the elements are identical and connected to each other at their mid-points, i.e. he used translational units. By combining the elements obliquely 3x3 and 4x4, and by placing the assemblies perpendicular to one another or with 60° angles, he constructed various spatial scissor structures. The second type of linkage is composed of struts having same length and eccentrically joined, i.e. he used polar units. The third and the fourth types consist of struts having different lengths; but intermediate joints are located at the midpoint in the third type while the struts are eccentrically joined in the last type.

Zanardo [14] studied translational and polar units to build singly and doubly curved surfaces. He did not use different names for the scissor units; but he designated his structures as “articulated lattice systems.” Krishnapillai [15] was the first one to use the term “Scissor-like-element (SLE)” for the elements and the term “structural units” for the module composed of SLEs. He also mentioned the lines (later it is called as “unit lines” by other researchers) defined by each SLE which can be either parallel to each other (as in translational units) or converged to a single focal point (as in polar and angulated units). In his patent, Krishnapillai developed regular polygon-shaped structural units in which the polygons can be either triangle, square or hexagon. Using these structural units with different combinations, various flat and curved structures can be generated.

In his Ph.D. thesis, Gantes [16] used Krishnapillai’s terminology for the definition of the elements and units. He classified the units according to their morphologies. He presented four different units: polygonal units, trapezoidal units, prismatic units and pyramidal units. Polygonal and trapezoidal units can be used to construct flat and curved structures. For flat structures, polygonal units can be either equilateral triangle, square or regular hexagon. For
curved structures, polygonal units can only be used to build circular vaults and spherical domes. The main difference between the polygonal units used for flat and curved structure is that the corresponding upper and lower nodes of the polygon lying in parallel planes remain parallel in flat structures whereas they intersect at a single point in curved structures. Gantes also used the trapezoidal units to create flat deployable structures. The geometric shapes created using polygonal and trapezoidal units are limited, because they allow generating only surfaces with constant curvature. In order to build more free forms that are architecturally more desirable and structurally more efficient, Gantes developed prismatic and pyramidal units. Compared to polygonal and trapezoidal units, it is seen that folded plate structures can be constructed by using prismatic units since the slope of the roof can be changed while it is not possible in polygonal and trapezoidal units. On the other hand, pyramidal units can be used to build arches and domes with arbitrary curvatures whereas polygonal units allow creating only circular arches and spherical domes.

Apart from the well-known terms used for the elements and units, different definitions have been also used by the researchers. For instance, Kaveh and Davaran [17] named the elements as “uniplets” while the unit is called as “duplet/scissor-link unit.” Similar definitions were also used by Babaei and Sanaei [18]. They classified the duplets into two as “regular (rectangular) duplets” and “irregular (trapezoidal) duplets.” A flat structure is created using rectangular duplets while a dome structure can be built using trapezoidal duplets. It is also possible to create arbitrary curvatures with trapezoidal duplets.

Aforementioned researchers have developed numerous planar and spatial scissor structures using the scissor units. However, their proposals were limited to certain geometric shapes. In 1990, Hoberman extended the applications of scissor structures by inventing a new scissor element called “angulated element” [19]. Because the element is kinked rather than straight, it allows deploying the structure from its perimeter towards the center. In [19], Hoberman defines the “scissor pairs” as combining two “angulated elements” with arbitrary “strut angles” at the “central pivots”. He calls the two terminal joints on one side of the scissor pair as “paired terminal pivot points,” and the angle between the “normal lines” connecting the two paired terminal pivot points of a scissor pair is called a “normal angle”. The normal angle remains constant when the strut angles of both angulated elements of a scissor pair are chosen as complement of the normal angle. The reason why he calls the lines on the two sides of a scissor pair as normal lines is that these lines represent the lines normal to the curves approximated by the scissor structure. This seemingly simple difference in the terminology actually makes a big difference for some scissor structures, because normal lines (not necessarily passing through the terminal joints) are geometrically much meaningful compared to the unit lines, which by definition have to pass through the terminal joints. Hoberman patented a method for building loop assemblies formed by angulated elements. Using that element and the unit composed of two angulated elements, he built many impressive examples of scissor structures for dome, arch, spherical, helicoid and hypar surfaces. The angulated unit is discussed in detail in the next section under the loop-based scissor structures.

After the invention of the angulated elements, a new classification has been added to the literature by the researchers. While some researchers called the new unit as “angulated unit,” the others directly called it as “Hoberman’s unit.” Using this unit, various scissor structures have been developed. In 1997, You and Pellegrino [20] presented a proof of the fact that in Hoberman’s design, which consists of multi-rows of concentric linkages formed with angulated units, the angulated elements maintain a constant angle during the deployment of the structure and thereof, these elements can be replaced with a new single element called “multi-angulated rod.” When the multi-angulated rod is used to construct deployable structures, it reduces the number of links. You and Pellegrino also discovered “generalized angulated elements (GAEs)” composed of interconnected angulated rods in order to generate scaling structures. They presented two types of GAEs that can be attached to the adjacent elements by either isosceles triangles (Type I GAE) or similar triangles (Type II GAE). Type I GAE is formed by angulated rods with equal semi-length and different kink angles whereas Type II GAE is formed by angulated rods with proportional semi-lengths and equal kink angles.

Langbecker [21] divided the scissor units into three: translational unit, polar unit and Hoberman’s unit. Atake [22] developed two types of three dimensional scissor units that are based on “pyramid type” and “sliced skew prism.” For the first type, he presented
three units called triangular, square and pentagonal pyramid units. For the second type, he proposed two skew type units based on sliced tetrahedron and octahedron. Hanaor and Levy [6] identified two basic scissor units: “peripheral scissors” and “radial scissors.” They proposed two sub-groups for each unit as triangular and rectangular prisms.

Similar to Langbecker’s classification, De Temmerman [23] reviewed the scissor units under three categories: translational unit, polar unit and angulated unit. He defined the scissor units based on the unit lines that connect the lower and upper end nodes of the unit. He identified two types of translational units, named as “plane translational unit” and “curved unit.” In both types, the elements are straight and linked at their mid-points; but the elements used in a plane translational unit are identical while they are different in a curved unit. By connecting the plane units to each other, a planar translational scissor linkage called the “lazy-tong” is obtained. A curved linkage is created with the curved units and a polar linkage is obtained by connecting the polar units together. The name of the linkage is given based on the unit type. Translational and curved units demonstrate curvilinear deployment whereas angulated units provide radial deployment.

Van Mele [24] and Jung et al. [25] also classified the scissor units as translational, polar and angulated units while Chen et al. [26] described four primary scissor units that are translational units, polar units, angulated units and generalized angulated units. As different from the former researchers, Yu and Luo [27] used the terms “straight-rod hinge unit” and “angular-rod hinge unit” for the scissor units. Similarly, Lu et al. [28] used the term “macro elements” for the elements. They presented four types of scissor units: straight beams, angulated beams, depth multi-angulated beam (DMAB) and the breadth multi-angulated beam (BMAB). The first and second types are respectively typical translational and angulated units. DMAB is the same as You and Pellegrino’s multi-angulated rod. BMAB, on the other hand, is an element described in one of Hoberman’s patents [29].

The authors of this article also published a paper about the scissor structures in which three primary scissor units are defined and planar scissor structural mechanisms (SSM) composed of them are introduced [30]. In that paper, four different types of rectilinear SSMs were presented. Two different types of curvilinear SSMs and two different SSMs with angulated units are obtained by changing the bar lengths and the locations of the intermediate joint. The unit lines connecting the corresponding upper and lower nodes remain parallel during the deployment process in rectilinear SSMs. The fact that, when the identical bars are connected to each other at their midpoints, a simple lazy-tong linkage is obtained also noted by De Temmerman [23]. If different bar lengths are linked at their midpoints, translation along an inclined axis is obtained instead of horizontal axis. It is also possible to use more than two different bar lengths in a translational scissor linkage. In this case, the SSM still realizes translation, but it cannot be compactly folded into a bundle. If arbitrary bar lengths are used, intermediate joints have to be placed eccentrically and all of them should lie on the same axis. In curvilinear SSMs, the circular shape is generated with either identical bars or arbitrary-length bars. In both cases, scissor hinges are placed eccentrically and lie on an arch. In SSMs with angulated units, either a radially deployable closed ring or a rectilinear linkage is obtained. It is also possible to create other types of SSMs by combining different scissor units based on a general deployability condition.

Rosenberg [31] presented three primary scissor units in his work and used the terms “center scissor-pair” for translational unit, “off-center scissor-pair” for polar unit and “angulated scissor-pair” for angulated unit. This terminology may be regarded as favorable compared to the conventional one, because “translational” and “polar” terms should better be associated with the type of motion of the assembly rather than the type of the scissor pair. For instance, as already depicted, there are several types of scissor units other than the translational units shown in Figure 1a that can be used for translational motion. In addition, some assemblies comprising the so-called polar units shown in Figure 1b do not result in polar deployment. Because these units allow generating single degree-of-freedom (DOF) structures that have limited shapes and deployment behaviors, Rosenberg attempted to extend the possible applications of such structures by offering a new transformable component that provides a range of variable shapes and alternatives. For this purpose, he combined two off-center scissor pairs in a novel manner and developed the new component called as “double scissor-pair” that enables generating a variety of non-uniform shapes controlled by a single actuator. The double scissor-pair is created by mirroring the off-center scissor pairs. By this means,
two compatible components are obtained which can be combined in arrays to build planar structures and in different ways to build three-dimensional structures. Even though the double scissor-pair realizes a non-uniform transformation, the unit lines connecting the mirrored pairs remain parallel to each other during the transformation process. Because the transformation of the double scissor-pair is predetermined, Rosenberg investigated new solutions that allow the users to choose the type of transformations. He modified the scissor-hinge of the double scissor-pair by using a slider at the connection point, which provides the required translational motion to move the scissor-hinge from the center to off-center position or vice versa. This solution enabled to incorporate an additional DOF to the double scissor-pair.

In order to construct transformable structures that enable to change the shape of the whole structure without changing the span, Akgün [32] proposed a two-dimensional scissor unit called as “modified scissor-like element (M-SLE)” that comprises of additional revolute joints and bars attached to the basic scissor unit. Then, he applied the same principle to constitute three-dimensional structures. For this purpose, he proposed “spatial scissor-like elements (S-SLEs),” “modified spatial scissor-like elements (M-S-SLEs)“ and “hybrid spatial scissor-like elements (HS-SLEs).” These units increase the transformation capacity of the scissor structures by allowing transformation from rectilinear geometries to various curved shapes without changing the span.

Bai et al. [33] proposed a new element called “angulated-straight element (ASE)” by combining basic scissor unit and Hoberman’s angulated unit, which moves in radial direction with a fixed angle. Because ASE and Hoberman element have same kinematic characteristics, they called these units as “radial-scaling element (RSE).” According to the shapes of the linkages (the lengths of the bars and the positions of scissor hinge), ASE is classified into two: “equilateral ASE” and “non-equilateral ASE.” By using different combinations, two types of equilateral ASEs and three types of non-equilateral ASEs are created. Compared to non-equilateral ASEs, the scaling ratios of equilateral ASEs are larger. Equilateral ASE can be combined in two different ways. By connecting ASE to equilateral scissor unit, a new type is obtained which is named “hybrid-radial-scaling element (HRSE).” Bai et al. classified the HRSE into two as Type I HRSE and Type II HRSE. While Type I HRSE cannot be folded completely due to the lengths of the bars and the configuration of ASE, Type II HRSE can fold completely.

Zhang et al. [34] classified the scissor units according to the angles between the unit lines. To build the scissor structure, they used three types of scissor units. The first type is called as a “parallel unit” in which unit lines remain parallel during deployment. The second type is “isogonal unit” and the third is “symmetric unit.” Both of these latter units are composed of angulated elements; but the difference between them is that the angle between the unit lines keeps constant in the isogonal unit while it changes in the symmetric unit during the motion.

2.2. Loop-Based Method

Loop-based method is another approach to design scissor structures. Compared to the unit-based method, there are fewer papers on the loop-based method. In Table 2, a review based on the scissor loops is given.

Hoberman’s patented work in 1990 was a great contribution to the area of scissor structures [19]. Invention of the angulated element gave rise to many researches and novel structures. In the descriptions of the patent, he described his invention as “a method for constructing reversibly expandable truss-structures that provides for an extremely wide variety of geometries.” The structure composed by this method is to be able to support itself and deploy while maintaining the initial curved geometry. This property of the structure is explained to be related to the unchanging values of some critical angles.

In the patent, a polygonal form is converted into a structure by approximation of distances between adjacent scissor pair pivots with the corresponding side of the polygon. This structure is defined as a “loop-assembly” for the first time in the literature. As it was claimed in the patent, the fully deployed form of the radially deployable structure was a scaled version of the initial design polygon thereby the kink angles of the angulated elements were the same as the corresponding angle of the polygon corner. In his next patent [35], Hoberman presented nested multiple rings of loop assemblies. The outermost pivot points of the loop-assemblies were connected to the adjacent rings’ pivots. Although the resulting structure had excessive number of joints and elements as proven by You and Pellegrino [20], it was yet capable of radial deployment.
Many years later, during a lecture at MIT in 2013, Hoberman presented the details of his method [36]. In the lecture notes, identical loops defined as hinged rhombs are placed on various geometries to design the scissor pairs in the end. While the simplest structures such as the translational lazy-tong can be formed by placing the rhombs on a line, placing them on a curved form such as arc, circle and ellipse results in radially deployable structures. Another form used in [36] is an arbitrary polygon. For placing the rhombs, aforementioned method in patent [19] is used. The rhombs are scaled to match the side lengths of the polygonal form while their angles are constant. Resulting loop-assembly is capable of scaling.

A noteworthy aspect of You and Pellegrino’s [20] study in relation to loop-based design is that they mentioned the loop formations in between adjacent scissor units as “each closed loop is a parallelogram.” They even made a remark on how the same structure could be assembled by using loops. Even though they did not base their designs solely on the loops, their research paved the way for the advancement of the method.

In the mentioned loop assemblies of Hoberman, it can be seen that the final deployed form of the structure is used as the starting point of the design with loop assembly method. The following studies also fall within this concept of deductive design strategy. In 2005, Liao and Li approached to the matter as a scalable planar graph design [37]. They started with the fully deployed state of the desired structure and selected nodes on the form, which correspond to intermediate joints. The distance between each adjacent node is equally divided into two. The arms on both sides of the intermediate joint make up an element of a scissor pair, and each element is connected to the next one with a revolute joint. Actuating this open link chain by the same amount in either direction yields chiral kinematic doublets. When the doublets are joined at the intermediate joints, the result is the partially deployed structure with rhombi loops.

Kiper and Söylemez [38] made use of the Cardan motion as their starting point for scaling arbitrary polygonal shapes. As Liao and Li, they considered the target geometry as the fully deployed state of the structure to be designed. They stated that the three joints of the identical angulated element constituting a scissor pair should be on a circle. Each side of the polygon is bisected, and each corner together with the bisection points on either side is placed on a circle. As the corner points of the polygon is assumed to be the intermediate joints of the angulated elements, when the circle with the joint locations of the angulated element is rotated around that point, a partially deployed structure with rhombi loops is obtained.

In 2014, Bai et al. [39] investigated the loop forms emerging between the scissor pairs. They began by analyzing the possible geometries in accordance with the foldability conditions laid out by Escrig and Varejcarcel [40]. The research showed four geometries applicable to closed-loop scalable structures: rhombus, kite, parallelogram and general tetragon. Using the vertices of these loops as joints, various connection alternatives were sought. While rhombus form yields only two alternatives, kite and parallelogram yield six alternative connections, though only two are usable for the structures. All the scissor units resulting from the study are either Type I or Type II GAEs.

The authors of this article have also conducted several studies on loop-assemblies. Yar et al. [41] expanded the use of kite loops by forming a transformable scissor structure rather than a deployable one. The loop-assembly of kites in a rectilinear fashion resulted in the parallel positioning of the unit lines of angulated scissor pairs. When the structure is actuated, it is capable of bending into a spiral form in either direction, transforming from a convex to a concave form while its loops also transform back and forth between a kite and a dart shape. Same loop-assembly method was also applied with concave kite, i.e. dart loops. Similarly, the dart loop structure can bend in either direction.

Another study of the authors is by Gür et al [42] where they made the use of a loop that was not considered previously in scissor structures: the anti-parallelogram. Also known as crossed-parallelogram or contra-parallelogram, it is a complex quadrilateral that intersects itself, though the intersection point does not provide a joint. An alternating (one above, one below) loop-assembly on a circular form similar to Hoberman’s was examined. The units formed from the assembly are Type II GAEs. Same loop-assembly pattern was examined with arbitrarily scaled versions of the same loop, which yielded radial deployment as well. This assembly also has Type II GAEs.
Gür et al. [43] also investigated structures approximating free-form curves. The curve is handled as the target form at fully deployed state and loops are scaled in between discrete nodes on the curve. It is seen that such a structure is scalable and the units are Type II GAEs.

The studies starting with the final form reveal that the loop-based method can give exact dimensions, angles and joint positions of the links in a practical way whether the final form is a regular geometry or a free-form curve. The designer can choose between various forms of loops in order to produce the linkage. Depending on the selection of the loop, the linkage might result in doing a scaling, curvature changing or transforming motion.

During the process, it is also possible to uncover some unique link forms or combinations of different kinds of links that can work together in the deployable structure, as mentioned in the next section. Therefore, this method rules out the strict additive approach of the unit-based method and rather give way to unforeseen combinations and results.

3. CONFUSION IN TERMINOLOGY

In the previous section, the unit-based and the loop-based design approaches, and the examples in the literature have been examined in detail. Generally, most studies have made use of the scissor units to assemble the structures in different geometries. The units and the structures have been named according to the hinge positions or the characteristics of the motion. In many studies, it has been stated that the translational units generate only linear deployment, the polar units always define curves and the angulated units provide constant angles. According to this approach, curvilinear motions are generated by using only translational scissor units and curvature-varying motions are obtained with either polar or angulated scissor units. However, there are some exceptional cases that have not been explained and classified in the previous studies. These cases do not comply with the deployment behaviors of the basic scissor linkages defined in the literature. The aim of this section is to expose the lacking points of the existing studies in the literature and explore the variance in the way the scissor pair is assembled which can change the motion characteristics of the structure.

In the literature, it is mostly accepted that the polar scissor units are used to obtain arch-shape structures. However, a translational scissor structure can be obtained by polar scissors as well. When a scissor pair is repeated by mirroring vertically or horizontally within the assembly, the motion changes. If a polar scissor pair is followed by its vertically mirrored version in alternating order, it results in a translational movement in an inclined direction. The loop is composed of non-adjacent short and long edges, forming a parallelogram (Figure 2).

Another example stepping out of the general characteristics of its scissor unit is composed of kite and dart loops. If the angulated units are generated from the assembled loops, a novel planar scissor linkage transforming between concave and convex forms is obtained [41] (Figure 3). Unlike the previous studies presenting the scaling deployment capability of the angulated scissor pair, this scissor linkage offers more flexibility in geometric form. According to common studies in the literature, a scissor structure can be either translational or polar but not both at the same time. By this example, a scissor structure meeting both geometries can be obtained.

In the literature, it is mostly accepted that the polar scissor units are used to obtain arch-shape structures.
Anti-parallelogram loop-assembly also yields unique properties [43]. A special case arises when the loops are positioned such that the long edge of a loop is collinear with the short edge of the adjacent loop. This condition allows the structure to be made up of straight rod elements with eccentric central hinges, i.e. polar units. When actuated, the movement is observed to be transformation between concave and convex forms (Figure 4).

**Figure 4: Polar scissor units forming a translational linkage**

An analysis of the structure shows the variation of the assembly of units compared to the linkage making angular deployment, i.e. the polar linkage (Figure 5).

**Figure 5: Polar units’ assembly comparison**

In accordance with these discussions, it is concluded that “polar scissor units” is an unfortunate naming because they can be used for non-polar motions as well. Therefore, it is better to name the whole assembly according to whether the motion is translational or polar rather than using these terms to name the scissor units.

Also, the word polar in general refers to two different things as in polar coordinates comprise radial and angular coordinates. In this regard, the polar linkage as described by Temmerman [23] and many others is mainly used for changing the angle of a circular arc shaped assembly while the radius is not the focus and mostly negligible, whereas the radially deploying linkages such as Hoberman’s [19] are used for changing the radius of a circular arc while preserving the subtended angles.

Therefore, it is better to name scissor structures by the transformation of the curve shape that the structure represents. First, the type of transformation should be clarified. Based on the available examples in the literature usually the compact and deployed form of the deployable structure is similar. Hence, there are scaling/dilation type deployable structures and angular deployable structures (Figure 6). Deployable structures with non-similar compact and deployed forms can be named as transformable structures. Actually, a form transformation does not necessarily have a compact and deployed form; hence, in general transformable structures are not necessarily deployable.

**Figure 6: Deployable v.s. transformable motion:**

- (a) scaling/dilation type deployable
- (b) angular deployable
- (c) transformable

For deployable structures that can be idealized as the transformation of a planar curve, the critical point is how the curvature changes during the transformation. A convenient way to analyze the change in curvature is to observe the relative positions of normal lines at convenient locations of the curve during the transformation. In a scaling deployable structure, the compact and deployed forms of the curve are identical up to scale, and the relative angle between adjacent normal lines does not change. On the other hand, in an angular deployable structure the arc length of a circular arc...
is altered where the angles between normal lines change during the deployment. Scaling type deployment has two special cases as radial deployment (changing radius) and linear or translational (changing length) deployment (Figure 7). In both cases, the angles between adjacent normal lines are preserved while the arc lengths are changing with the same ratio. Linear deployment can be thought of as a limit case of radial deployment such that the radius tends to infinity or the angle between the adjacent normal lines approaches to zero. Hence, linear deployment may not necessarily handled as a separate case in design. A general scaling motion can be a combination of linear and radial scaling motion as depicted in Figure 6a.

The confusion of terminology for the scissor units is explained with several exceptional cases. It is emphasized that the motion type (translational, radial, etc) should be specified for the whole structure rather than the scissor units. A classification for the motion types is presented. We think there is still lack of proper terminology for scissor units.

Compared to the loop-based method, the unit-based method is more common in which the shape of the whole structure is formed by the selected scissor units. The deployment behavior of the whole structure directly depends on deployment capability of the used scissor units. Thus, the selection of the unit is very important. In this method, the designer should choose the type of scissor unit beforehand. Then, the structure is created by connecting the units to each other with generally revolute joints. However, it is hard to predict at the beginning whether the whole scissor structure complies with the deployability condition or not. In most cases, the designers have to develop new solutions to generate different scissor structures due to the geometric limitations of the proposed units. Moreover, since the primary units are limited to certain geometric forms, the designer should investigate different design approaches to generate more free forms. Of course, there are some proposals in the literature generating more arbitrary curvatures and free forms; but it requires finding proper scissor units at the beginning, which allow obtaining the final desired configuration. Because the scissor unit is the main input instead of the final geometry of the structure, this method requires much work and time.

On the other hand, the main input is the final geometry of the structure in the loop-based method. It does not restrict the designer to choose the type of scissor unit at the beginning. Rather, the type and the number of scissors can be selected later according to the loops. Therefore, this method seems to be more easily applicable than the unit-based method for the designer to constitute the desired scissor structure. In addition, the proposed examples of the loop-based design show that this method is more promising than the other method since it allows generating new linkages with different motions that have not been discovered before. Moreover, it provides to construct transformable scissor structures with single degree-of-freedom that are not limited two certain predefined geometric forms as in the existing deployable scissor structures.

**Figure 7:** Radial deployable motion; b) linear deployable motion

Whether unit based or loop based method is utilized, the type of transformation should be specified first. In the loop-based method, there seems no problem about terminology in naming the loops (rhombus, parallelogram, etc.). However, in unit-based method the names for the scissor units should rather be selected by considering the geometry of the pair of links rather than implying the transformation to be obtained in the assembly. Proper terminology for scissor units is still an open problem for us.

**4. CONCLUSION AND DISCUSSION**

In this paper, two geometric design methods used in the literature to construct deployable and transformable scissor structures have been analyzed by discussing the current classifications and the used terminologies for scissor structures. Rather than listing the research in the literature as type-based, the key studies have been tabulated to expose the difference in terminology that causes complexity.
ACKNOWLEDGMENTS

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REFERENCES


### Table 1: Unit-based scissor structures

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### Units for Flat and Curved Structures

- **Polygonal units**
- **Trapezoidal unit**
- **Prismatic unit**
- **Pyramidal unit**

### Units for Structures with Arbitrary Geometry

- **Duplets / Scissor-link units**

### Type of Linkage(s) / Mechanism(s)

- **Type I GEA: consisting of two isosceles triangles connected by two parallelograms**
- **Type II GEA: consisting of two similar triangles connected by two parallelograms**
- **Foldable ring structure with identical angulated rods**
- **Foldable angular structure with multi-angulated rods**

### Types of Units

- **Translational unit**
- **Polar unit**
- **Hoberman’s unit**

### Radial Scissor Units

- **Triangular prism**
- **Rectangular prism**
- **Triangular antiprism**
- **Rectangular prism**
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**Note:** The images are placeholders and should be replaced with actual images from the document.
Table 2: Loop-based scissor structures

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- **Polyhedral Approximation with Rhombic Loops**
  - No specific name
  - Spherical truss structure

- **Retractable Structure with Rhombic Loops**

- **Rhombic Loops**

- **Kite Loops**

- **Parallelogram Loops**

- **Dart Loops**

- **Identical Anti-Parallelogram Loops**

- **Similar Anti-Parallelogram Loops**

- **Similar Anti-Parallelogram Loops**

- **Solid Polyhedral Scissor Loops**

- **Deploidy**