

## ADAPTIVE ACTUATOR FAILURE COMPENSATION FOR COOPERATING MULTIPLE MANIPULATOR SYSTEMS

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**Abstract:** This paper presents adaptive actuator failure compensation for a cooperating multiple manipulator system with uncertain actuator failures in the task space. Advantages of designing control schemes in task spaces are emphasized, applications of task space control in robotics are discussed and a short review on control algorithms for cooperating multiple manipulator systems is given. Dynamic equations of motion of the multiple manipulator system in the task space are derived, and the adaptive actuator failure compensation problem is formulated. A compensation controller structure is proposed, for which adaptive parameter update laws are developed. The adaptive control scheme is able to compensate for the uncertainties arising from both the system parameters and the actuator failures. Based on Lyapunov stability analysis, the closed-loop signal boundedness and the convergence of the tracking error to zero are ensured. *Copyright © 2003 IFAC*

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### 1. INTRODUCTION

Many robotic applications such as moving a massive object, handling flexible payload, or assembling applications are not feasible for one manipulator, because of the complexity of the application. In these cases, cooperating multiple manipulators are needed to handle the common object. Legged vehicles and multi-fingered hands can also be categorized as cooperating multiple manipulator systems. With a set of closed kinematic chains, for instance, the manipulators are holding a common rigid object (Figure 1), multiple manipulator sys-

tems have more complexity in control design due to the dynamic interaction between the manipulators, because the system has a set of kinematic and dynamic constraints, where manipulators are controlled cooperately to avoid internal stress on the payload. The controller should be designed to ensure the load sharing and compensate for the variation of the payload.

Manipulator task space controllers have been studied before such as adaptive control design (Feng and Palaniswami, 1993), (Jiang *et al.*, 1994) and PID control design (Cheah *et al.*, 1999). In (Feng and Palaniswami, 1993) an adaptive control algorithm is designed for task space control of manipulators where the inverse of the Jacobian is not

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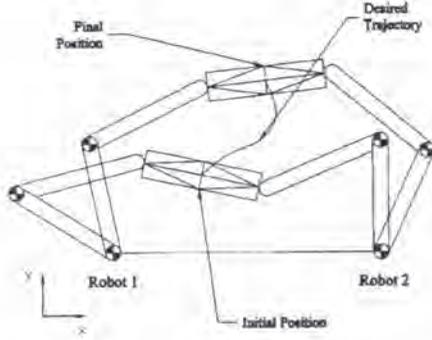


Fig. 1. Cooperating multiple manipulators.

required and the requirement of bounded inverse of the inertia matrix is eliminated, using knowledge of the joint acceleration vector. In (Jiang *et al.*, 1994) an iterative learning algorithm is used with adaptive controller design to eliminate the computation need for real time parameter identification, but learning algorithms can not handle large modeling uncertainties and external disturbance. By formulating the control problem in task space, the need for solving the inverse kinematics problem is eliminated, but these control schemes still require the Jacobian matrix to be known. In (Cheah *et al.*, 1999) a PID control algorithm is designed to compensate for the uncertainties in the Jacobian matrix, where an estimator is used to obtain an approximation of the Jacobian matrix.

In our research the object is manipulated with multiple manipulators not only because it requires multiple manipulators to be moved but also to ensure that if actuator failures occur the remaining manipulators will be able to accomplish moving the object as desired. In this paper, an adaptive scheme for a cooperating multiple manipulator system with actuator failures in the task space is proposed, where system stability and tracking error convergence are achieved without detecting the failed actuator or prior knowledge of failure. An adaptive actuator failure compensation controller for a platform manipulator was designed in (Kececi and Tao, 2002), but the interaction between the expandable legs and the upper platform was ignored. In our study, the effect of a manipulator on other manipulators in the cooperating manipulator system is also considered.

The paper is organized as follows. Dynamic equations of the cooperating manipulators system in the task space are derived in Section 2. In Section 3 an adaptive control scheme is developed to compensate for uncertainties arising from actuator failures. Lyapunov stability analysis proves boundedness of the closed-loop signals and asymptotic tracking of a reference trajectory for the object. Simulation results for the designed control algorithm are presented in Section 4.

## 2. SYSTEM DYNAMIC EQUATIONS

When cooperating multiple manipulators are moving an object, they form a closed kinematic chain, where the system is constrained by holonomic and nonholonomic constraints between the manipulators themselves and the object. The dynamic modeling of the cooperating multiple manipulator system in task space is derived in this section. It is assumed that each manipulator, which is non-redundant with the same  $n$  degree of freedom, does not enter any singular configuration and there is no relative motion between the object and the manipulator end-effectors.

The dynamic equation of the  $i^{th}$  manipulator with  $n$  DOF in a multiple manipulator system is given as (Xie *et al.*, 1999):

$$D_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i - J_i^T F_i \quad (1)$$

for  $i = 1, 2, \dots, m$ , where  $m$  is the number of manipulators in the multiple manipulator system,  $q_i$ ,  $\dot{q}_i$  and  $\ddot{q}_i$  are joint variable position, velocity and acceleration vectors,  $D_i(q_i) \in R^{n \times n}$  is the inertia matrix,  $C_i(q_i, \dot{q}_i) \in R^{n \times n}$  is the Coriolis and centrifugal terms,  $g_i(q_i) \in R^{n \times 1}$  is the gravity term,  $\tau_i \in R^{n \times 1}$  is the generalized torque vector,  $J_i^T$  is the Jacobian matrix defined as  $J_i = \partial h_i(q_i)/\partial q_i$  with  $h_i$  being the forward kinematic output function for the  $i^{th}$  manipulator to be defined in (4), and  $F_i$  is the generalized end-effector force vector for the  $i^{th}$  manipulator.

Equation of motion for the common object is formulated as

$$D_o(x_c)\ddot{x}_c + C_o(x_c, \dot{x}_c)\dot{x}_c + g_o(x_c) = A^T F, \quad (2)$$

where  $x_c \in R^{n_c \times 1}$ ,  $\dot{x}_c$  and  $\ddot{x}_c$  are position, velocity and acceleration vectors of the center of the mass of the object,  $n_c$  is the dimension of the position vector of the mass of the object,  $D_o(x_c) \in R^{n_c \times n_c}$  is the inertia matrix of the object,  $C_o(x_c, \dot{x}_c) \in R^{n_c \times n_c}$  is the Coriolis and centrifugal terms of the object,  $g_o(x_c) \in R^{n_c \times 1}$  is the gravity term,  $F = [F_1^T, \dots, F_m^T]^T$  is the generalized end-effector force vector and  $A \in R^{nm \times n_c}$  is the Jacobian matrix defined by

$$A = [A_1^T, \dots, A_m^T]^T, \quad A_i = \frac{\partial \pi_i(x_c)}{\partial x_c}, \quad (3)$$

in which the constraint equations are defined as

$$x_i = h_i(q_i) = \pi_i(x_c),$$

$$\dot{x}_i = J_i \dot{q}_i = A_i \dot{x}_c, \quad i = 1, 2, \dots, m, \quad (4)$$

where  $x_i \in R^n$  is the position of the  $i^{th}$  manipulator in the Cartesian coordinates,  $h_i$  is the forward kinematic output function of the  $i^{th}$  manipulator and  $\pi_i$  is the transformation matrix from object frame to the  $i^{th}$  manipulator end-effector frame.

When the dynamic equation of motion of the  $i^{th}$  manipulator is written in the object coordinate, it is formulated as (Jean and Fu, 1993)

$$\begin{aligned} \bar{D}_i(x_c)\ddot{x}_c + \bar{C}_i(x_c, \dot{x}_c)\dot{x}_c + \bar{g}_i(x_c) \\ = E_i^T(x_c)\tau_i - A_i^T F_i, \end{aligned} \quad (5)$$

where  $E_i(x_c) = J_i^{-1}A_i$ ,  $\bar{D}_i = E_i^T D_i E_i$ ,  $\bar{C}_i = E_i^T C_i E_i + E_i^T D_i \dot{E}_i$  and  $\bar{g}_i = E_i^T g_i$ .

By summing the manipulator dynamic equations (5) with the object dynamic equation (2), the dynamic equation of the system in the task space is formulated as

$$D'(x_c)\ddot{x}_c + C'(x_c, \dot{x}_c)\dot{x}_c + g'(x_c) = E^T(x_c)\tau, \quad (6)$$

where  $D' = \sum_{i=1}^m \bar{D}_i + D_o$ ,  $C' = \sum_{i=1}^m \bar{C}_i + C_o$ ,  $g' = \sum_{i=1}^m \bar{g}_i + g_o$ ,  $E = [E_1^T, E_2^T, \dots, E_m^T]^T \in R^{nm \times n_c}$  and  $\tau = [\tau_1^T, \tau_2^T, \dots, \tau_m^T]^T \in R^{nm \times 1}$ .

### 3. ADAPTIVE ACTUATOR FAILURE COMPENSATION

In this section, the failure compensation problem for a cooperating manipulator system is formulated and an adaptive control scheme is designed to ensure system stability and asymptotic tracking of a reference for the object in the task space. By using a direct adaptive design, it is expected that there is no need for fault detection and isolation algorithms in order to ensure the desired system performance in the presence of failures.

#### 3.1 Problem Formulation

In a multiple manipulator system, the actuator failure problem is formulated as follows: at an unknown time instant, some actuators at the joints of some manipulators may fail during operation. There can be up to  $nm - n_c$  failures in the multiple manipulator system (6), that is, at least  $n_c$  independent actuators at the joints of some manipulators are left for guaranteeing the control objective, because the manipulators with  $n$  DOF are independent of each other.

The actuator failure is modeled as

$$\tau_{ij}(t) = \bar{\tau}_{ij}, \quad t \geq t_{ij}, \quad (7)$$

for the  $i^{\text{th}}$  manipulator  $j^{\text{th}}$  joint,  $i \in \{1, 2, \dots, m\}$ ,  $j \in \{1, 2, \dots, n\}$ , where the failure time instant  $t_{ij}$  and the constant value of  $\bar{\tau}_{ij}$  are unknown.

In case of actuator failures, the actual input  $\tau$  can be expressed as

$$\tau(t) = v(t) + \sigma(\bar{\tau} - v(t)) \quad (8)$$

where  $v(t) \in R^{nm}$  is the vector of applied control inputs to be determined,  $\bar{\tau} = [\bar{\tau}_{11}, \bar{\tau}_{12}, \dots, \bar{\tau}_{1n}, \dots, \bar{\tau}_{m1}, \dots, \bar{\tau}_{mn}]^T$  is the vector of torques produced by the failed actuators,  $\sigma = \text{diag}\{\sigma_{ij}\}$  is an  $nm \times nm$  diagonal matrix identifying the unknown failure pattern with  $\sigma_{ij} = 1$ , if the actuator at the  $j^{\text{th}}$  joint of the  $i^{\text{th}}$  manipulator fails, otherwise,  $\sigma_{ij} = 0$ .

When some actuators are failed in a certain failure pattern  $\sigma$ , the manipulator dynamic equation (6) becomes

$$\begin{aligned} D'(x_c)\ddot{x}_c + C'(x_c, \dot{x}_c)\dot{x}_c + g'(x_c) \\ = E^T(x_c)\sigma\bar{\tau} + E^T(x_c)(I - \sigma)v(t). \end{aligned} \quad (9)$$

The objective of adaptive compensation control is to adjust the remaining control inputs to achieve the desired system performance, when there are up to  $nm - n_c$  actuator failures at manipulator joints with unknown failure time instants, failure parameters and failure locations, in addition to system parameter uncertainties. More specifically, the control objective is to design a feedback control law  $v(t)$  for the dynamic systems (9) to ensure that all closed-loop system signals and parameter estimates are bounded, and that in the task space, the object position  $x_c(t)$  asymptotically tracks a given reference  $x_{cd}(t)$ .

#### 3.2 Adaptive Controller Design

Because of the uncertainties in actuator failures and manipulator system parameters, an adaptive design is used to compensate for actuator failures and system uncertainties.

The advantage of designing the controller in the task space is that there is no need to measure the forces acting on the end-effectors of the manipulators, since these internal forces are eliminated in the dynamical modeling. However, the task space control of a manipulator system requires knowledge of the common object position and velocity vectors,  $x_c$  and  $\dot{x}_c$ , which can be measured by using vision systems (Zergeroglu *et al.*, 2001).

Define the tracking error  $e_x \in R^{n_c}$  and the filtered tracking errors  $r_x \in R^{n_c}$  and  $v_x \in R^{n_c}$  as

$$e_x = x_c - x_{cd}, \quad r_x = \dot{e}_x + \lambda e_x, \quad v_x = \dot{x}_{cd} - \lambda e_x, \quad (10)$$

where  $x_{cd}$  is the reference for  $x_c$  and  $\lambda \in R^{n_c \times n_c} > 0$  is a gain matrix.

The closed-loop equation (9) can be expressed as

$$\begin{aligned} D'(x_c)\dot{r}_x + C'(x_c, \dot{x}_c)r_x = -Y_x(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_x \\ + E^T(x_c)\sigma\bar{\tau} + E^T(x_c)(I - \sigma)v(t), \end{aligned} \quad (11)$$

where

$$\begin{aligned} Y_x(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_x \\ = D'(x_c)\dot{v}_x + C'(x_c, \dot{x}_c)v_x + g'(x_c), \end{aligned} \quad (12)$$

$\theta_x \in R^{n_s}$  is a unknown system parameter vector and  $Y_x \in R^{n_c \times n_s}$  is a matrix of known functions.

**Failure parameterization.** Suppose that for any up to  $nm - n_c$  actuator failures, there is no singular configuration in the multiple manipulator system, which implies that the matrix  $E^T(x_c)(I - \sigma)$  has full row rank for  $\forall \sigma \in \Sigma$ , where

$\Sigma = \{\sigma_i | i=1, 2, \dots, N\}$  is a set of the failure patterns  $\sigma_i$  for  $i=1, 2, \dots, N$  with  $N = \sum_{i=0}^{nm-n_c} \binom{nm}{i}$ . Since actuator failures can be identified by their failure patterns, we use  $\sigma_{\bar{i}}$  with some  $\bar{i} \in \{1, 2, \dots, N\}$  to represent one specific failure case among all up to  $nm-n_c$  actuator failure cases. It is also noted that only one actuator pattern in  $\Sigma$  will happen at a time.

Introducing  $\bar{E}_i(x_c) = (I - \sigma_i)E(x_c)$  for  $i=1, 2, \dots, N$ , we define  $Y = [\bar{E}_1(\bar{E}_1^T \bar{E}_1)^{-1} Y_x, \bar{E}_2(\bar{E}_2^T \bar{E}_2)^{-1} Y_x, \dots, \bar{E}_N(\bar{E}_N^T \bar{E}_N)^{-1} Y_x] \in R^{nm \times Nn_c}$ ,  $H = [\bar{E}_1(\bar{E}_1^T \bar{E}_1)^{-1}, \bar{E}_2(\bar{E}_2^T \bar{E}_2)^{-1}, \dots, \bar{E}_N(\bar{E}_N^T \bar{E}_N)^{-1}] \in R^{nm \times Nn_c}$ .

**Control law.** Suppose that at time  $t$ ,  $p$  number of actuators are failed in the pattern  $\sigma = \sigma_{\bar{i}}$  with  $\tau_j = \bar{\tau}_j$  (since there is no need to specify the location of the failed actuators as at which joint of which manipulator in our design, we use  $\tau_j$  to denote the  $j^{\text{th}}$  torque in  $\tau$ ), for  $j=j_1, j_2, \dots, j_p$ ,  $\{j_1, j_2, \dots, j_p\} \subset \{1, 2, \dots, nm\}$ . The adaptive controller is designed for  $v_j(t)$ ,  $j=1, 2, \dots, nm$ , as

$$v_j(t) = Y_j(x_c, \dot{x}_c, v_x, \dot{v}_x) \hat{\theta}_j - H_j(x_c) \hat{\rho}_j - H_j(x_c) \hat{K}_j r_x, \quad (13)$$

where  $Y_j$  is the  $j^{\text{th}}$  row of  $Y$ ,  $H_j$  is the  $j^{\text{th}}$  row of  $H$ ,  $\hat{\theta}_j \in R^{Nn_c}$  and  $\hat{\rho}_j \in R^{Nn_c}$  are the estimates of  $\theta_j = [\theta_{j1}^T, \theta_{j2}^T, \dots, \theta_{jN}^T]^T$  with  $\theta_{ji} \in R^{n_c}$  and  $\rho_j = [\rho_{j1}^T, \rho_{j2}^T, \dots, \rho_{jN}^T]^T$  with  $\rho_{ji} \in R^{n_c}$  for  $i=1, 2, \dots, N$ , satisfying the matching conditions

$$\begin{cases} \theta_{j\bar{i}} = \theta_x, \rho_{j\bar{i}} = E^T \sigma_{\bar{i}} & \text{if } \sigma = \sigma_{\bar{i}}, \bar{i} \in \{1, 2, \dots, N\}, \\ \theta_{ji} = 0, \rho_{ji} = 0 & \text{for } i=1, 2, \dots, N \text{ and } i \neq \bar{i}, \end{cases}$$

$\hat{K}_j = [\hat{K}_{j1}, \hat{K}_{j2}, \dots, \hat{K}_{jN}]^T \in R^{Nn_c \times n_c}$  with  $N$  diagonal matrices  $\hat{K}_{ji} \in R^{n_c \times n_c}$ ,  $i=1, 2, \dots, N$ , whose diagonal elements  $\hat{K}_{jil}$ ,  $i=1, 2, \dots, N$ ,  $l=1, 2, \dots, n_c$ , are estimates of parameters  $K_{jil} = K_{0l}$  which are some positive constants, if  $i=\bar{i}$ , otherwise,  $K_{jil} = 0$ .

**Adaptive scheme.** The parameter update laws of  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{jil}(t)$  are designed as

$$\dot{\hat{\theta}}_j = -\Gamma_{\theta_j} Y_j^T(x_c, \dot{x}_c, v_x, \dot{v}_x) E_{(j)}(x_c) r_x, \quad (14)$$

$$\dot{\hat{\rho}}_j = \Gamma_{\rho_j} H_j(x_c) E_{(j)}(x_c) r_x, \quad (15)$$

$$\dot{\hat{K}}_{jil} = \gamma_{jil} H_{jil}(x_c) r_{zl} E_{(j)}(x_c) r_x, \quad (16)$$

for  $j=1, 2, \dots, nm$ ,  $i=1, 2, \dots, N$ , and  $l=1, 2, \dots, n_c$ , where  $H_{jil}(x_c)$  is the  $[(i-1)n_c + l]^{\text{th}}$  component of  $H_j(x_c)$  and  $r_{zl}$  is the  $l^{\text{th}}$  component of  $r_x$ .

From the controller structure (13), the closed-loop equation (11)  $\sigma = \sigma_{\bar{i}}$  is derived by

$$\begin{aligned} & D'(x_c) \dot{r}_x + C'(x_c, \dot{x}_c) r_x \\ &= -Y_x \theta_x + E^T \sigma_{\bar{i}} + \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T Y_j \hat{\theta}_j \\ & \quad - \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T H_j \hat{\rho}_j - \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T H_j \hat{K}_j r_x \\ &= -Y_x \theta_x + E^T \sigma_{\bar{i}} + \bar{E}_{\bar{i}}^T Y \theta_0 - \bar{E}_{\bar{i}}^T H \rho_0 + \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T Y_j \hat{\theta}_j \end{aligned}$$

$$\begin{aligned} & - \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T H_j \hat{\rho}_j - \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T H_j \hat{K}_j r_x - K_0 r_x \\ &= \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T(x_c) Y_j(x_c, \dot{x}_c, v_x, \dot{v}_x) \hat{\theta}_j \\ & \quad - \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T(x_c) H_j(x_c) \hat{\rho}_j \\ & \quad - \sum_{j \neq j_1, \dots, j_p} E_{(j)}^T(x_c) H_j \hat{K}_j(x_c) r_x - K_0 r_x, \quad (17) \end{aligned}$$

where  $\theta_0 = \theta_1 = \theta_2 = \dots = \theta_{nm}$ ,  $\rho_0 = \rho_1 = \rho_2 = \dots = \rho_{nm}$ ,  $\hat{\theta}_j = \hat{\theta}_j - \theta_j$  and  $\hat{\rho}_j = \hat{\rho}_j - \rho_j$  are the parameter estimate errors,  $\hat{K}_j = [\hat{K}_{j1}, \hat{K}_{j2}, \dots, \hat{K}_{jN}]^T \in R^{Nn_c \times n_c}$  in which  $\hat{K}_{ji} \in R^{n_c \times n_c}$  is a diagonal matrix with  $\hat{K}_{jil} = \hat{K}_{jil} - K_{jil}$  as its diagonal elements for  $i=1, 2, \dots, N$ ,  $l=1, 2, \dots, n_c$ ,  $K_0 = \text{diag}\{K_{01}, K_{02}, \dots, K_{0n_c}\} > 0$ , and  $E_{(j)}$  is the  $j^{\text{th}}$  row of  $E$ .

### 3.3 Stability Analysis

Suppose that failures happen at time instants  $t_k$ ,  $k=1, 2, \dots, M$ , and  $0 < t_1 < t_2 < \dots < t_M$  (at each time instant  $t_k$ , there may be more than one actuator failures at some joints of some manipulators). We consider such a Lyapunov function as

$$\begin{aligned} V &= V_k = \frac{1}{2} r_x^T D'(x_c) r_x \\ & \quad + \frac{1}{2} \sum_{j \neq j_1, \dots, j_p} \hat{\theta}_j^T \Gamma_{\theta_j}^{-1} \hat{\theta}_j + \frac{1}{2} \sum_{j \neq j_1, \dots, j_p} \hat{\rho}_j^T \Gamma_{\rho_j}^{-1} \hat{\rho}_j \\ & \quad + \frac{1}{2} \sum_{j \neq j_1, \dots, j_p} \sum_{i=1}^N \sum_{l=1}^{n_c} \gamma_{jil}^{-1} \hat{K}_{jil}^2 \end{aligned} \quad (18)$$

for each time interval  $(t_k, t_{k+1})$ ,  $k=0, 1, \dots, M$ , with  $t_0 = 0$  and  $t_{M+1} = \infty$ , corresponding to a certain failure pattern as  $\{j_1, j_2, \dots, j_p\}$ , where  $\Gamma_{\theta_j} = \Gamma_{\theta_j}^T > 0$ ,  $\Gamma_{\rho_j} = \Gamma_{\rho_j}^T > 0$  and  $\gamma_{jil} > 0$ .

Differentiating  $V$  with respect to time in the interval  $(t_k, t_{k+1})$  along (17) yields

$$\begin{aligned} \dot{V} &= \frac{1}{2} r_x^T [\dot{D}'(x_c) - 2C'(x_c, \dot{x}_c)] r_x \\ & \quad + \sum_{j \neq j_1, \dots, j_p} r_x^T E_{(j)}^T(x_c) Y_j(x_c, \dot{x}_c, v_x, \dot{v}_x) \hat{\theta}_j \\ & \quad - \sum_{j \neq j_1, \dots, j_p} r_x^T E_{(j)}^T(x_c) H_j(x_c) \hat{\rho}_j \\ & \quad - \sum_{j \neq j_1, \dots, j_p} r_x^T E_{(j)}^T(x_c) H_j(x_c) \hat{K}_j r_x - r_x^T K_0 r_x \\ & \quad + \sum_{j \neq j_1, \dots, j_p} \hat{\theta}_j^T \Gamma_{\theta_j}^{-1} \dot{\hat{\theta}}_j + \sum_{j \neq j_1, \dots, j_p} \hat{\rho}_j^T \Gamma_{\rho_j}^{-1} \dot{\hat{\rho}}_j \\ & \quad + \sum_{j \neq j_1, \dots, j_p} \sum_{i=1}^N \sum_{l=1}^{n_c} \gamma_{jil}^{-1} \dot{\hat{K}}_{jil} \hat{K}_{jil}, \quad (19) \end{aligned}$$

where the first term results in zero from the skew-symmetric property of  $\dot{D}(q) - 2C(q, \dot{q})$ . With the adaptive update laws (14)–(16), the time-derivative of the Lyapunov function  $V$  becomes

$$\dot{V} = -\tau_x^T K_0 r_x \leq 0. \quad (20)$$

When new actuator failures occur, the Lyapunov function  $V = V_k$  changes with failures into  $V_{k+1}$ . Hence  $V$  is not continuous at the time instants  $t_k$ ,  $k = 0, 1, \dots, M$ . Except for a finite number ( $M$  as indicated here) of discontinuous points,  $V$  is differentiable with a negative time derivative, that is,  $V$  decreases with time in each time interval  $(t_k, t_{k+1})$  when there is no actuator failures during this time span.

Starting from the first time interval  $[t_0, t_1)$ , we see that  $V(t) \leq V(t_0)$  from  $\dot{V} \leq 0$  for  $\forall t \in [t_0, t_1)$ . It is concluded that all signals are bounded for  $t \in [t_0, t_1)$ , including the estimates  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{ju}(t)$  for  $j = 1, 2, \dots, nm$ ,  $i = 1, 2, \dots, N$ ,  $l = 1, 2, \dots, n_c$ .

At time  $t = t_1$ , when some actuators fail, that is, the control signals to some joints are stopped by some constant torques with unknown values,  $V$  changes abruptly from  $V_0$  to  $V_1$ . First of all, based on a new failure pattern, the unknown parameters  $\theta_j$ ,  $\rho_j$  and  $K_{ju}$  change into a set of new  $\theta_j$ ,  $\rho_j$  and  $K_{ju}$  with finite values. It is also noted that some of the parameter estimates  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{ju}(t)$  with some  $j \in \{1, 2, \dots, nm\}$  are removed from the Lyapunov function  $V$  because their corresponding actuators are not working anymore. Since  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{ju}(t)$  are continuous and are finite at time  $t_1$  and the unknown parameters  $\theta_j$ ,  $\rho_j$ ,  $K_{ju}$  have finite values to satisfy the matching conditions under the current failure pattern, the change of  $V$  is a finite value jumping, which means that  $V(t_1^+) = V_1(t_1)$  is bounded.

Repeating the argument above, we establish the boundedness of  $r(t)$ ,  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{ju}(t)$  for some  $j$  corresponding to the remaining actuators in the time interval  $(t_1, t_2)$  and prove that  $V(t_2^+) = V_2(t_2)$  is bounded. Continuing in the same way, we have that  $V(t) \leq V(t_k^+)$  for  $\forall t \in (t_k, t_{k+1})$  with a finite  $V(t_k^+)$ ,  $k = 0, 1, \dots, M$ . Therefore, we conclude that  $V(t)$  is piecewise continuous and bounded.

Recall that any  $n_c$  actuators of the  $nk$  actuators are independent and assumed to guarantee the nonsingular property, that is, each row  $E_{(j)}(x_c)$  of  $E(x_c)$  can be represented by a linear combination of any other  $n_c$  rows of  $E(x_c)$ . From the adaptive update laws (14)–(16), we thus know that for the  $j^{\text{th}}$  designed control input  $v_j(t)$ , the adaptive laws of its estimates  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{ju}(t)$  are a linear combination of the adaptive laws for the estimates of any other  $n_c$  control inputs. On the

other hand, at any time there remain at least  $n_c$  actuators for achieving the control objective. Hence at least  $n_c$  sets of  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{ju}(t)$  with some  $j$  remain in the Lyapunov function  $V$ , which implies that  $\hat{\theta}_j(t)$ ,  $\hat{\rho}_j(t)$  and  $\hat{K}_{ju}(t)$  some  $j \in \{1, 2, \dots, nm\}$  (no less than  $n_c$  different  $j$ ) for each  $i = 1, 2, \dots, N$  and  $l = 1, 2, \dots, n_c$  are bounded for  $\forall t \in [0, \infty)$ . Since at least  $n_c$  sets of them with some  $j$  are bounded for  $\forall t \in [0, \infty)$ , the others are also bounded in the sense that the estimates obey their adaptive laws, which are linear combinations of the remaining  $n_c$  sets, with different initial values. Notice that even if those estimate signals may not be applied to the system if the corresponding actuators are failed, the adaptive laws of them are still calculated in computing chips virtually. It follows that all closed-loop signals are bounded for both the real signals applied to the manipulator system and virtual signals calculated in computing chips.

Considering the last time interval  $(t_M, \infty)$  with a finite  $V(t_M^+)$ , we see that it follows from (20) that  $r_x(t) \in L^2$ . On the other hand, from the boundedness of the closed-loop signals, it can be shown that  $\dot{r}_x(t) \in L^\infty$  so that  $\lim_{t \rightarrow \infty} r_x(t) = 0$ , from which it follows that  $\lim_{t \rightarrow \infty} e_x(t) = 0$  and  $\lim_{t \rightarrow \infty} \dot{e}_x(t) = 0$ .

Thus, stability in the Lyapunov sense and asymptotic tracking:  $\lim_{t \rightarrow \infty} e(t) = 0$  are established for the adaptive actuator failure compensation design of the cooperating multiple manipulator system in the task space, despite actuator failures.

#### 4. SIMULATION RESULTS

In order to demonstrate the effectiveness of the control algorithm, a simulation study is performed. For a cooperating manipulator system consisting of 2 two-link manipulators and a third joint at the end-effector to ensure the orientation of the object, the objective is to move the common object from the initial point (1,1) to the final point (2,2) together in 10 seconds. At the third second, the actuator at the second joint of the second manipulator fails and does not apply any torque to the joint. The failure causes a transient response on the error in Cartesian space and after the failure, the first manipulator starts to carry the object. Figure 2 and Figure 3 show the common object position and velocity errors respectively. The joint torque inputs to the manipulators are shown in Figure 4 and Figure 5. The specifications of the manipulators, selected as identical to simplify the calculation, are  $m_1 = 4kg$  is the mass of the first link,  $l_1 = 2m$  is the length of the first link,  $m_2 = 2kg$  is the mass of the second link,  $l_2 = 1m$  is the length of the second link,  $J$  is the Jacobian matrix of the manipulator,  $m_{obj} = 1kg$  is

the mass of the object,  $l_{obj} = 0.5m$  is the length of the object,  $I_{obj} = 4kgm^2$  is the moment of inertia of the object.

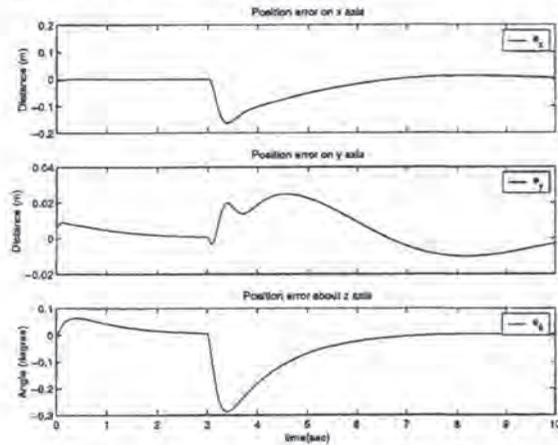


Fig. 2. Common object position error.

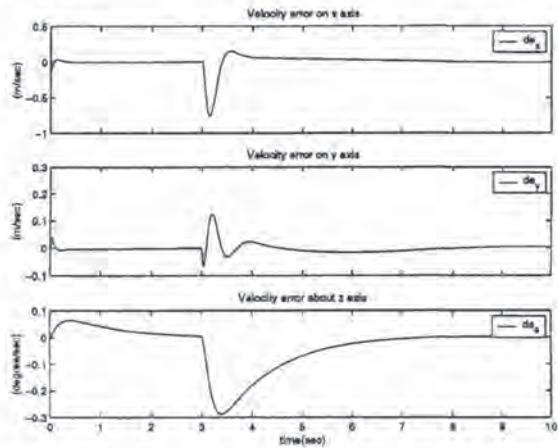


Fig. 3. Common object velocity error.

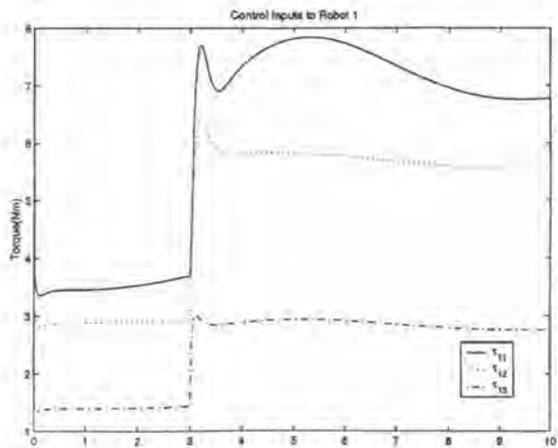


Fig. 4. Joint torques for Robot 1.

## 5. CONCLUSIONS

This paper presents a problem formulation and a solution to adaptive actuator failure compensa-

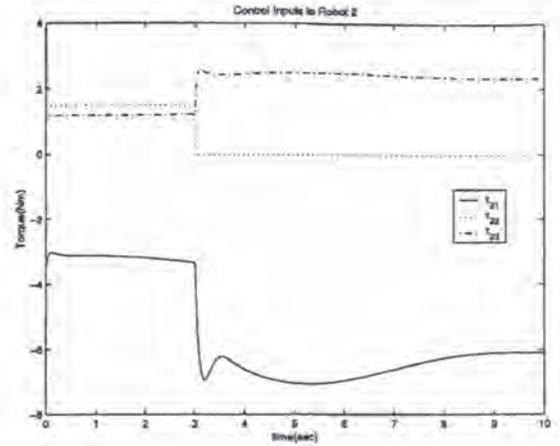


Fig. 5. Joint torques for Robot 2.

tion for a cooperating multiple manipulator system. The advantages of task space control of manipulator systems are discussed and the adaptive control scheme is analyzed for the compensation of the uncertainties arising from the actuator failures as well as parameter uncertainties in the system. The position and velocity errors of the common object are ensured to converge to zero asymptotically, in addition to closed-loop signal boundedness, despite the unknown actuator failures.

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